On some geometric characteristics of the orbit foliations of the co-adjoint action of some 5-dimensional solvable Lie groups

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ABSTRACT:

In this paper, we discribe some geometric charateristics of the so-called MD(5,3C)-foliations and MD(5,4)-foliations, i.e., the foliations formed by the generic orbits of co-adjoint action of MD(5,3C)-groups and MD(5,4)-groups.

Key words: *K-representation, K-orbits, MD-groups, MD-algebras, foliations*.

1. INTRODUCTION

It is well-known that Lie algebras are interesting objects with many applications not only in mathematics but also in physics. However, the problem of classifying all Lie algebras is still open, up to date. By the Levi-Maltsev Theorem [5] in 1945, it reduces the task of classifying all finite-dimensional Lie algebras to obtaining the classification of solvable Lie algebras.

There are two ways of proceeding in the classification of solvable Lie algebras: *by dimension* or *by structure*. It seems to be very difficult to proceed by dimension in the classification of Lie algebras of dimension greater than 6. However, it is possible to proceed by structure, i.e., to classify solvable Lie algebras with a specific given property.

We start with the second way, i.e, the structure approach. More precisely, by Kirillov's Orbit Method [4], we consider Lie algebras whose correponding connected and simply connected Lie groups have co-adjoint orbits (K-orbits) which are orbits of dimension zero or maximal dimension. Such Lie algebras and Lie groups are called *MD-algebras* and *MD-groups*, respectively, in term of Diep [2]. The problem of classifying general MD-algebras (and corresponding MD-groups) is still open, up to date: they were completely solved just for dimension $n \leq 5$ in 2011.

There is a noticeable thing as follows: the family of maximal dimension K-orbits of an MD-group forms a so-called *MD-foliation*. The theory of foliations began in Reeb's work [7] in 1952 and came from some surveys about existence of solution of differential equations [6]. Because of its origin, foliations quickly become a very interesting object in modern geometry.

When foliated manifold carries a Riemannian structure, i.e., there exists a Riemannian metric on it, the considered foliation has much more interesting geometric characteristics in which are *totally goedesic* or *Riemannian* [8]. Such foliations are the simplest foliations can be on an given Riemannian manifold and have been investigated by many mathematicians. In this paper, we follow that flow to consider some geometric characteristics of foliations formed by K-orbits of indecomposable connected and simply connected MD5-groups whose corresponding MD5-algebras having first derived ideals are 3-dimensional or 4-dimensional and commutative.

This paper is organized in 5 sections as follows: we introduce considered problem in Sections 1; recall some results about MD(5,3C)-algebras and MD(5,4)-algebras in Section 2; Section 3 deals with some results about MD(5,3C)-foliations and MD(5,4)-foliations; Section 4 is devoted to the discussion of some geometric characteristics of MD(5,3C)-foliations and MD(5,4)-foliations; in the last section, we give some conclusions.

2. MD(5,3C)-ALGEBRAS AND MD(5,4)-ALGEBRAS

Definition 2.1 ([see 4]). Let G be a Lie group and $\mathcal{G}$ its Lie algebra. We define an action $Ad : G \to \mathrm{Aut}(\mathcal{G})$ by

$$Ad(g) := \left(L_g \circ R_{g^{-1}}\right)_*,$$

where L_g and R_g are left-translation and right-translation by an element g in G, respectively. The action Ad is called *adjoint representation* of G in $\mathcal{G}$.

Definition 2.2 ([see 4]). Let $\mathcal{G}^*$ be the *dual space* of $\mathcal{G}$. Then, Ad gives rise an action $K : G \to \mathrm{Aut}(\mathcal{G}^*)$ which is defined by $\langle K(g)F, X\rangle := \langle F, Ad(g^{-1})X\rangle$ for every $F\in \mathcal{G}^*$, $X\in \mathcal{G}$, $g\in G$; where the notation $\langle F, X\rangle$ denotes the value of linear form F at left-invariant vector field X. The action K is called *co-adjoint representation* or *K-representation* of G in $\mathcal{G}^*$ and each its orbit is called an *K-orbit* of S in $\mathcal{G}^*$.

