MAXIMUM- INSCRIBED AND MINIMUM- CIRCUMSCRIBED FITTING FOR CO-ORDINATE MEASURING MACHINE

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ABSTRACT: *This paper describes algorithms that fit geometric shapes to data sets according to maximum- inscribed (MI) and minimum- circumscribed (MC) fit. We use these fits to build the CMM's (Coordinate Measuring Machine) software in cases of circle, sphere and cylinder. For each case, we obtain the fit by two methods: first, by (relative easy) least squares fit method and then refine by MI and MC fit method. Although, the later method is substantially more complicated than the former one, Its results are used to make comparision with the the results of least squares method in order to give more options in the CMM software. In the near future we will continue to develop MI and MC fit with an effective algorithm- Simulated Annealing algorithm.*

1. INTRODUCTION

This paper summarizes works carried out in Coordinate Measuring Machine Manufacturing project. The aim of our project was to introduce the reference algorithm to solve the fitting problem. There are four fitting algorithms which are commonly used in CMM are Least Squares Fit, Minimum Circumscribed Fit, Maximum Inscribed Fit and Chebyshev Fit (Minimum Zone Fit). Table 1 describes the applicability to basic geometric shapes.

Table 1. Applicability to geometric shapes.

	Least-Squares	Min-Zone	Min-Circumscribed	Max-Inscribed
Line	X	X		
Plane	X	X		
Circle	X	X	X	X
Sphere	X	X	X	X
Cylinder	X	X	X	X
Cone	X	X		

Coordinate measuring machine (CMM) software has long used least-squares fit calculations to construct resolved geometry representations of measured features. The problem, however, is that the ASME Y14.5M-1994 and the ISO 1101 standards on dimensions and tolerances never specify the use of a least-squares fit. Besides, the size of features such as circles, cylinders, or spheres require a mating envelope (Maximum Inscribed or Minimum Circumscribed) fit for tolerances of location, orientation, and dimension, when used as a datum feature. The geometric forms considered in this paper are circle, sphere and cylinder. Minimum Circumscribed Fit (or Maximum Inscribed Fit) was applied to calculate the dimension tolerance and the position tolerance of outer (or inner) features.

2. MINIMUM CIRCUMSCRIBE FIT

Minimum circumscribe fit defines a geometric object as small as possible, while containing all measured points of the actual feature. The minimum circumscribed fit is used to calculate the size, orientation tolerance and position tolerance of outer features, such as a shaft, because it measures the smallest mating part that the feature fits inside. The least- squares fit is also used in the case of outer features. However, it is not mean for industry or manufacturing. It has to make sure that the minimum circumscribed fit is not used on partial arcs of a circle or on partial hemispheres of a sphere because the result does not represent the surface of the feature.

