Mathematical model in assesment of saltwater intrusion in Saigon – Dong Nai river system (Southern Vietnam) due to sea level rise

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ABSTRACT:

SaiGon –DongNai (SG-DN) river system plays a vital role in developing the southern key economic triangle including Ho Chi Minh City, DongNai and BinhDuong provinces. Saltwater intrusion results from many factors and complex movements in SG–DN river system, in the midst of which are sea level rise and water regulation of upstream reservoirs. Theses causes have gradually changed the hydraulic regimes of the river system. As a result, saltwater intrusion has become seriously. In this article, the authors used mathematical models to investigate the change of saltwater boundary of the river system before and after the impact of sea level rise and the regulatory regime of the reservoirs. The findings contributed to the predicted scenarios where sea level rise and salinity boundary could be controlled through the regulation of upstream reservoirs.

***Keywords:** sanility intrustion, sea-level rise,mathematical model.*

1. INTRODUCTION

SG-DN river system includes DongNai river’s mainstream and four major tributary rivers that are La Nga river, Be River, SaiGon River and Vam Co River. The SG-DN river basin is the home of many South East provinces and part of the highland provinces as well, with its total area about 40,000 km^2.

Saline intrusion is an extremely important and more noticeable issue in the low basin of SG-DN river. As featuring a deep river bed, we have discovered that a gentle riverbed slope with high water level amplitude of daily tide leads to the rising tide of saltwater intrusion upstream, especially in the mid and late dry season (March and April). Particularly, in recent years, along with the impact of climate change and sea level rise, the calculations and predictions of saltwater intrusion have become more pressing. Many studies, therefore, have lately reflected substantial improvement by using mathematical methods and algorithms. These methods have given the simulation results almost close to the actual measured ones. Eventually, the prediction outcomes through mathematical calculations are reasonably accurate.

Implementation of computing saltwater intrusion in this article used software F28 (written and developed by Dr. Le Song Giang) with calculations based on the solution of differential equations describing motion of water in the environment by finite volume method, thereby ensuring the principle of conservation of mass and momentum. The results of this article have showed the salinity intrusion under the rising sea level effect with the participation of households in *their* irrigation upstream monitoring stations along the *downstream* river of the system.

2. MODEL DECRIPTION

2.1 Method

Flows and salt-water intrusion in SG-DN river system is calculated by the integration of one-dimensional and two- dimensional models: the former for small rivers and canals. Flows and chemical transport of this model is described by Saint-Venant equations (1) (2) (Vreugdenhil,1989) and transport equations (3) :

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial s}=q_l \qquad (1)$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial s}\left(\frac{Q^2}{A}\right)+gA\frac{\partial \eta}{\partial s}+gA\frac{|Q|Q}{K^2}-V_a q_l=0 \qquad (2)$$

Where η – the water level; Q, A and K – the flow rate, the cross section area and the flow rate module for 1D flow. q_l – lateral inflow per unit length of river; V_a – axial component of velocity of lateral inflow of q_l)

At the river nodes (Figure 1) where tributaries connect with each other, the preserve volume of water equations in the form of the integral is used:

$$\frac{\partial W_J}{\partial t}+\sum_i Q_i=\sum_i \int_{L_i} q_{l_i}\,dl \qquad (3)$$

Where W_J – the volume of node J; L – the control contour of the node; and q_n – the normal to the control contour component of flow rate per unit width.

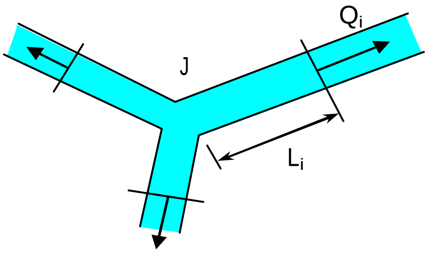


Fig.1: Node of river diagram

Mainstreams and estuaries are modeled in 2D model. Flows and chemical transport model are described by shallow water equations (4) (5) and 2D transport equations (10)

$$\frac{\partial \eta}{\partial t}+\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}=q_v \qquad (4)$$

$$\frac{\partial \mathbf{q}}{\partial t}+\frac{\partial f(\mathbf{q})}{\partial x}+\frac{\partial g(\mathbf{q})}{\partial y}=\mathbf{b}(\mathbf{q}) \qquad (5)$$

Where η – the water level; $\mathbf{q}=\left[q_x,q_y\right]^T=D\mathbf{U}$ – the vector of flow rate per unit width in 2D flow; $\mathbf{U}=\left[u_x,u_y\right]^T$ – the vector of depth-averaged velocity; D – the water depth; $\mathbf{F}(\mathbf{q})$ – the flux vector of flow rate per unit width; and $\mathbf{b}(\mathbf{q})$ – the vector of external forces.