Definition 2.3 ([see 2]). An n*-dimensional MD-group* or *MDn-group* is an n-dimensional solvable real Lie group such that its K-orbits in K-representation are orbits of dimension zero or maximal dimension. The Lie algebra of an MDn-group is called *MDn-algebra.*

Remark 2.4. The family $\mathcal{F}$ of maximal dimension K-orbits of G forms a partition of $V = \cup\{\Omega : \Omega \in \mathcal{F}\}$ in $\mathcal{G}^*$. This leads to a foliation as we will see in the next section.

Definition 2.5 ([see 2]). With an MDn-algebra $\mathcal{G}$, the $\mathcal{G}^1 := [\mathcal{G}, \mathcal{G}]$ is called *the first derived ideal* of $\mathcal{G}$. If $\dim \mathcal{G}^1 = m$, then $\mathcal{G}$ is called an *MD(n,m)-algebra*. Furthermore, if $\mathcal{G}^1 \cong \mathbb{R}^m$, i.e., $\mathcal{G}^1$ is abelian, then $\mathcal{G}$ is called an *MD(n,mC)-algebra.*

It is well known that all Lie algebras with dimension $n \le 3$ are always MD-algebras. For $n = 4$, the problem of classifying MD4-algebras was solved by Vu [10]. Recently, the similar problem for MD5-algebras also has been solved. In this section, we just consider a subclass consists of MD(5,3C)-algebras and MD(5,4)-algebras. More specifically, we have the following results.

Proposition 2.6 ([10, Theorem 3.1]).

1)There are 8 families of indecomposable MD(5,3C)-algebras which are denoted as follows: $\mathcal{G}_{5,3,1(\lambda_1,\lambda_2)}$, $\lambda_1, \lambda_2 \in \mathbb{R}\setminus\{0,1\}, \lambda_1 \ne \lambda_2$; $\mathcal{G}_{5,3,2(\lambda)}$, $\lambda \in \mathbb{R}\setminus\{0,1\}$; $\mathcal{G}_{5,3,3(\lambda)}$, $\lambda \in \mathbb{R}\setminus\{1\}$; $\mathcal{G}_{5,3,4}$; $\mathcal{G}_{5,3,5(\lambda)}$, $\lambda \in \mathbb{R}\setminus\{1\}$; $\mathcal{G}_{5,3,6(\lambda)}$, $\lambda \in \mathbb{R}\setminus\{0,1\}$; $\mathcal{G}_{5,3,7}$; $\mathcal{G}_{5,3,8(\lambda,\varphi)}$, $\lambda \in \mathbb{R}\setminus\{0\}$, $\varphi \in (0,\pi)$.

2)There are 14 families of indecomposable MD(5,4)-algebras which are denoted as follows: $\mathcal{G}_{5,4,1(\lambda_1,\lambda_2,\lambda_3)}$, $\mathcal{G}_{5,4,2(\lambda_1,\lambda_2)}$, $\mathcal{G}_{5,4,3(\lambda)}$, $\mathcal{G}_{5,4,4(\lambda)}$, $\mathcal{G}_{5,4,5}$, $\mathcal{G}_{5,4,6(\lambda_1,\lambda_2)}$, $\mathcal{G}_{5,4,7(\lambda)}$, $\mathcal{G}_{5,4,8(\lambda)}$, $\mathcal{G}_{5,4,9(\lambda)}$, $\mathcal{G}_{5,4,10}$, $\lambda, \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}\setminus\{0,1\}$; $\mathcal{G}_{5,4,11(\lambda_1,\lambda_2,\varphi)}$, $\mathcal{G}_{5,4,12(\lambda,\varphi)}$, $\mathcal{G}_{5,4,13(\lambda,\varphi)}$, $\lambda, \lambda_1, \lambda_2 \in \mathbb{R}\setminus\{0\}$,

$\varphi \in (0;\pi)$; $\mathcal{G}_{5,4,14(\lambda,\mu,\varphi)}$, $\lambda, \mu \in \mathbb{R}$, $\mu > 0$, $\varphi \in (0;\pi)$.

