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THE IMPACT OF UNCERTAINTY ON SUBSIDENCE AND COMPACTION PREDICTION

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ABSTRACT: *This paper presents the stochastic approach using Monte Carlo simulation as applied to compaction and subsidence estimation in an offshore oil and gas deep-water field in the Gulf of Mexico. The results reveal both the impact of using probability distributions to estimate compaction and subsidence for a disk shaped-homogenous reservoir as well as taking into account Young's modulus, Poisson's ratio and the reduction of pore fluid pressure.*

The uncertainty reservoir model is also compared with numerical simulation using commercial software – Eclipse 300. The stochastic-based simulation results confirm that the deterministic results obtained from the coupled geomechanical – fluid flow model are in the range of acceptable distribution for stochastic simulation. The sensitive analysis shown that Young's modulus has more impact on compaction than Poisson's ratio. The results also presented that values of Young's modulus in this deep-water field in Gulf of Mexico lying beyond 140,000psi are insignificant to compaction and subsidence. Based on output results of compaction and subsidence with the stochastic model, potential reservoirs presenting subsidence and compaction are described as an uncertainty range within distribution of Young's modulus, Poisson's ratio and the reduction of pore fluid pressure in large-scale regional model.

Keywords: *Risk Analysis, Subsidence, Compaction, Monte Carlo simulation and Uncertainty*

1. INTRODUCTION OF SUBSIDENCE, COMPACTION AND OBJECTIVES

Sub-surface compaction due to fluid withdrawal from a reservoir (oil, gas or water) has been well documented worldwide over the last few decades. Compaction of a reservoir can also lead to subsidence at the ground surface or seafloor. Examples have been observed in Venezuela [1], the Gulf of Mexico [2, 3] and Gippsland Basin [4]. In the Cooper Basin – Australia, this problem was first investigated by Ta & Hunt [5]. The compacting reservoir can enhance oil and gas production but it can cause excessive stress at the well casing and within the completion zone where collapse of structural integrity could leads to failure and lost production. In addition, surface subsidence also results in problems at the wellhead within pipeline system and platform foundations.

The need for more sophisticated prediction approaches in assessing the impact of subsidence and compaction on production management of the reservoir has led to a continuous improvement of numerical models employed an approach in using the continuum poroelastic theory. For example, the use of advanced models for accurate prediction of land subsidence were documented [6, 7]. However, although sophisticated poroelastic constitutive models have been developed for a realistic description of the actual rock mass behavior [8, 9, 10], the geomechanical analysis of producing fields is usually performed deterministically so that solutions to poroelastic equations are subsequently limited. To overcome the limitation of the deterministic approach which would require an extensive medium characterization, neither supported by the available data nor allowed by the available resources, the properties of rock heterogeneity at the field and regional scale have been incorporated stochastically into geostatistical models [11, 12]. While geostatistical models have been extensively used over the last few decades for modeling flow and transport into random porous media, only a limited number of works have addressed the influence of using stochastic approaches to assess the effect of rock properties on geomechanical behaviors of reservoir [13]. In particular, there are few studies that have been incorporated a stochastic-based approach when analyzing rock heterogeneous as applied to compaction and subsidence problem. In addition, some of the most important parameters controlling the compaction caused by pore fluid pressure drawdown in a depleting reservoir usually have ignored magnitude variation when modeling geomechanical parameters such as Young's modulus (E), Poisson's ratio (ν), and even reduction of pore fluid pressure (Δp_f) mainly as a result of limited field data.

This paper addresses the impact on subsidence and compaction prediction when taking into account uncertainty of E, ν and ΔpB_{fB} as applied to a deep-water petroleum field in the Gulf of Mexico (i.e. location not revealed due to confidentially). The reservoir model modeled stochastically is compared with the commercial numerical software-Eclipse 300. Finally, potential reservoirs where subsidence and compaction could happen are presented in term of describing the range of E and ν within a stochastic characterization of a large-scale regional reservoir model

1.1. Stochastic Approach - Monte Carlo Simulation

In most engineering applications, the deterministic model is dominant over stochastic-based models where a single output value is obtained for every input value for all variables. The assumption made is that the input variable is known; in reality many input variables have uncertainty attached to them, hence the need to be modeled as stochastically [14]. Murtha [15] defined risk as "Potential gain or losses associated with each particular outcomes" and uncertainty as "the range of possible outcome". Risk and uncertainty estimate the input parameter as a range instead of a single point.

The area of risk analysis is designed to handle uncertain variables through stochastic models using the Monte Carlo simulation method. In this study, Monte Carlo simulation is applied for evaluation of the compacting reservoir based on the analytical geomechanical-fluid flow equation (General rock properties shown in Table 1).