The vectors of flux, $\mathbf{F}(\mathbf{q})$, and of external forces, $\mathbf{b}(\mathbf{q})$, have the forms:

$$\mathbf{F}=\begin{bmatrix} f(\mathbf{q}) \\ g(\mathbf{q}) \end{bmatrix}=\begin{bmatrix} q_x\mathbf{U}-A_H D\,\partial\mathbf{U}/\partial x \\ q_y\mathbf{U}-A_H D\,\partial\mathbf{U}/\partial y \end{bmatrix} \qquad (6)$$

$$\mathbf{b}(\mathbf{q})=\begin{bmatrix} -gD\,\partial\eta/\partial x-\left(\tau_{bx}-\tau_{wx}\right)/\rho+fq_y \\ -gD\,\partial\eta/\partial y-\left(\tau_{by}-\tau_{wy}\right)/\rho-fq_x \end{bmatrix} \qquad (7)$$

Where A_H – the eddy viscosity; f – Coriolis parameter; (τ_{wx}, τ_{wy}) – the shear stress on water surface due to the wind; (τ_{bx}, τ_{by}) – the bottom stress;.

The bottom stress is calculated using Manning's formula:

$$\left(\frac{\tau_{bx}}{\rho},\frac{\tau_{by}}{\rho}\right)=\frac{gn^2}{D^{1/3}}\sqrt{u_x^2+u_y^2}\left(u_x,u_y\right) \qquad (8)$$

While the eddy viscosity is calculated using Elder's formula (Hervouet, 2003):

$$A_H=6u_*D \qquad (9)$$

Where n – the Manning's roughness coefficient; u_* - the bottom shear stress velocity.

$$\frac{\partial (DC)}{\partial t}+\frac{\partial}{\partial x}(q_x C)+\frac{\partial}{\partial y}(q_y C)= \frac{\partial}{\partial x}\left(\varepsilon_H D\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon_H D\frac{\partial C}{\partial y}\right)+ DS_C+q_v C_{qv} \qquad (10)$$

Where C - average depth concentration of solutes or suspended; $\mathbf{q}=\left[q_x,q_y\right]^T=D\mathbf{U}$ – the vector of flow rate per unit width in 2D flow; $\mathbf{U}=\left[u_x,u_y\right]^T$ – the vector of depth-averaged velocity; D – the water depth; q_v – lateral inflow per unit surface; ε_H - eddy diffusion coefficient; C_{qv} - concentration in lateral inflow; S_C - source term, express speed production or destruction of dissolved (or suspended);

The vectors eddy diffusion coefficient ε_H, have the forms:

$$\varepsilon_H = \sigma . A_H \qquad (11)$$

Where σ - prandtl number; A_H – the eddy viscosity.

1D model and 2D model integated in common node, used a common water level and soveled from transport equations (3):

$$\frac{\partial W_J}{\partial t}+\sum_i Q_i=\sum_i \int_{L_i} q_{l_i} dl \qquad (3)$$

These basic equations are solved by finite volume method, in which the 2D mesh is non-structure with quadrangle elements.

2.2 Model building

2.2.1. Channel network of river for computing

SG-DN river system is modeled as a network with 195 compute nodes, 220 branches. The branches are divided into 1.496 sections, which lengths are approximately 1.000 meters, and 1.716 sections.

Mainstreams and estuaries are divided into 2D quadrilateral grid with 30.859 elements and 33.733 nodes.

River and sea bottom topography built from the base map and topographic map of SG-DN river system was created/drawn by Ho Chi Minh City People's Committee.

2.2.2. Input parameters

- Base map, topographic map of SG-DN river system.
- Coordinates of nodes and branches channel topology table, element.
- Coordinates cross-sections and shapes.
- Salinity over time.
- Sea level on planet steps of the model.
- Discharge flow of water reservoirs for irrigation in the basin.

