A NON- STATIC COSMOLOGICAL MODEL IN THE VECTOR MODEL FOR GRAVITATIONAL FIELD

Vo Van On

Thu Dau Mot University

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ABSTRACT: *In this paper, based on the vector model for gravitational field we obtained the modified Friedman equations, which were similar to the classical Friedman equations but were added a term of energy – momentum tensor of gravitational field. Non- static flat cosmological model in this model was similar to General Theory of Relativity (GTR) 's model but the expansive rate in the vacuum age was difference with General Theory of Relativity 's model.*

Keywords: *non-static flat cosmological model; modified Friedman equations; expansive ages*

1. INTRODUCTION

In the previous papers [1, 2, 3, 4], we have constructed a vector model for gravitational field and also obtained the modified Einstein 's equation in this model as follows

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda = -\frac{8\pi G}{c^4} T_{Mg.\mu\nu} + \omega T_{g.\mu\nu} \qquad (1)$$

where $T_{Mg,\mu\nu}$ is the energy – momentum tensor of matter,

$T_{g,\mu\nu}$ is the energy – momentum tensor of gravitational field.

In this paper we shall use this equation to deduce the modified Friedman 's equations and investigate a non – static flat cosmological model. The outline of the paper is organized as follows : Sec. I, Introduction; in Sec. II, we determine the average strength of gravitational field in the universe and modified Friedman 's equations; in Sec. III, we investigate the expansive ages of the universe; finally, we summarize our results in Sec IV.

2. THE FRIEDMAN - ROBERTSON – WALKER METRIC AND THE MODIFIED FRIEDMAN EQUATIONS

We consider a cosmological solution in this model. We assume that matter distribute homogenous and isotropic in the Universe. This is the Cosmological principle (the Copernican principle). With this assumption, the metric of the Universe has the standard Friedman- Robertson – Walker form[5, 6] on the co – moving coordinate system

$$ds^2 = -c^2dt^2 + R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \qquad (2)$$

Where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. $R(t)$ is the scale factor, we can see it as the radius of the

Universe at the moment t. The constant k can be $0, \pm 1$ depending on the curvature of the Universe.

From the Friedman – Robertson – Walker metric (the FRW metric), we also obtain the Hubble law for the red shift of the Universe as in GTR[5]

$$z \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e} = H_0 \frac{d}{c} \qquad (3)$$

where λ_0 is the wave-length of the photon received by us on the Earth, λ_e is the wave - length of this photon at a distant galaxy. H_0 is Hubble ‘s constant, d is the astronomical distance from us to the distance galaxy.

The modified Einstein ‘ s equation in this model is[1, 2]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda = -\frac{8\pi G}{c^4} T_{Mg.\mu\nu} + \omega T_{g.\mu\nu} \qquad (4)$$

With the above FRW metric, from the expression of the line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \qquad (5)$$

where $x^0 = ct, x^1 = r, x^2 = \theta, x^3 = \phi$, we have

$$g_{00} = -1, g_{11} = \frac{R^2}{1-kr^2}, g_{22} = R^2 r^2, g_{33} = R^2 r^2 \sin^2\theta \qquad (6)$$

and

$$g^{00} = -1, g^{11} = \frac{(1-kr^2)}{R^2}, g^{22} = \frac{1}{R^2 r^2}, g^{33} = \frac{1}{R^2 r^2 \sin^2\theta} \qquad (7)$$

From the FRW metric and the Christoffel symbol

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad} (\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \qquad (8)$$

the only non – zero components of the Ricci tensor are

$$R_{00} = \frac{3\ddot{R}}{c^2 R}, \quad R_{11} = -\frac{R\ddot{R} + 2\dot{R}^2 + 2kc^2}{c^2(1-kr^2)} \qquad (9)$$

$$R_{22} = -(R\ddot{R} + 2\dot{R}^2 + 2kc^2)\frac{r^2}{c^2}, \quad R_{33} = -(R\ddot{R} + 2\dot{R}^2 + 2kc^2)\frac{r^2}{c^2}\sin^2\theta \qquad (10)$$

The Ricci scalar is then

$$R = R^\mu_\mu = g^{\alpha\beta} R_{\beta\alpha} = -\frac{6}{c^2 R^2}(R\ddot{R} + \dot{R}^2 + kc^2) \qquad (11)$$

The energy- momentum tensor of the gravitational matter in the equation (4) has the form of the perfect fluid

$$T_{Mg.\mu\nu} = (\frac{p}{c^2} + \rho_g) U_\mu U_\nu + p g_{\mu\nu} \qquad (12)$$

For a motionless fluid $U^\mu = (c, 0)$. Therefore

$$T_{Mg.00} = \rho_g c^2 + p - p = \rho_g c^2 \qquad (13)$$

And $T_{Mg.ii} = p g_{ii}$ (14)

Thus, we have

$$T_{Mg.\mu\nu} = \begin{pmatrix} \rho_g c^2 & 0 & 0 & 0 \\ 0 & p\dfrac{R^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & pR^2r^2 & 0 \\ 0 & 0 & 0 & pR^2r^2\sin^2\theta \end{pmatrix}$$

