

A CALCULATION FOR COMPENSATING THE ERRORS DUE TO SPRINGBACK WHEN FORMING METAL SHEET BY SINGLE POINT INCREMENTAL FORMING (SPIF)

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ABSTRACT: *The question of compensating for the error of dimension due to springback phenomenon when forming metal sheet by SPIF method is being one of the challenges that the researchers of SPIF in the world trying to solve. This paper is only a recommendation that is based on the macro analysis of a sheet metal forming model when machining by SPIF method for calculating a reasonable recompensated feeding that almost all researchers have not been interested in yet:*

- Considering the metal sheet workpiece is elasto-plastic and the sphere tool tip is elastic, the authors attempt to calculate for compensating the error of dimension due to elastic deforming of the tool tip

- The metal sheet is clamped by a cantilever joint that has an evident sinking at the machining area that is also calculated to add to the compensating feeding value. The paper also studies the limited force for ensuring the elastic deforming at these working area of the sheet to eliminate all the unexpected plastic deforming of the sheet.

With two small but novel contributions, this study can help to take theoretical model for elastic forming of metal sheet closer to real situation.

Keywords: *SPIF method, sphere tool tip,*

1. INTRODUCTION

The deformation of manufacturing installations is an unavioded phenomenon in almost all pressing machines. In this technology, on one hand, we attempt to progress the plastic deformation of the workpiece as much as possible. On the other hand we have to restrict one of the manufacturing installations such as machine,

spindle, tools, clamping installations... to the minimum with in the purpose of increasing the accuracy of the products.

Especially in the Single Point Incremental Forming method, a recent technology of metal sheet forming, the unexpected deformation of the product after forming (The Springback phenomenon) is a critical question that the researchers in SPIF field are interesting.

The goal of this paper is to describe the analyzing calculation for providing the compensative feeding rate for remedying the damaging effects of the deformations of workpiece (metal sheet) and increasing the accuracy of the dimensions of the products.

In an acceptable hypothesis of the absolute rigidity of the spindle, carriage, the paper only concentrates in the calculation for compensation the deformation of the secondary installations for CNC milling machine when forming metal sheet in SPIF technology.

The compensative values are composed:

- Elastic deformations of the tangent surface of the punch and the metal sheet.
- Elastic deformations of the volume of the cantilever part of the punch.
- Elastic deformations of the clamping installation.
- Elastic deformations due to the elastic sinking of the sheet.

2. CALCULATING TOTAL COMPENSATION

2.1. Elastic deformations of the punch when machining

In figure 1, we can see the sphere tip punch that is mounted in the spindle of a CNC milling machine. To consider the absolute rigidity of the spindle and the carriage machine, their deformations, if exist, are infinitesimal, the deformation of the punch can be divided in 3 sections:

- Section 1: the deformation of the sphere surface of the tangent area (y_1) is equal to the depth t of feeding rate.
- Section 2: a part of sphere area (y_2) of the length of $D/2-t$ that has a variable section.
- Section 3: the tail of the punch to the clamping area of length (y_3)

$$\text{Hence } \varepsilon = \ln \frac{l}{l_0} = \ln \left(\frac{D(\varphi_{Max} - \varphi)}{D\sqrt{Dt - t^2} - D\sin\varphi} \right)$$

(1)

-At point A (φ_{max}) the strain $\varepsilon_A=0$

-At top C' of the punch ($\varphi=0$) the strain is $\varepsilon_{C'}$

$$\varepsilon_{C'} = \varepsilon_{max} = \ln \left(\frac{AC'}{AC} \right) = \ln \left(\frac{\frac{D}{2} \varphi_{Max}}{\sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{D}{2} - t\right)^2}} \right) = \ln \left(\frac{D \arccos\left(\frac{D - 2t}{D}\right)}{2\sqrt{Dt - t^2}} \right)$$

Since the elastic deformation is calculated by (1) we can apply Ludwid 's formula for calculating the elastic stress at an arbitrary tangent angle φ on the sphere section of the sheet.