Remark 2.7. In view of Proposition 2.6, we obtain 8 families of MD(5,3C)-groups and 14 families of MD(5,4)-groups. All groups of these families are indecomposable, connected and simply connected. For convenience, we will use the same indicates to denote these MD-groups. For example, $G_{5,3,4}$ is the connected and simply connected MD(5,3C)-group corresponding to $\mathcal{G}_{5,3,4}$.

3. MD(5,3C)-FOLIATIONS AND MD(5,4)-FOLIATIONS

Definition 3.1 ([see 1]). A *p-dimensional foliation* $\mathcal{F} = \{L_\alpha\}$ on an *n*-dimensional smooth manifold V is a family of *p*-dimensional connected submanifolds of V such that:

1) $\mathcal{F}$ forms a partition of V.

2)For every $x \in V$, there exist a smooth chart

$$\varphi = (\varphi_1, \varphi_2) : U \to \mathbb{R}^p \times \mathbb{R}^{n-p}$$

defined on an open neighborhood U of x such that if $U \cap L_\alpha \neq \varnothing$, then the connected components of $U \cap L_\alpha$ are described by the equations $\varphi_2 = \text{const}$. We call V the *foliated manifold*, each member of $\mathcal{F}$ *a leaf* and the number *n* – *p* is called the *codimension* of $\mathcal{F}$.

Let (V, g) be a Riemannian manifold and $\mathcal{F} = \{L_\alpha\}$ be a foliation on (V, g). We denote by $T\mathcal{F}$ and $N\mathcal{F}$ the tangent distribution and orthogonal distribution of $\mathcal{F}$, respectively.

Definition 3.2 ([see 6, 8]). A submanifold $L \subset V$ is called a *totally geodesic* if it satisfies one of equivalent conditions as follows:

1) Each geodesic of V that is tangent to L then it lies entirely on L.

2) Each geodesic of L is also a geodesic of V.

Definition 3.3 ([see 6, 8]). A foliation $\mathcal{F}$ on (V, g) is called *totally geodesic* (and $T\mathcal{F}$ is called *geodesic distribution*) if all leaves of $\mathcal{F}$ are totally geodesic submanifolds of V. If $N\mathcal{F}$ is geodesic distribution, then $\mathcal{F}$ is called *Riemannian*.

Remark 3.4. For any foliation $\mathcal{F}$ on (*V*, *g*), in the geometric viewpoint, we have

1) $\mathcal{F}$ is totally geodesic if each geodesic of V is either tangent to some leaf of $\mathcal{F}$ or not tangent to any leaf of $\mathcal{F}$.

2) $\mathcal{F}$ is Riemannian if each geodesic of V is either orthogonal to some leaf of $\mathcal{F}$ or not orthogonal to any leaf of $\mathcal{F}$.

Definition 3.5 ([see 1]). Two foliations $(V_1, \mathcal{F}_1)$ and $(V_2, \mathcal{F}_2)$ are said to be *equivalent* or have *same foliated topological type* if there exist a homeomorphism $h : V_1 \to V_2$ which sends each leaf of $\mathcal{F}_1$ onto each leaf of $\mathcal{F}_2$.

Proposition 3.6 ([see 10, 13, 14]). Let *G* be one of indecomposable connected and simply connected MD(5,3C)-groups (respectively, MD(5,4)-groups). Let $\mathcal{F}_G$ be the family of maximal dimensional K-orbits of *G*, and $V_G = \cup\{\Omega : \Omega \in \mathcal{F}_G\}$. Then, $(V_G, \mathcal{F}_G)$ is a *measureable foliation* (in term of Connes [1]) and it is called *MD(5,3C)-foliation* (respectively, *MD(5,4)-foliation*) associated to *G*.

Due to Proposition 2.6 and Remark 2.7, there are 8 families of MD(5,3C)-foliations and 14 families of MD(5,4)-foliations. Note that for all MD(5,3C)-groups (respectively, MD(5,4)-groups), V_G are diffeomorphic to each other. So, instead of $(V_{G_{i,...}}, \mathcal{F}_{G_{i,...}})$, we will write $(V_i, \mathcal{F}_{i,...})$. For example, $(V_3, \mathcal{F}_{3,4})$ is MD(5,3C)-foliation associated to $G_{5,3,4}$.