MINIMUM CIRCUMSCRIBED FIT FOR CIRCLE

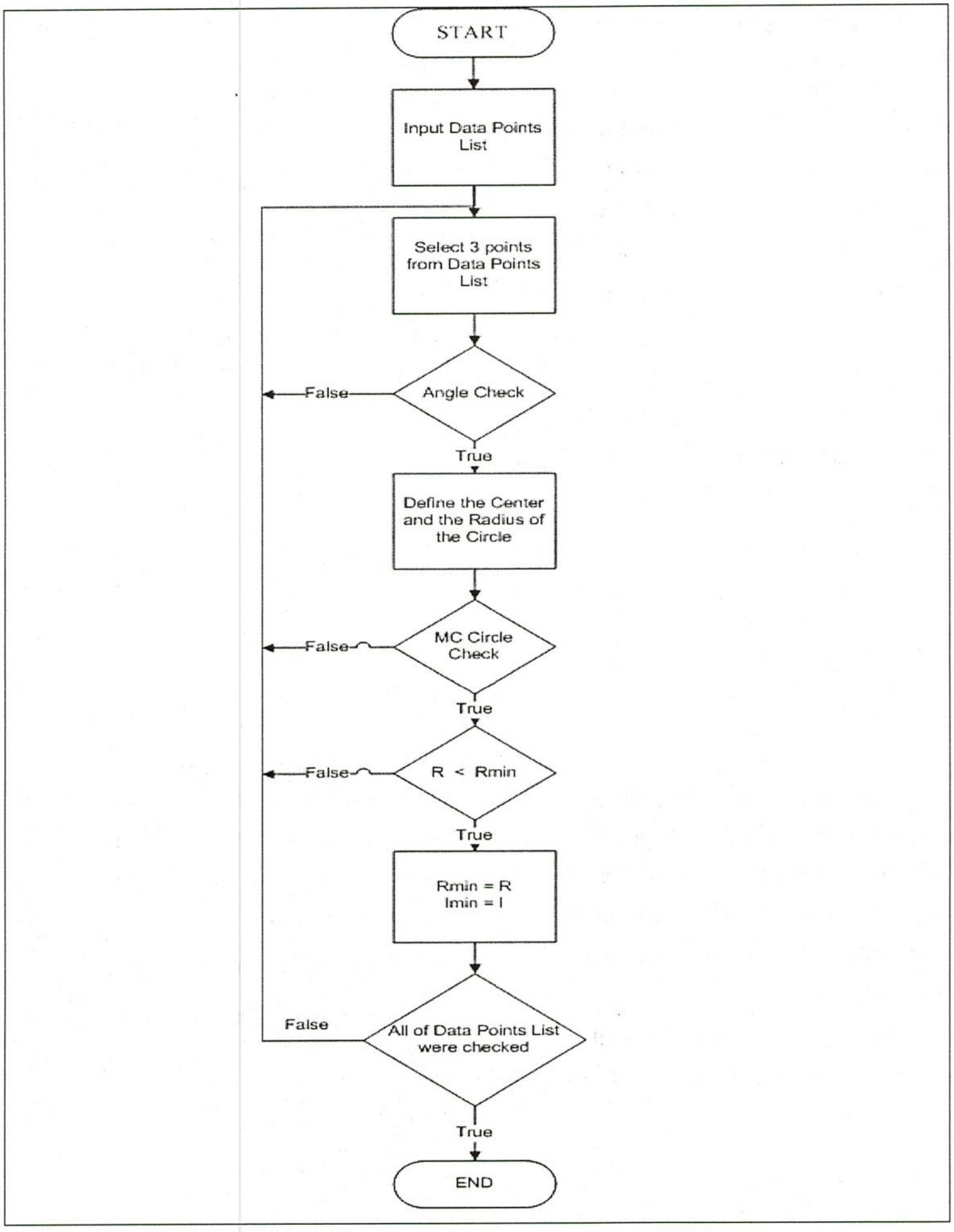
START
Input Data Points List
Select 3 points from Data Points List
Angle Check
False
True
Define the Center and the Radius of the Circle
MC Circle Check
False
True
R < Rmin
False
True
Rmin = R
Imin = I
All of Data Points List were checked
False
True
END

Fig 1.

2.1. Minimum Circumscribed Circle

In this paper, we introduce the use of minimum circumscribed fit to calculate the size (radius) and the position (center) of the 2D circle. The minimum circumscribed circle satisfies the following three conditions:

1) No data point lies outside the circle.
2) The circle touches three data points which form an acute or right triangle.
3) No circle of greater radius satisfies condition (1) and (2).

In the MC Circle Flow Chart (Fig 1), the Angle Check function is called to guarantee 3 points selected place in a whole circle. We will obtain more than one circle without the Angle Check function. Then, the MC Circle Check will define if there are any point place outside the circle which has just been built. We can imagine minimum circumscribed circle as in figure 2.

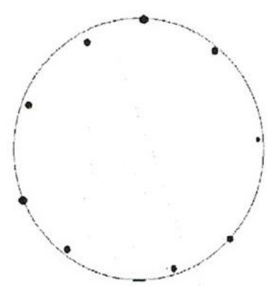

Fig. 2

The minimum circumscribed circle cannot be used directly for 3D data point list. With 3D data point list, we use least- squares fit to define a plane before using minimum circumscribed fit to find the minimum circumscribed circle.

A data point list might have more than one minimum circumscribed circle which has the same radius but different center position. In this case, the minimum circumscribed will return the position of the center of the circle which was obtained first.

