Using mathematical programming method to optimize amount of injection gas for Dai Hung field, Nam Con Son basin, offshore Vietnam

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ABSTRACT:

During oil – production by gaslift method, because of the limitation of injection gas rate, the optimization of distributing injection gas rate for group of well is universally interesting. There are some optimization methods for gaslift well in Vietnam, nearly all uses analytical mathematic method with solutions finding the extrema for multivariable function. This paper presents an optimization method for group of gaslift well to increase the oil rate for Dai Hung field, Nam Con Son Basin, offshore Vietnam.

Keywords: mathematical programming, optimization, gaslift, injection gas.

1. INTRODUCTION

Optimization, as a branch of mathematics, has many applications and widely effective. Therefore, the field of optimization is becoming increasingly diverse, with many different names such as Mathematics Programming, Optimal Control, Operation Research, Games Theory…

In mathematics, term optimization refers to the study of problems:

\textit{Given a function } f: A \rightarrow \mathbb{R}, \textit{find an element } x_0 \in A \textit{ such that } f(x_0) \leq f(x), \forall x \in A \textit{ or } f(x_0) \geq f(x) \forall x \in A.

One such material formulation is sometimes called a mathematical programming. Many practical problems and theoretical model can be generalized in that way.

The domain A of f is called the search space. Typically, A is a subset of Euclidean space \mathbb{R}^n, usually defined by a set of constraints that the elements of A must satisfy. The elements of A are called the candidate solutions. The function f is called an objective function. A feasible solution that minimizes or maximizes the objective function is called an optimal solution.

To apply to design optimal regime for exploitation of gas-lift system, we will seek objective function based on production and injection gas data of each well satisfy constraints and find value to achieve the objective function value optimal.

2. THEORETICAL BASIS

2.1 Methods of analysis and research

Optimizing gas-lift production has many different definitions, but the general approach is to make the highest profit factor. Thus, the goal of optimization is to exploit the maximum flow, the minimum individual cost and maximum profit. In this paper based on criteria optimal maximum flow and minimum operators own expense. Optimizing group of gas-lift well will clear the following practical problem:

+ Group wells for maximum flow rate while excluding limited amounts of injection gas.

+ Distribution of injection gas to group of gas-lift wells for maximum flow rate for a given amount of injection gas.
On each operation curve \( Q = f(V) \) has a feature called optimal flow-rate point, corresponding to the highest effective factor of gas-lift process, \( \eta \).

The factor \( \eta \) will reach a maximum value at the point where the ratio \((Q/V)\) peak. Also, at tangent point to curve \( Q(V) \) drawn from the origin of axis. In this mode, the ratio \((V/Q)\) is minimal, or the minimum amount of injection gas to raise a unit volume of fluid. The production rate corresponding to the maximum value of \( \eta \) is called optimal rate.

The more volume of injection gas, the more production rate we have because of the decrease of fluid density in tubing. But, this only increased to \( Q_{\text{max}} \) value, after that if volume of injection gas continues increasing the production rate will reduce because of slipping between injection gas and oil.

Therefore, the operation mode of the wells located between optimal mode \( (Q_{\text{opt}}) \) and maximum mode \( (Q_{\text{max}}) \) (Fig. 1)

![Figure 1. The effect of injection gas to production rate.](image)

### 2.2 Survey method

Basically, the survey method presented here is based on the change of production rate versus volume of injection gas.

**Survey procedures:**

- Reduced injection gas volume small enough so that the well still working stability, write down injection gas volume \( (V_{k1}) \), injection gas pressure \( (P_{k1}) \) and production rate \( (Q_{k1}) \).

- Increase the amount of injection gas to about 25%, write down injection gas volume, pressure and production rate when the well working stability. Continue to increase the amount of injection gas by 25% of each time and write down those parameters until the production rate decrease.

- Draw the relationships of the injection gas rate \( R = f(V_{k}) \), the injection gas pressure \( P_{k} = f(V_{k}) \), the production rate \( Q_{k} = f(V_{k}) \) and injection gas volume \( V_{k} \). (Fig. 2)

The more volume of injection gas, the more production rate we have because of the decrease of fluid density in tubing. But, this only increased to \( Q_{\text{max}} \) value, after that if volume of injection gas continues increasing the production rate will reduce because of slipping between injection gas and oil.

As shown in Fig.2, we can see point 1 at optimal mode (the curve \( Q_{k} = f(V_{k}) \) corresponding to point 2 the minimum injection rate (the curve \( R = f(V_{k}) \). This means that at this point the cost for injection gas volume is smallest. This is the optimal mode is determined from the perspective of energy.

The maximum production rate of each well can be done at point 3. The operation mode of a well between optimal mode (point 1) and maximum mode (point 3).