Table 1. Rock and model properties

Variables	Symbol	Values	Units
Distance from reservoir centre axis	a	10000	ft
Average reservoir radius	R	5000	ft
Reservoir depth of burial	D	10000	ft
Average reservoir thickness	h	160	ft
Dimensionless radial distance	$\rho=a/R$	2	--
Dimensionless depth	$\eta=D/R$	2	--
Bessel function	$A(\rho,\eta)$	--	--
Poisson's ratio	ν	--	--
Young's modulus	E	--	psi
Biot's constant	α	0.95	--
Reduction of pore fluid pressure	Δp_f	1500	psi
Bulk coefficient (base case)	C_b	2.56E-5	psi^{-1}
Rock density	ρ_s	128	lb/ft^3

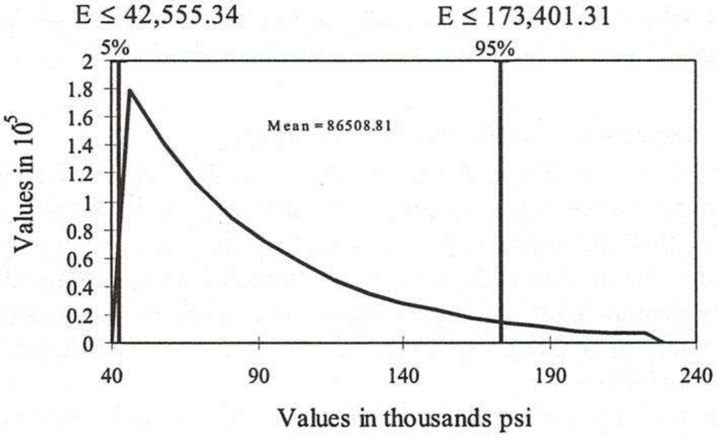
1.2. Numerical Reservoir Simulation

In this section, the coupled geomechanical–fluid flow model is used in a deep-water field in the Gulf of Mexico within using the Eclipse 300 reservoir simulator. The modeling built is simplistic and based on deterministic parameters. Theories used in calculating of compaction problems are based on the mass balance equation, Darcy's law of fluid flow, and Terzaghi's principal of effective stress [16]. Rock and fluid property constants used pertain to the Gulf of Mexico. Compaction calculations here are made along a vertical cross-section parallel through the model's center position as described further below.

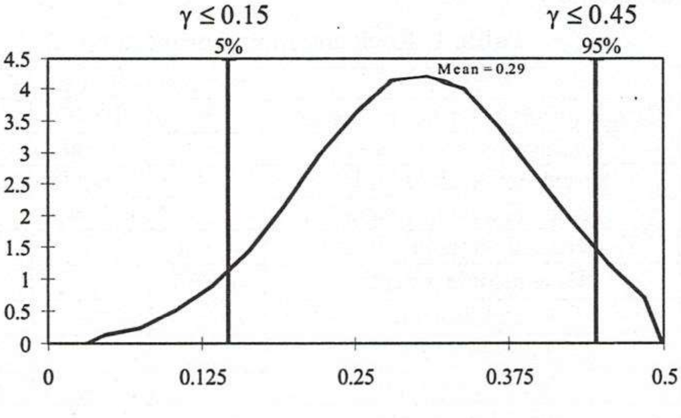
1.2.1 Reservoir Rock Properties

The reservoir itself was discretised into 8 layers. The model measures 10000 × 10000 × 160ft in the x, y and z directions that are meshed with three-dimensional cube grid with grid size 500 × 500 × 20ft in the x, y and z direction, respectively.

Geomechanical rock properties includes E, ν, Biot's constant (α) and density (ρs): these parameters describe a linearly elastic porous medium (Table 1). Here, the range of E and ν data come from two wells (Figure 1). The coupled numerical model can only be simulated with the deterministic values of parameters extracted from the distribution of E and ν parameters in which the mean and medium values are considered.



(a)



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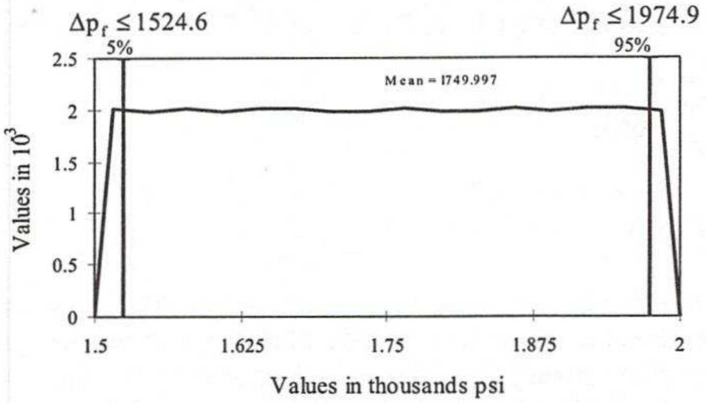


(c)

Figure 1. Distribution data for (a) Young's modulus - E, (b) Poisson's ratio – ν and (c) Reduction of pore fluid pressure ΔpBfB.

1.2.2. Fluid Properties

Generally, fluid properties are a function of composition, temperature, saturation and pressure, and will vary spatially and temporally. Deterministic values of key fluid properties used in the simulation are presented in Table 2. The simulation was run to ten years with a minimum time-step of one day and maximum of 500 days.