$$\sigma = k\varepsilon^n = k \cdot \ln \left(\frac{D(\varphi_{Max} - \varphi)}{D\sqrt{Dt - t^2} - D\sin\varphi} \right)^n \quad (2)$$

$$\rightarrow \ln\sigma = \ln(k \cdot \varepsilon^n)$$

$$\ln k + n \cdot \ln(\varepsilon) = \ln(k) + n \cdot \ln \left[\ln \left(\frac{D(\varphi_{Max} - \varphi)}{D\sqrt{Dt - t^2} - D\sin\varphi} \right)^n \right]$$

Formula (2) describes the elastic stress at an arbitrary point in arbitrary tangent area of sheet and punch. It has the same direction of strain. This means it has tangent direction with the sphere at an arbitrary line that makes an angle φ (Figure 2) with the axe of the punch. We can consider it the normal elastic stress in the tangent direction σ^T

$$\sigma_T = k \cdot \ln \left(\frac{D(\varphi_{Max} - \varphi)}{D\sqrt{Dt - t^2} - D\sin\varphi} \right)^n$$

The stress of the circumference direction $\sigma_T=0$ due to the non deformation on circumference.

Let's consider an infinitesimal cube volume in the tangent area in figure 2. According to Von Mises critical, we write down 3 main orthogonal stresses of the cube. From [7] we can find out the relationship among the main stresses:

$$\sigma_S = Y = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

with $\sigma_1 = \sigma_T$,

$$\sigma_2 = \sigma_R, \sigma_3 = \sigma_V = 0, \sigma_5 = Y = \frac{1}{\sqrt{2}} \sqrt{(\sigma_T - \sigma_R)^2 + \sigma_R^2 + \sigma_T^2} = \sqrt{\sigma_R^2 + \sigma_T^2 - \sigma_T \sigma_R}$$

$$\sigma_R^2 - \sigma_R \sigma_T + \sigma_T^2 - Y^2 = 0$$

$$\text{Condition } \Delta = \sigma_T^2 - 4(\sigma_T^2 - Y^2) = 4Y^2 - 3\sigma_T^2 \geq 0 \Rightarrow \sigma_T \leq \frac{2}{\sqrt{3}} Y$$

$$\sigma_R = \frac{\sigma_T \pm \sqrt{4Y^2 - 3\sigma_T^2}}{2}$$

With the condition of the positive of σ_R , we can eliminate the negative value:

$$\sigma_R = \frac{\sigma_T + \sqrt{4Y^2 - 3\sigma_T^2}}{2} \quad (4)$$

Replace (3) into (4) we have the normal stress on the sheet surface and with the law of Newton III it is also the normal stress on the spherical surface of the punch.

$$\sigma_R = \frac{k \cdot \ln \left(\frac{D(\varphi_{\text{Max}} - \varphi)}{2\sqrt{Dt - t^2} - D \sin \varphi} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D(\varphi_{\text{Max}} - \varphi)}{2\sqrt{Dt - t^2} - D \sin \varphi} \right)^{2n}}}{2} \quad (5)$$

Select "+" sign and interest in the worst case that is the maximum stress: it appears at the top C' of the punch ($\varphi=0$)

$$\sigma_{\text{Max}} = \frac{k \cdot \ln \left(\frac{D\varphi_{\text{Max}}}{2\sqrt{Dt - t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D\varphi_{\text{Max}}}{2\sqrt{Dt - t^2}} \right)^{2n}}}{2}$$

$$\text{In figure 2 } \varphi_{\text{Max}} = \frac{D - 2t}{D}$$

$$\text{Hence } \sigma_{\text{Max}} = \frac{k \cdot \ln \left(\frac{D - 2t}{2\sqrt{Dt - t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D - 2t}{2\sqrt{Dt - t^2}} \right)^{2n}}}{2} \quad (6)$$

The tangent strain is $\varepsilon = \sigma_R / E_p$, where E_p is Young's modulus of the punch

$$\varepsilon = \frac{k \cdot \ln \left(\frac{D(\varphi_{\text{Max}} - \varphi)}{2\sqrt{Dt - t^2} - D \sin \varphi} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D(\varphi_{\text{Max}} - \varphi)}{2\sqrt{Dt - t^2} - D \sin \varphi} \right)^{2n}}}{2E_p}$$