Proposition 3.7 ([see 10, 14]). With these notations as above, we have:

1)There exist exactly 2 topological types $\mathcal{F}_1$, $\mathcal{F}_2$ of 8 families of considered MD(5,3C)-foliations as follows:

$$\mathcal{F}_1 = \left\{ \left(V_3, \mathcal{F}_{3,1(\lambda_1,\lambda_2)}\right), \left(V_3, \mathcal{F}_{3,2(\lambda)}\right), \ldots, \left(V_3, \mathcal{F}_{3,7}\right) \right\},$$

$$\mathcal{F}_2 = \left\{ \left(V_3, \mathcal{F}_{3,8(\lambda,\varphi)}\right) \right\}.$$

2) There exist exactly 3 topological types $\mathcal{F}_3$, $\mathcal{F}_4$, $\mathcal{F}_5$ of 14 families of considered MD(5,4)-foliations as follows:

$$\mathcal{F}_3 = \left\{ \left(V_4, \mathcal{F}_{4,1(\lambda_1,\lambda_2)}\right), \left(V_4, \mathcal{F}_{4,2(\lambda)}\right), \ldots, \left(V_4, \mathcal{F}_{4,10}\right) \right\},$$

$$\mathcal{F}_4 = \left\{ \left(V_4, \mathcal{F}_{4,11(\lambda_1,\lambda_2,\varphi)}\right), \left(V_4, \mathcal{F}_{4,12(\lambda,\varphi)}\right), \left(V_4, \mathcal{F}_{4,13(\lambda,\varphi)}\right) \right\},$$

$$\mathcal{F}_5 = \left\{ \left(V_4, \mathcal{F}_{4,14(\lambda,\mu,\varphi)}\right) \right\},$$

where $V_3 \equiv \mathbb{R}^2 \times \left(\mathbb{R}^3\right)^*$, $V_4 \equiv \mathbb{R} \times \left(\mathbb{R}^4\right)^*$.

4. SOME GEOMETRIC CHARACTERISTICS OF MD(5,3C)-FOLIATIONS AND MD(5,4)-FOLIATIONS

Now, we describe some geometric characteristics of considered MD(5,3C)-foliations and MD(5,4)-foliations.

4.1. Foliations of the type $\mathcal{F}_1$

Choose $\mathcal{F}_{3,4}$ represents the type $\mathcal{F}_1$. From the geometric picture of K-orbits in [14,15], we see that the zero dimensional K-orbits are points in Oxy, the leaves of $\mathcal{F}_{3,4}$ are 2-dimensional K-orbits as follows:

$$\Omega_F = \left\{ \left(\alpha + \left(1 - e^a\right)\gamma; y; e^a\gamma; e^a\delta; e^a\sigma\right) : y, a \in \mathbb{R} \right\},$$

where $\gamma^2 + \delta^2 + \sigma^2 \neq 0$.

Recall that $G^*_{5,3,4} \equiv \mathbb{R}^5$. Let us identify Oz with $\{(0,0)\} \times \mathbb{R}.z \times \mathbb{R}.t \times \mathbb{R}.s$, i.e., each point on Oz has coordinate $(0,0,z,t,s)$. So we can see $G^*_{5,3,4}$ as $\mathbb{R}^3 \equiv Oxyz$. Then, all the leaves of $\mathcal{F}_{3,4}$ are half-planes $\{x + z = \gamma + \delta, z > 0 \text{ or } z < 0\}$ (Figure 1).