![Figure 2. The relationships between injection rate, injection pressure, and production rate versus injection volume](image)

### 2.3 The operation equation of each well

The form of the production rate of each well:

\[
Q = AV^2 + BV + C
\]

(1)

with:

- \( Q \): production rate \( (\text{m}^3/\text{d}) \)
- \( V \): injection gas rate \( (\text{m}^3/\text{d}) \)
- \( A, B, C \): the parameters
From the survey values and equation (1), we have:

\[
\begin{align*}
Q_1 &= AV_1^2 + BV_1 + C \\
Q_2 &= AV_2^2 + BV_2 + C \\
Q_3 &= AV_3^2 + BV_3 + C
\end{align*}
\] (2)

We get A, B, C from the solution of equation system (2).

2.4 Apply Mathematical Programming to optimizing gas-lift production method

As shown above, we will define the objective function and constraints and find out the value of injection gas volume to objective function reach the optimal one.

Assume that, we have a group of wells (include n wells). \(V_i\) is the injection gas volume of the well \(i (i = 1, 2, \ldots, n)\) and \(Q_i = A(V_i)^2 + B V_i + C\) is the operation of well \(i\). \(V_T\) is the total available volume of injection gas for group of wells.

From the survey method show in item 3, we will find out the \(V_{i\text{opt}}\) and \(V_{i\text{max}}\) of well \(i\). The problem now become:

Objective function: \(Q = \sum Q_i \rightarrow \max\)

Constraints: \[
\begin{align*}
\sum V_i &\leq V_T \\
V_{i\text{opt}} &\leq V_i \leq V_{i\text{critical}}
\end{align*}
\]

3. ĐẠI HƯNG FIELD CASE STUDY

Optimization using mathematical programming method for group of 4 wells in Dai Hung Field, DH-4X, DH-8P, DH-9P, DH-10P with given data in table 1 and 2 below:

Table 1. The situation before optimizing

<table>
<thead>
<tr>
<th>Well Name</th>
<th>Volume of injection gas (x10(^3) m(^3)/day)</th>
<th>Production Rate (m(^3)/day)</th>
<th>Water Cut (%)</th>
<th>Oil density (kg/m(^3))</th>
<th>Oil rate (m(^3)/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DH-4X</td>
<td>8</td>
<td>32</td>
<td>20</td>
<td>780</td>
<td>19.97</td>
</tr>
<tr>
<td>DH-8P</td>
<td>15</td>
<td>32</td>
<td>30</td>
<td>780</td>
<td>17.47</td>
</tr>
<tr>
<td>DH-9P</td>
<td>45</td>
<td>46</td>
<td>30</td>
<td>780</td>
<td>25.12</td>
</tr>
<tr>
<td>DH-10P</td>
<td>35</td>
<td>50</td>
<td>20</td>
<td>780</td>
<td>31.2</td>
</tr>
<tr>
<td>Total</td>
<td>103</td>
<td>160</td>
<td></td>
<td></td>
<td>93.76</td>
</tr>
</tbody>
</table>

Table 2. The result from gas-lift well survey

<table>
<thead>
<tr>
<th>Well Name</th>
<th>Volume of injection gas (x10(^3) m(^3)/day)</th>
<th>Production Rate (m(^3)/day)</th>
<th>Water Cut (%)</th>
<th>Oil density (kg/m(^3))</th>
<th>Oil rate (m(^3)/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DH-4X</td>
<td>5</td>
<td>20</td>
<td></td>
<td></td>
<td>12.48</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>32</td>
<td></td>
<td></td>
<td>19.97</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>38</td>
<td></td>
<td></td>
<td>23.71</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>45</td>
<td></td>
<td></td>
<td>29.08</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>46</td>
<td></td>
<td></td>
<td>28.70</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>44</td>
<td></td>
<td></td>
<td>27.46</td>
</tr>
<tr>
<td>DH-8P</td>
<td>9</td>
<td>18</td>
<td></td>
<td></td>
<td>9.83</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>26</td>
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<td>14.20</td>
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<td>40</td>
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<td>32</td>
<td>39</td>
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<td>21.29</td>
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<tr>
<td>DH-9P</td>
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<td>22</td>
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<td></td>
<td>12.01</td>
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<td>23</td>
<td>48</td>
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<td>30</td>
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<td></td>
<td>30.03</td>
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<td>35</td>
<td>56</td>
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<td>30.58</td>
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<td></td>
<td>40</td>
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<td></td>
<td>28.94</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>46</td>
<td></td>
<td></td>
<td>25.12</td>
</tr>
<tr>
<td>DH-10P</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td>9.36</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>50</td>
<td></td>
<td></td>
<td>31.2</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>65</td>
<td></td>
<td></td>
<td>40.56</td>
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<td></td>
<td>65</td>
<td>70</td>
<td></td>
<td></td>
<td>43.68</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>66</td>
<td></td>
<td></td>
<td>41.18</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>52</td>
<td></td>
<td></td>
<td>32.45</td>
</tr>
</tbody>
</table>