2.2.3. Boundary conditions:

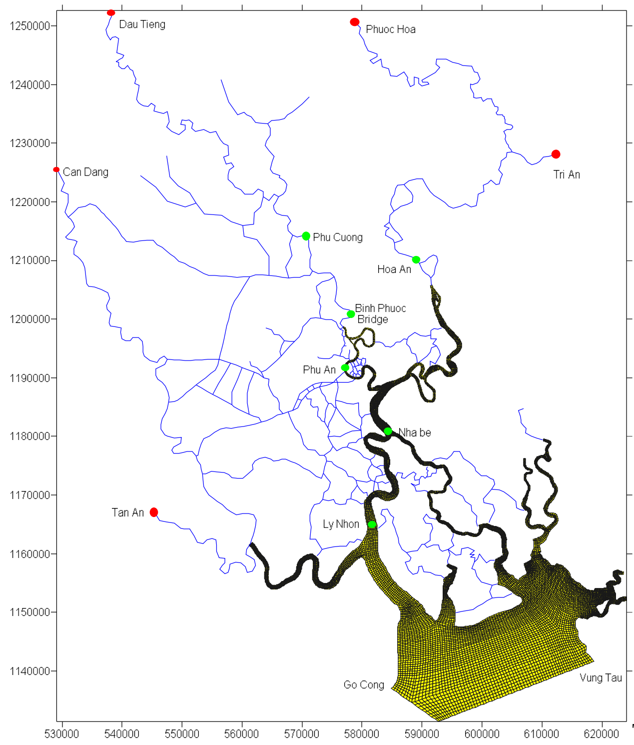
The flow of discharged reservoirs such as Dau Tieng, Tri An, Phuoc Hoa, Can Dang, Tan An and sea level at Vung Tau-Go Cong observation station are used as boundary conditions.

Salinity record at Vung Tau – Go Cong observation station when sea-water inflows is obtained using 34 g/l. This is an average salinity of the East Ocean.

The model has six important open boundaries. At the nodes of Tri An, Phuoc Hoa, Dau Tieng and Can Dang, the flow rates are given using measurement data. The measurement of water level is applied at Tan An node. At the boundary from Vung Tau to Go Cong, the water level is calculated from tidal constants.

2.2.4. Data model:

- Manning’s roughness coefficient for the river section and the 2D model



- Time step of the model

Fig.2 : The computational mesh and observation stations of SG-DN river system

2.3 Model calibration

2.3.1 Water level calibration

Manning's roughness coefficient (n) of the Equation (8) is the parameter of being calibrated. The results of Manning's roughness coefficient calibration are given in Table 1.

Table 1: Manning's roughness coefficient calibration

Station	Node	ID section	Initial Manning's roughness coefficient	Calibrated Manning's roughness coefficient
Bien Hoa	186	164 - 186	0.035	0.045
Ben Luc	470	470 - 494	0.034	0.02
Tan An	495	495 - 528	0.034	0.02
Phu An	30084	11	0.025	0.035
Nha Be	29490	10	0.02	0.033

Calculated water level used to calibrate from 12.00 a.m. on March 1, 2012 to 12.00 a.m. on March 15, 2012. The results of calibration for the observation stations are given in the figures below:

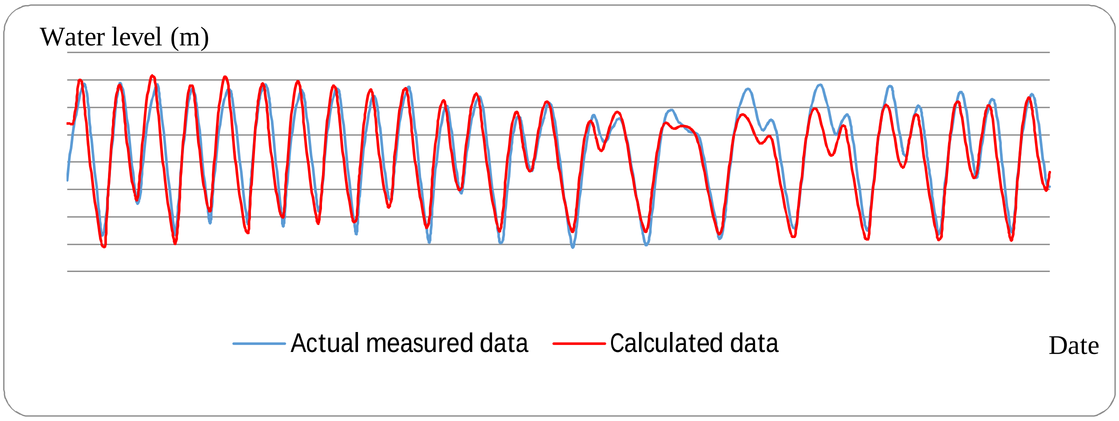


Fig. 3 : The results of water level calibration for the Bien Hoa observation stations