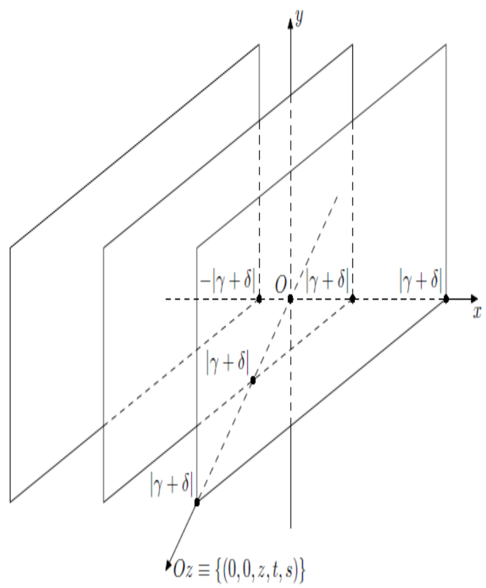

Figure 1. The leaves of $\mathcal{F}_{3,4}$

Because $V_3 \equiv \mathbb{R}^2 \times \left(\mathbb{R}^3\right)^*$ is Euclidean space, its totally geodesic submanifolds are only k-planes. Therefore, we have the following proposition.

Proposition 4.1. $\mathcal{F}_1$*-type MD(5,3C)-foliations are totally geodesic and Riemannian.*

4.2. Foliations of the type $\mathcal{F}_2$

Choose $\mathcal{F}_{3,8\left(1,\frac{\pi}{2}\right)}$ represents the type $\mathcal{F}_2$. From the geometric picture of K-orbits in [13, 14], we see that the zero dimensional K-orbits are points $F(\alpha,\beta,0,0,0)$ in Oxy, the leaves of $\mathcal{F}_{3,8\left(1,\frac{\pi}{2}\right)}$ are 2-dimensional K-orbits Ω_F = $\left\{ \left(\alpha + (\sin a)\gamma - (1 - \cos a)\delta; y; (\gamma + i\delta)e^{-ia}; e^a\sigma\right) : y, a \in \mathbb{R} \right\}$,

where $\gamma^2 + \delta^2 + \sigma^2 \neq 0$.

- Let us identify

$$Oy \equiv \{0\} \times \mathbb{R}.y \times \mathbb{R}.z \times \mathbb{R}.t \times \{0\}.$$

Then, $G^*_{5,3,8\left(1,\frac{\pi}{2}\right)} \equiv \mathbb{R}^5$ can be seen as $\mathbb{R}^3 \equiv$ *Oxys*. In this case, the leaves of $\mathcal{F}_{3,8\left(1,\frac{\pi}{2}\right)}$ are half-planes $\{x = \alpha, s > 0 \text{ or } s < 0\}$ (Figure 2).

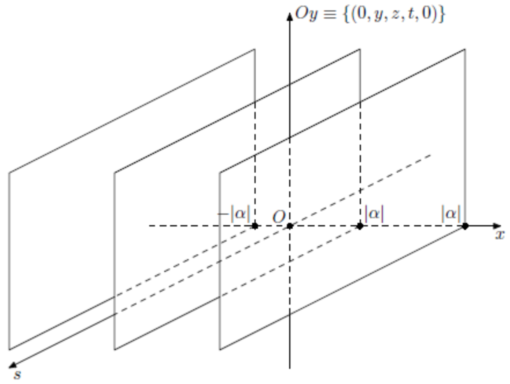

Figure 2. The leaves of $\mathcal{F}_{3,8\left(1,\frac{\pi}{2}\right)}$ in half 3-plane $\{z = t = 0, s > 0\}$

- Let us identify

$$Ot \equiv \mathbb{R}.x \times \{(0,0)\} \times \mathbb{R}.x - \alpha + \delta \times \{0\},$$

Then, $\mathcal{G}^*_{5,3,8\left(1,\frac{\pi}{2}\right)}$ can be seen as $\mathbb{R}^4 \equiv Oxyt$. In this case, the leaves of $\mathcal{F}_{3,8\left(1,\frac{\pi}{2}\right)}$ are rotating cylinderes (Figure 3).