From (6) we can calculate the maximum strain at the top of the punch (at $\varphi=0$)

$$\varepsilon_{Max} = \frac{k \cdot \ln\left(\frac{D-2t}{2\sqrt{Dt-t^2}}\right)^n + \sqrt{4Y^2 - 3k^2 \ln\left(\frac{D-2t}{2\sqrt{Dt-t^2}}\right)^{2n}}}{2E_p}$$

The tangent depth is t (Figure 2), we can calculate the displacement of the shorted dimension at tangent area $y_1=t \cdot \varepsilon_{Max}$:

$$y_1 = t \cdot \frac{k \cdot \ln\left(\frac{D-2t}{2\sqrt{Dt-t^2}}\right)^n + \sqrt{4Y^2 - 3k^2 \ln\left(\frac{D-2t}{2\sqrt{Dt-t^2}}\right)^{2n}}}{2E_p} \quad (7)$$

2.1.2. Elastic deformation of the volume of the cantilever part of the punch y_3 :

By the cantilever clamped section, this part of the punch is also pressed.

With its diameter D and the length L of the punch the pressed deformation is calculated as:

$$y_3 = \frac{\sigma_{ZMax}}{E_p} = \frac{P_{ZMax} \cdot L}{A \cdot E_p}$$

Axial force P_Z is calculated in the downward feeding rate :

$$P_Z = \int_0^{\beta_{Max}} \sigma_R \cdot \cos \beta \cdot ds = \int_{r=0}^{r=\sqrt{Dt-t^2}} \sigma_R \cdot \cos \beta \cdot ds = \int_0^{\sqrt{Dt-t^2}} \sigma_R \cdot \cos \beta \cdot 2\pi r dr$$

Calculate its maximum value when σ_R reaches its critical value in (6)

$$\cos \beta = \sqrt{1 - 4 \frac{r^2}{D^2}}$$

$$P_{ZMax} = -\frac{2\pi D^2}{4.3} \sigma_{Max} \cdot \left[\left(1 - 4 \frac{r^2}{D^2}\right)^3 \right]_0^{\sqrt{Dt-t^2}} = \frac{\pi D^2}{6} \sigma_{Max} \left[1 - \left(1 - 4 \frac{Dt-t^2}{D^2}\right)^3 \right]$$

$$P_{ZM} = 2\pi \int_0^{\sqrt{Dt-t^2}} \sigma_{Max} \cdot \sqrt{1 - 4 \frac{r^2}{D^2}} \cdot r dr = -\frac{\pi D^2}{4} \sigma_{Max} \cdot \int_0^{\sqrt{Dt-t^2}} \sqrt{1 - 4 \frac{r^2}{D^2}} d\left(1 - 4 \frac{r^2}{D^2}\right)$$

$$P_{ZMax} = \frac{\pi D^2}{6} \sigma_{Max} \left\{ 1 - \left[\left(\frac{2t}{D}\right)^2 - 2\left(\frac{t}{D}\right) + 1 \right]^3 \right\} = \frac{\pi D^2}{6} \sigma_{Max} \left[1 - \left(1 - \frac{2t}{D}\right)^6 \right]$$

$$P_{ZMax} = \frac{\pi D^2}{6} \sigma_{Max} \left[1 - \left(1 - \frac{2t}{D}\right)^3 \right]$$

Replace (6) into:

$$P_{ZMax} = \frac{\pi D^2}{12} \left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left[1 + \left(\frac{2t}{D} - 1 \right)^3 \right] \quad (8)$$

The shorted pressed displacement y_1 in Z direction [7] is:

$$y_3 = \frac{P_{Zmax} L}{AE_p} = \frac{\frac{\pi D^2}{12} \left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left[1 - \left(1 - \frac{2t}{D} \right)^3 \right]}{\frac{\pi D^2}{4} E_p}$$

$$y_3 = \frac{\left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left[1 + \left(\frac{2t}{D} - 1 \right)^3 \right] L}{3E_p} \quad (9)$$