2.2. Minimum Circumscribed Sphere

MINIMUM CIRCUMSCRIBED FIT FOR SPHERE

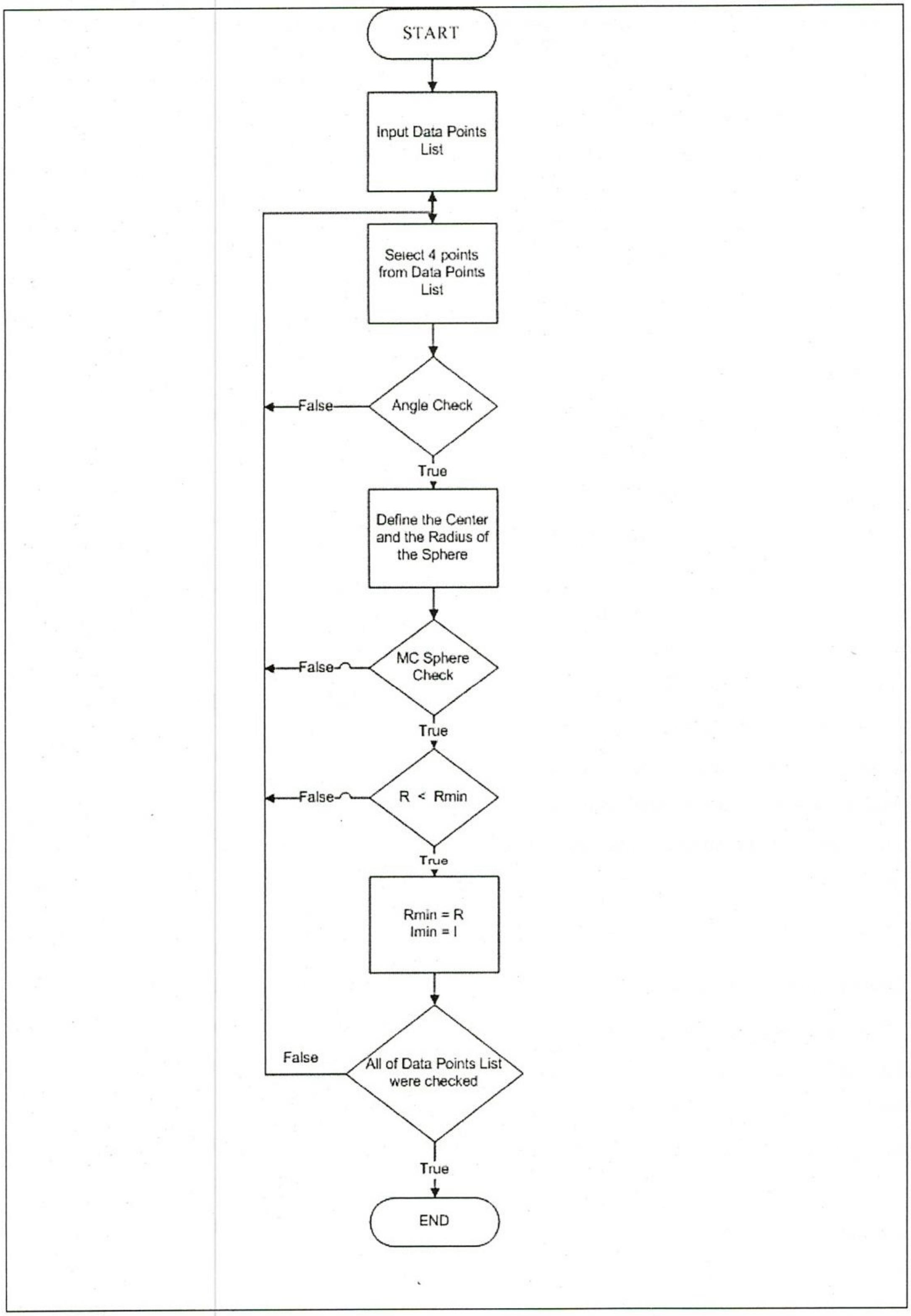
START
Input Data Points List
Select 4 points from Data Points List
Angle Check
False
True
Define the Center and the Radius of the Sphere
MC Sphere Check
False
True
R < Rmin
False
True
Rmin = R
Imin = I
All of Data Points List were checked
False
True
END

Fig. 3

The minimum circumscribed sphere satisfies the following three conditions:

1) No data point lies outside the sphere.
2) The sphere touches four data points where any three of which construct an acute or right triangle.
3) No sphere of greater radius satisfies condition (1) and (2).

Firstly, we select four data points from the data point list. Then, we build 4 triangles with 4 data points which have been selected. If all of triangle are acute or right triangle, the Angle Check function returns true. We obtain the solution of the system of equation (4) and define the center and the radius of the sphere. However, solving this system of equation can be reached without computer supporting. In this case, we use Maple to obtain the result. The formula of the result which were solved by Maple will be used in "define the center and the radius of the sphere" function of the software.

$$(x_i - x_o)^2 + (y_i - y_o)^2 + (z_i - z_o)^2 = R^2 \qquad (4)$$

With i varies from 1 to 4.

The "MC Sphere Check" function will return true if there is no data point lies outside the sphere. The minimum circumscribed sphere is only updated if $R<R_{min}$.

In some cases, there are more than one sphere satisfy all of conditions and have the same radius. However, minimum circumscribed only returns the first sphere obtained.

2.3. Minimum Circumscribed Cylinder

Conditions of the minimum circumscribed cylinder are similar to conditions of minimum circumscribed circle and minimum circumscribed sphere. However, defining the axis and the radius of the cylinder is a serious problem. We cannot reach the solution of this equations system (5) by popular computing software as MATLAB or Maple.

$$R=\sqrt{[(y_i-y)c-(z_i-z)b]^2+[(x_i-x)c-(z_i-z)a]^2+[(x_i-x)b-(y_i-y)a]^2} \qquad (5)$$

With i varies from 1 to 7.