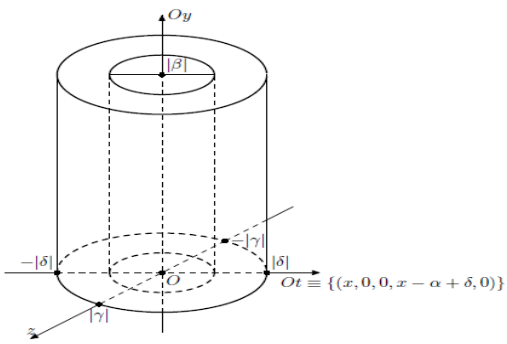

Figure 3. The leaves of , $\mathcal{F}_{3,8\left(1,\frac{\pi}{2}\right)}$ in hyperplane 6.1. $x - t = \alpha - \delta$

- Let us identify

$$Oy \equiv \{0\} \times \mathbb{R}.y \times \{(0,0)\} \times \mathbb{R}.s,$$

and Ot as above. Then, $\mathcal{G}^*_{5,3,8\left(1,\frac{\pi}{2}\right)}$ can be seen as $\mathbb{R}^3 \equiv Oyzt$ and the leaves of $\mathcal{F}_{3,8\left(1,\frac{\pi}{2}\right)}$ are cylinderes whose generating curves are parallel to *Oy*-axis, directrices are helices $\{z + it = (\gamma + i\delta)e^{-ia}, s = e^a\sigma\}$ in *Oyzt.*

It is clear that there exist some leaves of $\mathcal{F}_{3,8\left(1,\frac{\pi}{2}\right)}$ which are not totally geodesic submanifolds of V_3. Therefore, we have the following proposition.

Proposition 4.3. *$\mathcal{F}_2$-type MD(5,3C)-foliations are not totally geodesic.*

4.3. Foliations of the type $\mathcal{F}_3$

Choose $\mathcal{F}_{4,5}$ represents the type $\mathcal{F}_3$. From the geometric picture of K-orbits in [10], for *F*(α,β,γ,δ,σ) in V_4, the leaves of $\mathcal{F}_{4,5}$ are 2-dimensional K-orbits as follows:

$$\Omega_F = \left\{\left(x; e^a\beta; e^a\gamma; e^a\delta; e^a\sigma\right) : x, a \in \mathbb{R}\right\},$$

where $\beta^2 + \gamma^2 + \delta^2 + \sigma^2 \neq 0$. Let us indentify Oz with $\mathbb{R}.z \times \mathbb{R}.\frac{\delta}{\gamma}z \times \mathbb{R}.\frac{\sigma}{\gamma}z$. Then, $\mathcal{G}^*_{5,4,5} \equiv \mathbb{R}^5$ can be seen as $\mathbb{R}^3 \equiv Oxyz$ and the leaves of $\mathcal{F}_{4,5}$ are half-planes $\gamma y = \beta z$ which rotate around Ox (Figure 4).

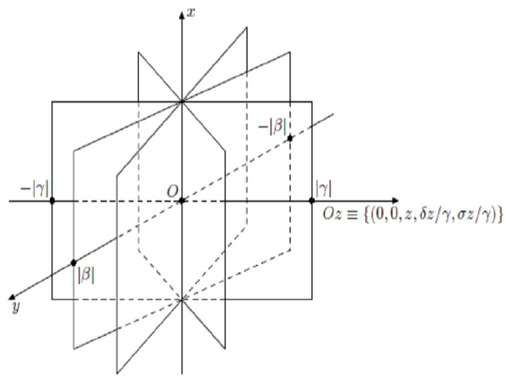

Figure 4. The leaves of $\mathcal{F}_{4,5}$

Proposition 4.4. *$\mathcal{F}_3$-type MD(5,4)-foliations are totally geodesic and Riemannian.*

4.4. Foliations of the type $\mathcal{F}_4$

Choose $\mathcal{F}_{4,12\left(1,\frac{\pi}{2}\right)}$ represents the type $\mathcal{F}_4$. From geometric picture of K-orbits in [10], for *F*(α,β,γ,δ,σ) in V_4, the leaves of $\mathcal{F}_{4,12\left(1,\frac{\pi}{2}\right)}$ are 2-dimensional K-orbits as follows:

$$\Omega_F = \left\{\left(x; (\beta + i\gamma)e^{-ia}; e^a\delta; e^a\sigma\right) : x, a \in \mathbb{R}\right\},$$

where $|\beta + i\gamma|^2 + \delta^2 + \sigma^2 \neq 0$. They are surfaces given by the following cases:

- Let us identify Ox with $\mathbb{R}.x \times \{(0,0)\} \times \mathbb{R}.t \times \mathbb{R}.s$. Then, we can see $G^*_{5,4,12(1,\frac{\pi}{2})} \equiv \mathbb{R}^5$ as $\mathbb{R}^3 \equiv Oxts$ and the leaves of $F_{4,12(1,\frac{\pi}{2})}$ are half-planes $\sigma t = \delta s$ which rotate around Ox (Figure 6).