(15)

With one index raised this tensor takes the more convenient form

$$T^{\mu}_{Mg.\nu} = diag(-\rho_g c^2, p, p, p) \quad (16)$$

We consider the second term in the right hand side of equation (4). This is the energy – momentum tensor of gravitational field, its expression is[1, 2]

$$T_{g.\mu\nu} \equiv -\frac{2}{\omega\sqrt{-g}}\frac{\delta S_g}{\delta g^{\mu\nu}} = (E^{\alpha}_{g.\mu} D_{g.\nu\alpha} - \frac{1}{4} g_{\mu\nu} E^{\alpha\beta}_{g} D_{g.\alpha\beta}) \quad (17)$$

where

$$E_{g.\mu\nu} = \begin{pmatrix} 0 & -E_{gx}/c & -E_{gy}/c & -E_{gz}/c \\ E_{gx}/c & 0 & H_{gz} & -H_{gy} \\ E_{gy}/c & -H_{gz} & 0 & H_{gx} \\ E_{gz}/c & H_{gy} & -H_{gx} & 0 \end{pmatrix} \quad (18)$$

It is the strength tensor of gravitational field

and

$$D_{g.\mu\nu} = \frac{1}{\mu_g} E_{g.\mu\nu} = \frac{c^2}{4\pi G} E_{g.\mu\nu} \quad (19)$$

Now we determine the average strength of gravitational field in the Universe

E_g. Because of the homogenous and isotropic distribution of matter in the Universe, the average strength of gravitational field is constant in the Universe. Indeed, we consider a spherical surface with the radius r in the Universe. Because of the symmetric property, the gravitational field caused by all matter outside of this spherical surface at an any point inside of this surface is zero. If we consider a point M on this surface, the gravitational field is only caused by all matter inside of this surface. Denoting the gravitational mass of all matter inside of the surface is $M_{g.r}$, the strength of gravitational field is $E_{g.r}$, we have

$$E_{g.r} = -\frac{GM_{g.r}}{r^2} = -G\rho_g \frac{4\pi r^3}{3r^2} = -\frac{4G\rho_g \pi r}{3} \quad (20)$$

Due to $M_{g.r} = \rho_g \frac{4}{3}\pi r^3$ (21)

But we also have $\rho_g = \frac{3M_g}{4\pi R^3}$ (22)

Because of the uniform distribution of matter in the Universe.

Where M_g is the gravitational mass of the Universe. R is the radius of the Universe at the moment t.

Substituting (22) into (20), we have

$$E_{g.r} = -\frac{GM_g}{R^2} \times \frac{r}{R} \quad (23)$$

Where the function $f(r) \equiv \frac{r}{R}$ takes the value from 0 to 1. Because of the uniform distribution of matter in the Universe, we can take the average value of the function $f(r)$ is 1/2

$$f(r) = 1/2 \quad (24)$$

Thus, the average strength of gravitational field is the same throughout the Universe and its expression is

$$E_{g.r} = -\frac{GM_g}{2R^2} \quad (25)$$

The strength tensor of gravitational field in X- direction is

$$E_{g.\mu\nu} = \begin{pmatrix} 0 & -E_{gx}/c & 0 & 0 \\ E_{gx}/c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \frac{E_g}{c}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (26)$$

Note that there is not the magneto – gravitational field.

With

$$E_g^{\mu\tau} = E_{g.\alpha\beta} . g^{\mu\alpha} g^{\tau\beta} \quad (27)$$

We have

$$E_g^{\mu\tau} = \frac{E_g}{c}\frac{1-kr^2}{R^2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (28)$$

We determine the energy – momentum tensor of gravitational field

$$T_{g.\mu\nu} = (D_{g.\mu}{}^{\alpha} E_{g.\nu\alpha} - \frac{1}{4} g_{\mu\nu} D_g^{\alpha\beta} E_{g.\alpha\beta}) \quad (29)$$

or

$$T_{g.\mu\nu} = \frac{c^2}{4\pi G}[g^{k\alpha} E_{g.\mu k} E_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} E_g^{\alpha\beta} E_{g.\alpha\beta}]$$

(30)

The $\mu\nu = 00$ component is

$$T_{g.00} = \frac{c^2}{4\pi G}[\frac{E_0^2}{c^2}\frac{1-kr^2}{R^2} - \frac{1}{4}(-1)(-2\frac{E_0^2}{c^2}\frac{1-kr^2}{R^2}] = \frac{1}{8\pi G}\frac{E_g^2}{}\frac{1-kr^2}{R^2}$$

(31)

The $\mu\nu = 11$ component is

$$T_{g.11} = \frac{c^2}{4\pi G}[g^{00} E_{g.10} E_{g.10} - \frac{1}{4} g_{11} E_g^{\alpha\beta} E_{g.\alpha\beta}]$$

$$= -\frac{E_g^2}{8\pi G} \quad (32)$$

We now find the modified Friedman equations in this model.