We find the axis of the cylinder by least-squares fit. Then, we use the "no data point placed outer condition" to find the radius of the cylinder.

3. MAXIMUM INSCRIBED FIT

The maximum inscribed fit expands the feature of perfect form as large as possible, while the feature places inside the measured point of the actual feature. The maximum inscribed fit is used to calculate the size, orientation tolerance and position tolerance of an inner feature, such as a hole, because it measures the largest mating part that fits inside the feature. The maximum inscribed fit should not be used on partial arcs of a circle or on partial hemispheres of a sphere, because the feature expands to infinity.

The calculation of maximum inscribed fit is similar to minimum circumscribed fit. It has only different from minimum circumscribed in the "check function". The "MC check function" is no data point lies outside the feature. In the opposite, the "MI check" is no data point lies inside the feature.

3. EVALUATION

To describe the result of minimum circumscribed fit and maximum inscribed fit we use an example with 6 data points which are listed in table 2.

Table 2

Data Point	x	y
1	2	0
2	1	1.1
3	0	0.1
4	1.002	-1
5	1.72	0.82
6	0.28	-0.82

The results which are returned by the software are recorded in table 3.

Table 3

	The Position of Center		Radius
	x	y	
Minimum Circumscribed Fit	0.990061	0.00872727	1.09132
Maximum Inscribed Fit	0.997619	0.00237451	1.00238
Least- Squares Fit	0.98232	0.034328	1.047713

The software also returns the image of three circle as Figure 4

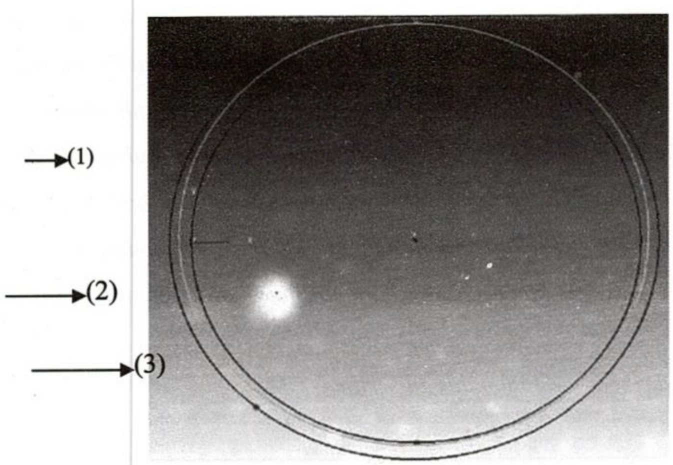

Fig 4.

In fig 6, circle (2) represents least- squares circle, circle (1) represents maximum inscribed circle, circle (3) represents minimum circumscribed circle.

4. CONCLUSION

We have successfully applied minimum circumscribed fit and maximum inscribed fit for circle, sphere and cylinder in the software of coordinate measuring machine (CMM) which we are manufacturing. Generally, the computation of circle or sphere has many advantages. However, the computation of cylinder rised a serious problem. Besides, these algorithm which has $O(n^3)$ (n is number of data points) complexity is only suitable for data point list which does not have too many points (less than 300 points). In the case when data point list has too many points, we recommend using the optimal method to simplify the problem. Furthermore, we will apply simulated annealing algorithm and Chebyshev fit method for our software.

NỘI SUY NỘI TIẾP LỚN NHẤT VÀ NGOẠI TIẾP BÉ NHẤT TRÊN MÁY ĐO TỌA ĐỘ

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TÓM TẮT: *Bài báo trình bày giải thuật nội suy theo phương pháp nội tiếp lớn nhất(MI) và ngoại tếp bé nhất(MC). Chúng tôi dùng các phương pháp nội suy này để xây dựng phần mềm cho CMM(máy đo tọa độ) trong trường hợp của đường tròn, mặt cầu và mặt trụ.Trong mỗi trường hợp, chúng tôi thực hiện bằng hai phương pháp: đầu tiên dùng phương pháp bình phương cực tiểu(least squares), sau đó dùngMI hoặc MC. Mặc dù việc tính toán bằng MI và MC phức tạp hơn so với bình phương cự tiểu nhưng chúng tôi dùng kết quả này để so sánh với kết quả có được từ phương pháp bình phương cực tiểu đồng thời đưa ra nhiều lựa chọn cho người dùng trong phần mềm của chúng tôi. Chúng tôi cũng đang cải tiến MI và MC bằng cách kết hợp với thuật toán Simulated Annealing để nâng cao hiệu quả tính toán.*

Từ khóa: *maximum inscribed, minimum circumscribed, CMM.*

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