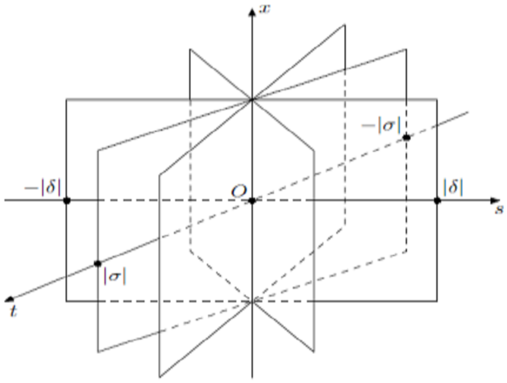

Figure 6. The leaves of $F_{4,12(1,\frac{\pi}{2})}$ in 3-plane

$y = z = 0$

- Let us identify Ox with $\mathbb{R}.x \times \{(0,0)\} \times \mathbb{R}.t \times \mathbb{R}.s$. Then, $G^*_{5,4,12(1,\frac{\pi}{2})}$ can be seen as $\mathbb{R}^3 \equiv Oxyz$ and the leaves of $F_{4,12(1,\frac{\pi}{2})}$ are rotating cylinders (Figure 7).

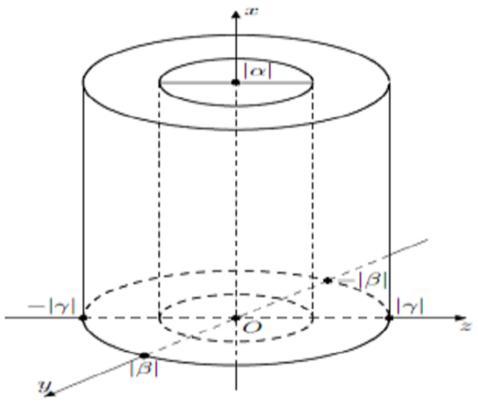

Figure 7. The leaves of $F_{4,12(1,\frac{\pi}{2})}$ in 3-plane

$t = s = 0$

- Let us identify Ox with $\mathbb{R}.x \times \{(0,0)\} \times \mathbb{R}.t \times \mathbb{R}.\frac{\sigma}{\delta}t$. In this case, the leaves of $F_{4,12(1,\frac{\pi}{2})}$ are rotating cylinders (Figure 8).

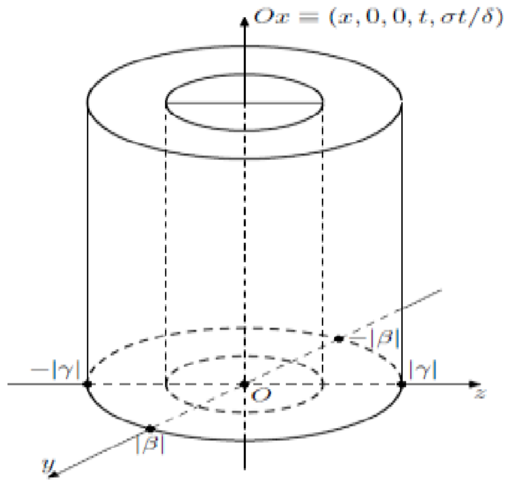

Figure 8. The leaves of $F_{4,12(1,\frac{\pi}{2})}$ in 3-plane

$\{t = e^a\delta, s = e^a\sigma\}$

Proposition 4.6. *F_4-type MD(5,4)-foliations are not totally geodesic.*

4.5. Foliations of the type F_5

Choose $F_{4,14(0,1,\frac{\pi}{2})}$ represents the type F_5. From geometric picture of K-orbits in [10], for $F(\alpha,\beta,\gamma,\delta,\sigma)$ in V_4, the leaves of $F_{4,14(0,1,\frac{\pi}{2})}$ are 2-dimensional K-orbits Ω_F as follows:

$$\left\{\left(x; (\beta + i\gamma)e^{-ia}; (\delta + i\sigma)e^{-ia}\right) : x, a \in \mathbb{R}\right\},$$

where $|\beta + i\gamma|^2 + |\delta + i\sigma|^2 \neq 0$. They are surfaces given by each case as follows:

- Let us identify Oz with $\{(0,0)\} \times \mathbb{R}.z \times \mathbb{R}.t \times \mathbb{R}.s$. The leaves of $F_{4,14(0,1,\frac{\pi}{2})}$ are rotating cylinders (Figure 9).