2.1.3. Calculating the strain y_2 on the surface of section 2 (the area that is not contacted to the sheet).

From the figure 2, equation of the profile $x^2 + y^2 = \frac{D^2}{4}$

The horizontal radius in tangent area changes in $[-(D/2-t), 0]$

$$r^2 = x^2 = \frac{D^2}{4} - y^2$$

$$\text{Area of this section } \left[A_y = \pi r^2 = \pi \left(\frac{D^2}{4} - y^2 \right) \right]$$

Dis-placement du in differential axial dy :

$$du = \frac{P_{ZMax} dy}{E_p A_y}$$

Total displacement is:

$$y_2 = \int_{-\frac{D}{2}-t}^0 du = \int_{-\frac{D}{2}-t}^0 \frac{P_{ZMax} dy}{E_p A_y} = \frac{P_{ZMax}}{\pi E_p} \int_{-\frac{D}{2}-t}^0 \frac{dy}{\left(\frac{D^2}{4} - y^2\right)}$$

$$y_2 = \frac{P_{ZMax}}{\pi E_p} \int_{-\frac{D}{2}-t}^0 \frac{dy}{\left(\frac{D}{2} - y\right)\left(\frac{D}{2} + y\right)} = \frac{P_{ZMax}}{\pi D E_p} \int_{-\frac{D}{2}-t}^0 \left(\frac{1}{\left(\frac{D}{2} - y\right)} + \frac{1}{\left(\frac{D}{2} + y\right)} \right) dy$$

$$y_2 = \frac{P_{ZMax}}{\pi D E_p} \left[\ln \frac{\frac{D}{2} + y}{\frac{D}{2} - y} \right]_{-\frac{D}{2}-t}^0 = \frac{P_{ZMax}}{\pi D E_p} \left(0 - \frac{-t}{D-t} \right) = \frac{t \cdot P_{ZMax}}{\pi D (D-t) E_p}$$

Replace (8) into we have the displacement of spherical area y_2 is :

$$y_2 = \frac{t \cdot D \left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left[1 + \left(\frac{2t}{D} - 1 \right)^3 \right]}{12(D-t)E_p} \quad (10)$$

2.1.4. Total strain due by the elastic of the punch $y_p = y_1 + y_2 + y_3$

From (7), (9) and (10) we can calculate the total strain of the punch:

$$y_p = \frac{\left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left[1 + \left(\frac{2t}{D} - 1 \right)^3 \right] L}{3E_p}$$

$$+ \frac{t \cdot D \left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left[1 + \left(\frac{2t}{D} - 1 \right)^3 \right]}{12(D-t)E_p} \quad (11)$$

$$+ t \cdot \frac{k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}}}{2E_p}$$

2.2. Deformation generated by the sinking of the sheet when forming:

The maximum axial resultant P_{Zmax} can cause the sinking of the sheet. Let's observe figure 3 with the simple clamping plate (round in general case) but the shape of the sheet is

more complex. L_{Max} is the maximum distance from the gutter of the clamping plate to the minimum radius of the sheet. The sinking is extracted from the result 8-4 of [6]

$$as \ y_t = \frac{P_{Max} \cdot A_t \cdot L_{Max}^3}{8E_t I_t}$$

Replace (8) into it:

$$y_t = \frac{\pi D^2 \left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left[1 - \left(1 - \frac{2t}{D} \right)^3 \right] \cdot A_r \cdot L_{Max}^3}{96 E_r I_r} \quad (12)$$

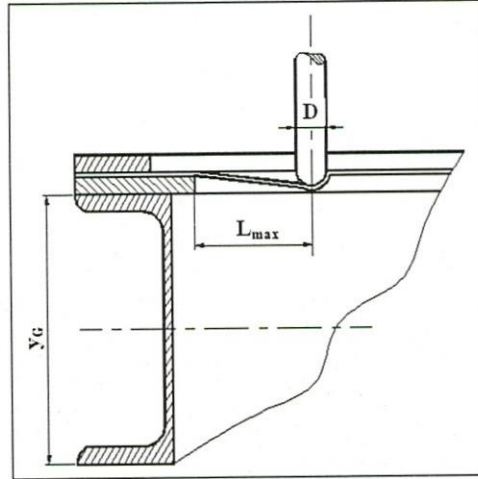


Figure 3: The sinking of the clamping plate and the rigidity of the carriage of the machine