The equation (4) for the (00) component is

$$R_{00}-\frac{1}{2}g_{00}R=-\frac{8\pi G}{c^4}T_{Mg.00}+\Lambda g_{00}+\omega T_{g.00}$$

$$\frac{3}{c^2}\left(\frac{\dot{R}}{R}\right)^2+3\frac{k}{R^2}=\frac{8\pi G}{c^2}\rho_g+\Lambda-\frac{\omega E_g^2}{8\pi G}\left(\frac{1-kr^2}{R^2}\right)$$

(33)

The equation (4) for the (11) component is

$$R_{11}-\frac{1}{2}g_{11}R=-\frac{8\pi G}{c^4}T_{Mg.11}+\Lambda g_{11}+\omega T_{g.11}$$

$$2\frac{\ddot{R}}{c^2R}+\frac{1}{c^2}\left(\frac{\dot{R}}{R}\right)^2+\frac{k}{R^2}=-\frac{8\pi G}{c^2}\frac{p}{c^2}+\Lambda-\frac{\omega E_g^2(1-kr^2)}{8\pi GR^2}$$

(34)

The different components also lead to the equation (34) due to the isotropy of the Universe. Thus, we obtain two the modified Friedman equations (33) and (34).

3. THE EXPANSIVE AGES OF THE UNIVERSE

We now consider the equation (33) for the expansive ages of the Universe : the vacuum – dominated age, the radiation – dominated age and the matter – dominated age.

3.1 The vacuum – dominated age

$$\rho_g=0,\Lambda=const \qquad (35)$$

Because the Universe is flat, we only consider the case $k=0$

Substituting

$$\Lambda=\frac{8\pi G}{c^2}\rho_V \qquad (36)$$

The equation (33) becomes

$$\left(\frac{\dot{R}}{R}\right)^2=\frac{8\pi G\rho_V}{3}-\frac{\omega G^2M_g^2c^2}{8.3.4.\pi R^6}=a-\frac{b}{R^6} \qquad (37)$$

where

$$a\equiv\frac{8\pi\rho_V}{3},b\equiv\frac{G^2M_g^2\omega c^2}{96\pi} \qquad (38)$$

we obtain the solution

$$R=Be^{\lambda t} \qquad (39)$$

where $B\equiv(a/b)^{1/9}.2^{-1/3}$, $\lambda\equiv1/(3A)$ (40)

We see that this model also give an inflation solution in the vacuum – dominated age like GTR but the expansive rate is different due to the constants B and λ in (39) are different from ones in GRT.

3.2 The radiation – dominated age

we substitute $\rho_g\rightarrow\rho_R=a.\frac{1}{R^4},\Lambda=0$ (41)

where a is a constant , it does not depend on the time and space.

Substituting (41) into (33) , we have

$$3\frac{\dot{R}^2}{R^2} = 8\pi G\rho_R - \frac{\omega E_g^2 c^2}{8\pi R^2} \qquad (42)$$

Or $\dot{R}^2 = \frac{A}{R^2} - \frac{B}{R^4}$ (43)

Where $A \equiv \frac{8\pi Ga}{3}, B \equiv \frac{\omega G^2 M_g^2}{96\pi}$ (44)

The solution of (43) is

$$R = Ct^{1/2} \qquad (45)$$

where $C = \sqrt{2}A^{1/4}$ (46)

We see that this model also give the expansive form like GTR in the radiation – dominated age.

3.3 The matter – dominated age

we substitute

$$\rho_g \to \rho_M = \frac{M_g}{\frac{4}{3}\pi R^3} = \frac{3M_g}{4\pi R^3} \qquad (47)$$

Substituting (47) into (33), we have

or

$$\dot{R}^2 = \frac{D}{R} - \frac{E}{R^4} \qquad (48)$$

Where $D \equiv 2GM_g$, $E \equiv \frac{\omega G^2 M_g^2 c^2}{96}$ (49)

The solution of (48) is

$$R = F.t^{2/3} \qquad (50)$$

Where $F \equiv \sqrt[3]{\frac{9}{4}D}$. (51)

Thus, this model also give the expansive form like GTR in the matter – dominated age.

4. CONCLUSION

In conclusion, based on the Vector model for gravitational field, we have

deduced the modified equations of Friedman and have studied the evolution of the Universe in this model. It showed that the evolution of the Universe in this model is the same with one in General theory of relativity but the rate is different in the vacuum age.

MỘT MÔ HÌNH VŨ TRỤ KHÔNG DỪNG TRONG MÔ HÌNH VÉCTƠ CHO TRƯỜNG HẤP DẪN

Võ Văn Ớn

Trường Đại học Thủ Dầu Một

TÓM TẮT: *Trong bài báo này, dựa trên Mô hình véctơ cho trường hấp dẫn chúng tôi thu được các phương trình Friedman cải tiến, nó tương tự với các phương trình Friedman cổ điển nhưng được bổ xung thêm một số hạng chứa tenxơ năng – xung lượng của trường hấp dẫn. Mô hình vũ trụ không dừng*

trong mô hình này cũng tương tự với mô hình vũ trụ không dừng trong lý thuyết Einstein nhưng tốc độ giãn nở của vũ trụ trong giai đoạn vacuum lại khác.

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