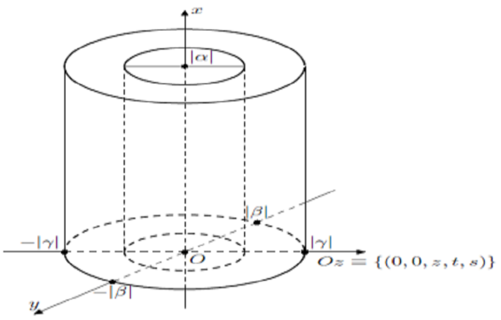

Figure 9. The leaves of $\mathcal{F}_{4,14\left(0,1,\frac{\pi}{2}\right)}$ in 3-plane $t = s = 0$

- Let us identify Ox with $\mathbb{R}.x\times\mathbb{R}.y\times\mathbb{R}.z\times\{(0,0)\}$. The leaves of $\mathcal{F}_{4,14\left(0,1,\frac{\pi}{2}\right)}$ are rotating cylinders (Figure 10).

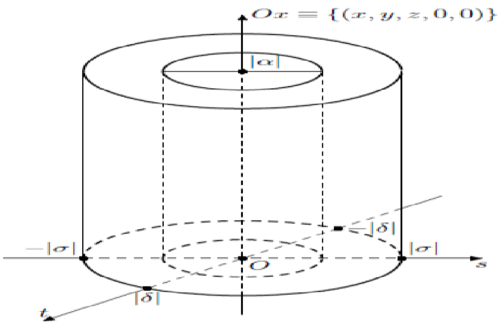

Figure 10. The leaves of $\mathcal{F}_{4,14\left(0,1,\frac{\pi}{2}\right)}$ in 3-plane $y = z = 0$

Finally, that are leaves

$$\Omega_F = \left\{\left(x;\left(\beta+i\gamma\right)e^{-ia};\left(\delta+i\sigma\right)e^{-ia}\right): x,a\in\mathbb{R}\right\}.$$

Each leaf is a cylinder whose generating curve is parallel to Ox-axis, directrix is a compact leaf of *linear foliation* $\mathcal{F}_{1,1}$ [6] on 2-dimensional torus $T^2 \approx S^1\times S^1$.

Proposition 4.7. $\mathcal{F}_5$-*type MD(5,4)-foliations are not totally geodesic*.

5. CONCLUSION

In this paper, we described some geometric characteristics of subclass of MD5-foliations: the subclass consists of MD(5,3C)-foliations and MD(5,4)-foliations. These results gave concrete examples of the simplest foliations on a special Riemannian manifold (Euclidean space). Recently, a special subclass consists of MD(n,1)-algebras and MD(n,n–1)-algebras has been classified for arbitrary n. Therefore, in another paper, we will consider a similar problem for the entire class of MD5-foliations; furthermore, for all MD(n,1)-foliations and MD(n,n–1)-foliations.

Về một số đặc trưng hình học của các phân lá quỹ đạo tạo bởi tác động đối phụ hợp của một vài nhóm Lie giải được 5-chiều

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TÓM TẮT:

Trong bài này, chúng tôi sẽ cho một vài đặc trưng hình học của các MD(5,3C)-phân lá và MD(5,4)-phân lá, tức là các phân lá tạo bởi các quỹ đạo đối phụ hợp ở vị trí tổng quát của các MD(5,3C)-nhóm và MD(5,4)-nhóm.

Từ khóa: *K-biểu diễn, K-quỹ đạo, MD-nhóm, MD-đại số, phân lá.*

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