2.3. The sinking due by the flexibility of the clamping plate y_G :

In figure 3 we can see the pressed part of the clamping plate y_G :

- The down clamping plate that is restricted by the square boundary with its side a and the diameter ϕ of upward clamping plate with a round hole inside (in the experimental condition $a=310$ and $\phi=250$)

- The foundation (Figure3) is composed of 2 C section steel bar. Name A_G is its section ($A_G= 5*310=1550\text{mm}^2$) and l_G is its height ($l_G=200\text{mm}$)

E_G is the Young's modulus of the clamping plates, we can calculate it as the following value:

$$y_G = \frac{P_{ZMax} L_G}{A_G E_G}$$

$$y_G = \frac{\pi D^2 \left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left[1 + \left(\frac{2t}{D} - 1 \right)^3 \right] J_G}{24 A_G \cdot E_G} \quad (13)$$

2.4. Total compensation:

Addition all the values in (11), (12), and (13) we get the total compensation:

$$y_{\Sigma} = y_P + y_T + y_G$$

$$y_{\Sigma} = \left[k \cdot \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^n + \sqrt{4Y^2 - 3k^2 \ln \left(\frac{D-2t}{2\sqrt{Dt-t^2}} \right)^{2n}} \right] \left\{ 1 + \left(\frac{2t}{D} - 1 \right)^3 \left[\frac{L}{3E_P} + \frac{t \cdot D}{12(D-t)E_P} + \frac{\pi D \cdot A_t \cdot L_{Max}^3}{96E_t I_t} + \frac{\pi D^2 \cdot I_G}{24A_G \cdot E_G} \right] + \frac{t}{2E_P} \right\} \quad (14)$$

3. CONCLUSION

By mean of analyzing, the paper could provides the total compensation due by elastic deformations of the punch, sheet, and clamping installations. In the experiment with material such as aluminum A 1050 H14, the concrete parameters such as $D=10\text{mm}$, $t=3\text{mm}$, $L=70\text{mm}$... with the application of equation

(14) we can get the total compensation value $y_{\Sigma}=2,73945\text{mm}$. It is a too big value that shows us the importance of springback after forming which could interfere to the errors of dimensions. In fact, all calculations that are described in this paper will be used for compensation in practice by the interfere into the specific software Pro/Engineer in the future.

TÍNH TOÁN BÙ TRỪ HIỆN TƯỢNG CO GIÃN KÍCH THƯỚC KHI TẠO HÌNH TẮM BẰNG PHƯƠNG PHÁP SPIF

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TÓM TẮT: Vấn đề bù trừ sai số kích thước thành phẩm gây ra do hiện tượng co giãn (Springback) sau khi tạo hình tấm kim loại bằng phương pháp SPIF (Single Point Incremental Forming) hiện đang là một trong những thách thức mà các nhà nghiên cứu công nghệ SPIF trên thế giới đang quan tâm và tìm cách giải quyết [1]. Bài báo này chỉ là một đề nghị nhỏ dựa trên phân tích giải tích vĩ mô mô hình gia công biến dạng dẻo tấm bằng phương pháp SPIF để đưa ra lượng bù dao hợp lý mà các nghiên cứu hiện nay chưa quan tâm đến:

- Xem phiôi tấm chịu biến dạng đàn dẻo còn chày có đầu hình cầu có biến dạng đàn hồi nhằm bù trừ cho biến dạng đàn hồi của chày.

- Tấm được kẹp chặt với liên kết ngàm có độ võng tại nơi chày ép tạo hình cũng được tính toán để đưa vào lượng bù trừ đồng thời bài viết cũng tính toán giới hạn lực tạo hình do các thông số gia công sao cho vùng lún của tấm còn nằm trong giới hạn đàn hồi và phục hồi trở lại sau khi tháo lực nhằm triệt tiêu sai số hình dáng phụ do hiện tượng dè không mong muốn.

Với 2 đóng góp nhỏ bé nhưng mới mẻ trên, bài toán lý thuyết dè trong tạo hình tấm được tiến gần hơn nữa với mô hình thật của một công nghệ gia công tấm hiện còn rất mới tại nước ta.

Từ khóa: phương pháp SPIF, tạo hình tấm

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