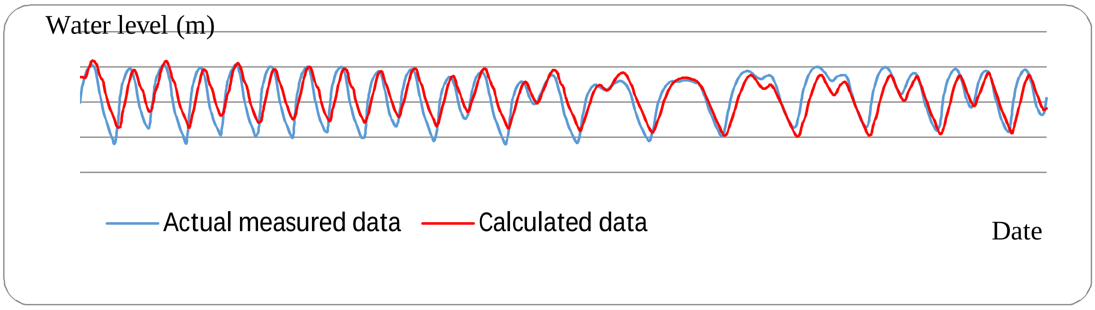


Fig. 4 : The results of water level calibration for the Ben Luc observation stations

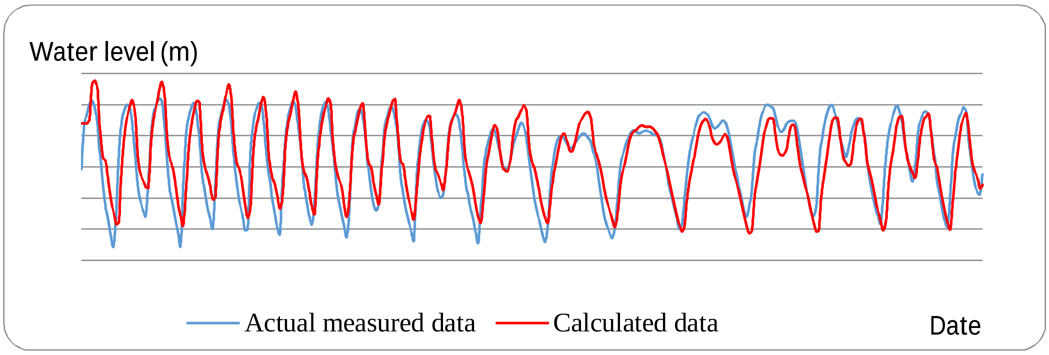


Fig. 5 : The results of water level calibration for the Tan An observation stations

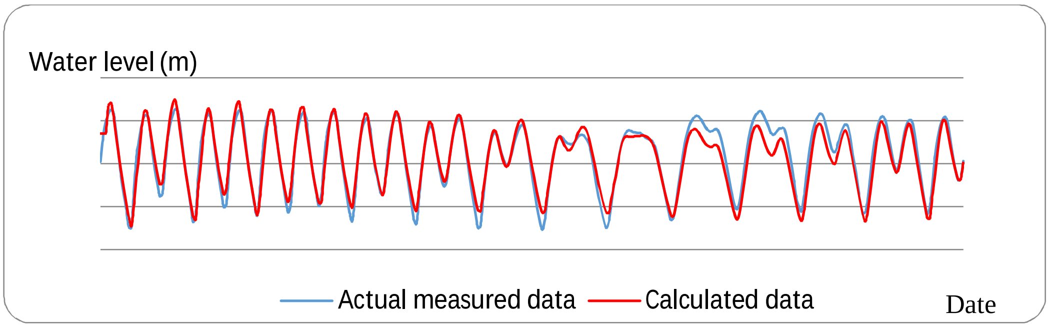


Fig. 6 : The results of water level calibration for the Phu An observation stations