3.1 Optimization for each of gas-lift well in Dai Hung Field

From table 2, and use eq. (1) and (2) we find out the \(Q_{\text{opt}}\) and \(V_{\text{opt}}\). From the peak of the curve \(Q(V)\) we also have \(Q_{\text{max}}\) and \(V_{\text{max}}\). The optimal operation mode for each well between optimal to maximum mode. The calculation results shown below:

Well DH-4X:
3.2 Simultaneous optimization for Group of 4 wells above

From data from table 2, using eq. (1) and (2) we have the operation equation for each well shown below:

- **Well DH-4X**: $Q = -0.2V^2 + 6.6V - 8$
- **Well DH-8P**: $Q = -0.111V^2 + 5V - 18$
- **Well DH-9P**: $Q = -0.076V^2 + 0.015V - 27.273$
- **Well DH-10P**: $Q = 0.021V^2 + 2.821V - 22.5$

Apply mathematical programming for this problem:

**Obj. function:**

$$Q = Q_{DH-4X} + Q_{DH-8P} + Q_{DH-9P} + Q_{DH-10P} \rightarrow \text{max}$$

**Constraints:**

$$V_{DH-4X} + V_{DH-8P} + V_{DH-9P} + V_{DH-10P} \leq 130$$
and

$$V_{(\text{opt})} \leq V_i \leq V_{(\text{critical})}$$

The result of solution we have:

- **Well DH-4X**: $V_{DH-4X} = 14.14$; $V_{DH-8P} = 18.24$;
- **Well DH-8P**: $V_{DH-8P} = 26.86$; $V_{DH-10P} = 43.76$
- **Well DH-9P**: $V_{DH-9P} = 26.86$; $V_{DH-10P} = 43.76$
- **Well DH-10P**: $V_{DH-10P} = 43.76$

Calculate the production rate for each well after optimization by this equation:

$$Q = (A.V^2 + B.V + C).(1-n).\rho$$

With: **n**: water cut; **\rho**: oil density
Table below show us the detail of calculation results:

**Table 3 - The results of calculation after optimization for the group wells**

<table>
<thead>
<tr>
<th>Well Name</th>
<th>Injection gas volume each well (x10^3 m^3/day)</th>
<th>Production rate after optimization (m^3/day)</th>
<th>Oil Production Rate after optimization (ton/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DH-4X</td>
<td>14.14</td>
<td>45.33</td>
<td>28.29</td>
</tr>
<tr>
<td>DH-8P</td>
<td>18.24</td>
<td>36.24</td>
<td>19.79</td>
</tr>
<tr>
<td>DH-9P</td>
<td>26.86</td>
<td>52.78</td>
<td>28.82</td>
</tr>
<tr>
<td>DH-10P</td>
<td>43.76</td>
<td>59.93</td>
<td>37.4</td>
</tr>
<tr>
<td>Total</td>
<td>103</td>
<td>194.28</td>
<td>114.29</td>
</tr>
</tbody>
</table>

After optimization of distribution the amount injection gas for the group wells we get the production rate increase by 34.28 m^3/day, oil-production increase by 20.53 ton/day. The comparison of production rate, volume of injection gas before and after optimization shown by the charts below:

- **Figure 3.** The comparison between volume of injection gas before and after optimization for each well

- **Figure 4.** The comparison between oil production rate before and after optimization for each well

4. **CONCLUSION**

With the limitation of injection gas about 103x10^3 m^3/day, after optimization the production rate increase by 34.28 m^3/day, and the oil rate increase by 20.53 ton/day. When compared with the real-production data from practice, the results of each is equivalent. This can prove the correctness of this method.
Ứng dụng phương pháp quy hoạch toán học tối ưu hóa lượng khí nén bơm ép cho mỏ Đại Hùng, bờ trầm cơn Côn Sơn, ngoài khu vực Việt Nam

- Phùng Đại Khánh
- Lê Thanh Liêm
- Hoàng Trọng Quang
  Trường Đại học Bách khoa, ĐHQG-HCM

TÔM TÁT:

Trong khai thác dầu bằng phương pháp gaslift, do lượng khí nén thường là có hạn, nên vấn đề tối ưu hóa việc phân bổ lượng khí nén cho các giếng trong một mỏ luôn được quan tâm. Đã có một số phương pháp tối ưu hóa cho nhóm giếng khai thác gaslift ở Việt nam, nhưng chủ yếu là sử dụng phương pháp giải tích toàn học với lời giải là tìm cực trị của hàm đa biến. Bài báo này sẽ đề cập đến việc áp dụng một phương pháp tối ưu hóa cho nhóm giếng khai thác gaslift - phương pháp Quy hoạch toàn học - ứng dụng cho mỏ Đại Hùng bờ trầm Côn Sơn Việt Nam.

Từ khóa: Quy hoạch toàn học, tối ưu hóa, gas-lift, khí bơm ép.

TÀI LIỆU THAM KHẢO