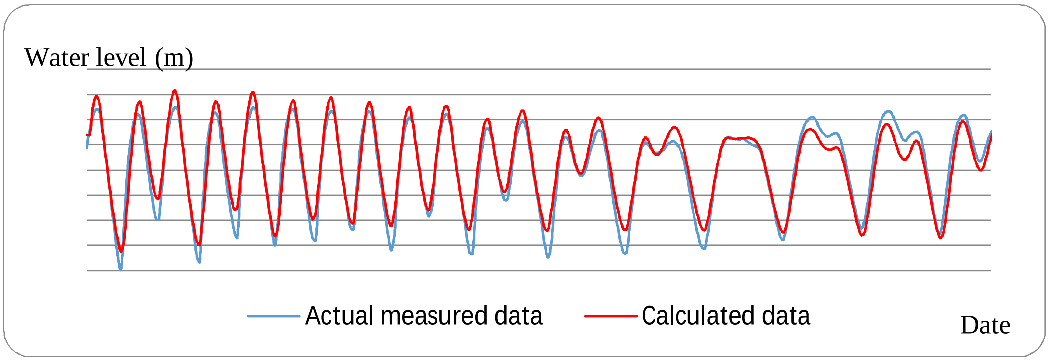


Fig. 7 : The results of water level calibration for the Nha Be observation stations

The comparison of calculated water level and actual water level measured at observation stations has proved that calculated water level amplitude is slightly smaller than the measured data in general, however it does provide a good analogy between the calculation and measurement results, and especially they appear in the same phase. (Figs. 3-7)

2.3.2 Salinity calibration

σ - prandtl number of the Equation (11) is the parameter of being calibrated. Salinity used to calibrate from 12.00 a.m. on March 8, 2012 to 12.00 a.m. on March 16, 2012. The results of calibration for the observation stations are given in the figures below:

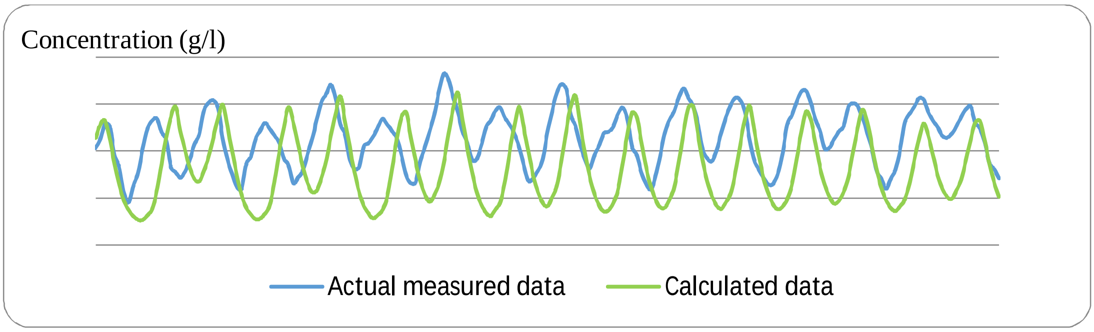


Fig. 8. The results of salinity calibration for the Nha Be observation stations

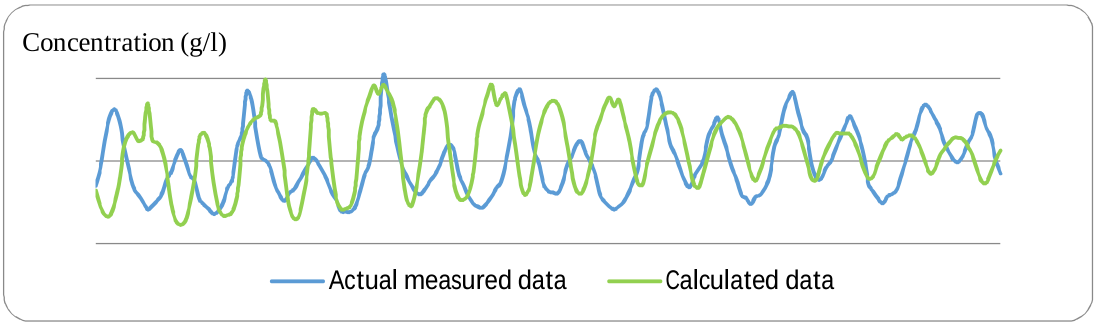


Fig. 9: The results of salinity calibration for the Phu An observation stations

Comparison of calculated salinity and actual measured salinity at observed stations shows that the calculated results have amplitude of oscillation higher than the measured data in general. Despite the little difference of phased similarity, the results are still accepted. (Figs. 8-9)

3. RESULTS AND DISCUSSION

This article presents the results and reviews of the salinity change of SG and DN river systems. The change occurred in March, 2012 at monitoring stations such as Vung Tau, Nha Be, Cat Lai, Thu Thiem, and An Hoa, according to the scenario of sea level rising by respectively 15 cm, 30 cm, 50 cm, 70 cm compared to the average of sea level at Vung Tau station.

The results of salinity spread calculated by the model are displayed in Table 3.

Figs. 10-12 describe the correlation of sea level rising and salinity at observation stations. As the results in Tab.3, we find that when sea level rises, the salinity will spread deeply into the upstream of the river. According to the Figs. 10-12 we learn that salinity spread rates appear differently, in which Nha Be station is impacted the most powerfully with the slope (a = 1.2778), the next one is Thu Thiem station with a = 0.9088 and Cat Lai station with a = 0.9009. However, Hoa An station is an important water resource for the city that has not been affected much with sea level rising by 75 cm.

Tab 3. Water levels and salinity concentration sea level rising by respectively 15cm, 30cm, 50cm, 75cm

Station	Delta h = 0 m		Delta h = 0.15 m		Delta h = 0.3 m		Delta h = 0.5 m		Delta h = 0.75 m	
	Water level	Concen-tration	Water level	Concen-tration	Water level	Concen-tration	Water level	Concen-tration	Water level	Concen-tration
	(m)	(g/l)	(m)	(g/l)	(m)	(g/l)	(m)	(g/l)	(m)	(g/l)
Vung Tau	(0.178)	34	(0.029)	33.98	0.121	33.98	0.321	33.99	0.571	33.99
Nha Be	0.974	9.576	1.108	12.54	1.295	13.26	1.505	14.23	1.740	15.12
Cat Lai	1.136	2.860	1.306	4.72	1.467	5.124	1.658	5.919	1.900	6.764
Thu Thiem	1.246	1.703	1.385	3.289	1.527	3.584	1.707	4.503	1.931	5.64
Hoa An	1.395	0.003	1.499	0.008	1.604	0.01076	1.752	0.01782	1.948	0.03029

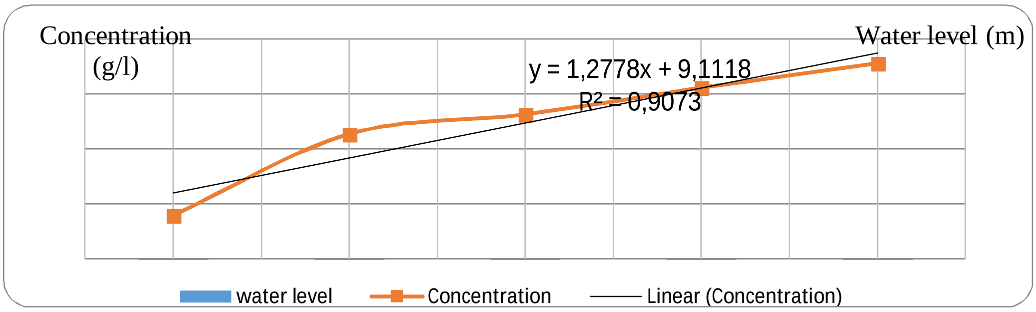


Fig. 10. The correlation of water level and salinity intrusion at Nha Be observation station

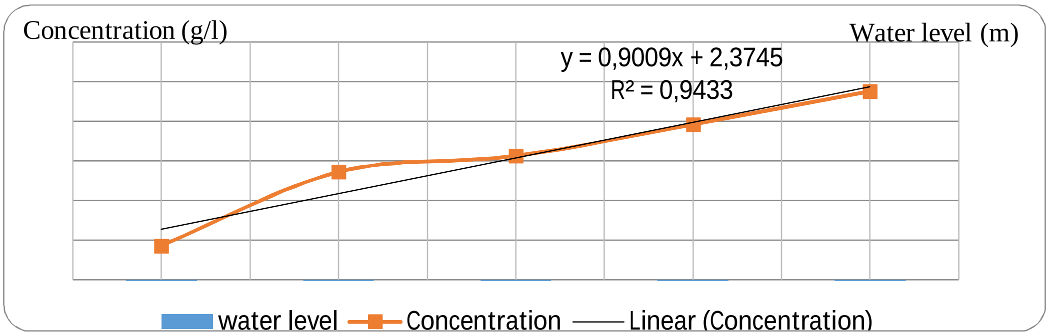


Fig. 11. The correlation of water level and salinity intrusion at Cat Lai observation station

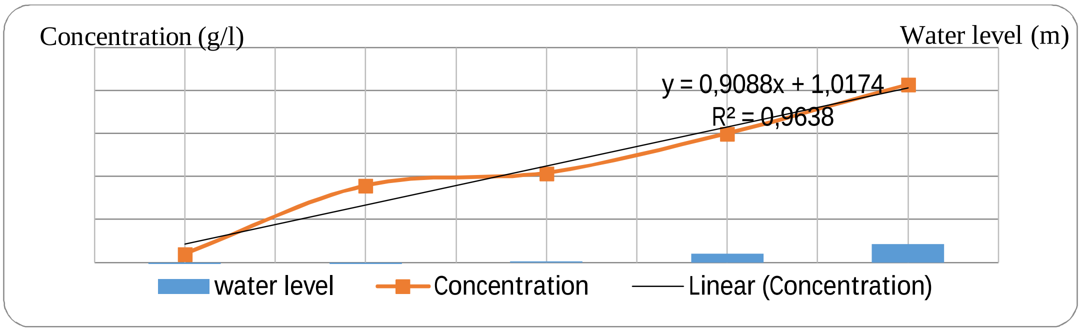


Fig. 12. The correlation of water level and salinity intrusion at Thu Thiem observation station

4. CONCLUSION

Results of water level calculated by the model reflects that the amplitude slightly smaller in comparison to the real measurement data, but the calculated salinity with the amplitude of oscillation is higher. Of the similarity phase, the calculated water levels have a coincident phase and fairly parallel between model results and measured water level results, but the calculated salinity just has a little difference. Through an examination of saltwater intrusion by sea level rising respectively 15 cm, 30 cm, 50 cm and 75 cm, the vibration amplitude in salinity has been measured at the stations near 300%, but Hoa An station considered as a main water supply has not been affected. On the other hand, it is also recognized that salinity regime of the basin downstream of SG-DN is quite sensitive to hydraulic river mode. Therefore, we can also rely on water regulation of upstream irrigation lake to improve salinity mode and to regulate the distribution of salinity in the downstream river.

Ứng dụng mô hình toán trong đánh giá xâm Nhập mặn của hệ thống sông Sài Gòn - Đồng Nai dưới tác động của nước biển dâng

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TÓM TẮT:

Hệ thống sông Sài Gòn-Đồng Nai đóng một vai trò quan trọng trong việc phát triển của tam giác kinh tế trọng điểm phía Nam bao gồm thành phố Hồ Chí Minh và các tỉnh Bình Dương, Đồng Nai. Quá trình xâm nhập mặn ở hệ thống sông Sài Gòn - Đồng Nai phụ thuộc vào nhiều yếu tố và diễn biến khá phức tạp. Một trong những yếu tố quan trọng và ảnh hưởng rất lớn tới quá trình xâm nhập mặn chính là chế độ thủy lực và sự điều tiết nước của các hồ thủy lợi ở thượng lưu. Mặt khác, dưới tác động của biến đổi khí hậu và nước biển dâng cũng làm xâm nhập mặn trở nên nghiêm trọng.

Trong bài viết này, tác giả sử dụng mô hình toán để nghiên cứu sự thay đổi ranh giới mặn của hệ thống sông trước và sau khi tác động của nước biển dâng và các chế độ điều tiết của các hồ chứa. Kết quả của bài báo này góp phần với các kịch bản dự báo nơi nước biển dâng và ranh giới mặn có thể được kiểm soát thông qua sự điều tiết của các hồ chứa thượng nguồn.

Từ khóa: xâm nhập mặn, nước biển dâng, mô hình toán.

TÀI LIỆU THAM KHẢO

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