Table 2. Fluid properties

Variables	Symbol	Initial values	Unit
Reservoir temperature	T_{res}	154.82	^{0}F
Reservoir pressure	P_{res}	11,580	psi
Oil viscosity at 9000psi	μ_o	0.53	cp
Initial water saturation	S_{iw}	0.25	--
Oil gravity	ρ_o	128	lb/ft^3
Water gravity	ρ_w	63.02	lb/ft^3
Bubble point pressure at T_{res}	P_b	5,400	psi

2. COMPUTATIONAL METHODOLOGY

Monte Carlo simulation was applied to the calculation of compaction and subsidence. This accounts for the fact that the key input parameters E and ν have not been exactly presented or properly calculated at the field scale. Reduction of Δpf related to fluid production has been taken into account. The practice of describing the input parameters with range is actually more realistic because it captures our absence of information in estimating the true value of the input parameter.

In an attempt to verify the consistency from Monte Carlo simulation, the simplest model was run for experiment-1 with all parameters required for the calculation fixed at the average or most likely value as presented in Table 1. For each of the next three experiments, calculations were used for the Monte Carlo simulation in which statistically generated values for each of the uncertain input parameters were used. Experiment-2 takes E as uncertain. The experiment-3 is the next experiment with addition of ν as uncertain. The fourth experiment is the last experiment with addition of pore fluid pressure reduction as uncertain.

Simplified coupled equations used here for stochastic-based simulations are based on nucleus-of strain equations from rock mechanics as described by Geertsma [17] and Holt [18].

The maximum vertical compaction (Δh) and subsidence (S) for a roughly disk-shaped oil and gas bearing reservoir formation with Cb, ν, R, h, and D (Table 1), can be estimated using the equations 1 and 2

$$\Delta h = \frac{1-\nu-2\nu^2}{(1-\nu)E}\Delta p_f h \tag{1}$$

$$S = \frac{C_b}{2}\Delta p_f h A(\rho,\eta) \tag{2}$$

The numerical results will be presented in the next section. The comparison process regarding numerical results are used to confirm that the most likely level of compaction and subsidence (i.e. that value of compaction arising from setting all parameters to their most likely value) is comparable to the 50-percentile result from the Monte Carlo simulation. In other words, the result of the deterministic model with simulator should then be comparable to the 50-percentile result for the Monte Carlo simulation of the same experiments.

2.1 Numerical Results

2.1.1 Compaction versus Poisson's Ratio

Fully coupled reservoir simulation shown that when fluid is withdrawn from the reservoir, the pore fluid pressure will be reduced. In turn, effective stress will be increased [19]. Subsequently, the reservoir will deform causing compaction as shown in the previous section. However, the impact of rock properties was not taken into account. Figure 2 presents a case showing the decrease in compaction for two reservoir models with different Poisson's ratio but the same Young's modulus. When Poisson's ratio increases from 0.21 (Case 1) to 0.29 in the sense the mean values of Poisson's ratio (case 2) extracted from the two-well dataset, the compaction at the well location reduces from 2.74ft to 2.39ft. Simultaneously, the compaction at the boundary also reduces from 2.58ft to 2.21ft. Therefore, higher Poisson's ratio causes a lower compaction. This result should be considered when planning of infrastructure development. A more sensitive analysis is investigated in the next section.

2.1.2 Compaction versus various Poisson's Ratio and Young's Modulus

In this work, several numerical tests are undertaken to investigate the influence of Young's modulus and Poisson's ratio on compaction. Table 3 shows minimum and maximum compaction in each run for various E - ν combinations.

As previously mentioned, it is clear that compaction is lower where the reservoir has a higher Poisson's ratio. In addition, compaction also reduces substantially when Young's modulus increases. For example, when Young's modulus increases from 68000psi (base case) to 86500psi (mean value of Young's modulus), maximum compaction at production location falls from 3.27 to 2.74ft with the same Poisson's ratio 0.21. In conclusion, high Poisson's ratio and high Young's modulus reservoir cause a much lower compaction.

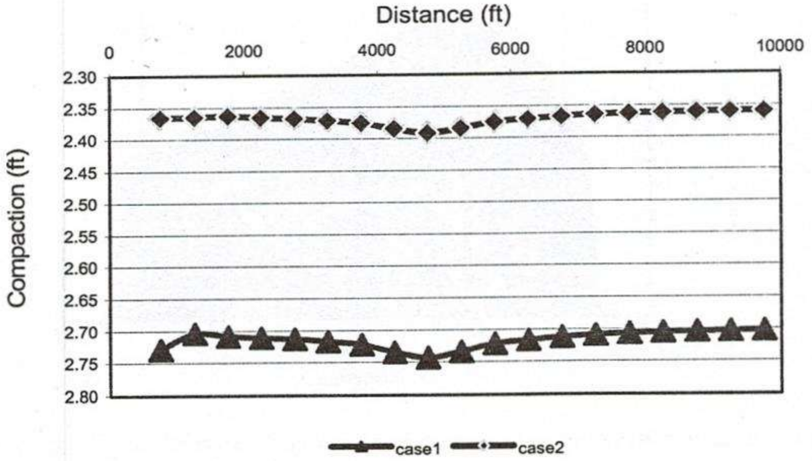


Figure 2. The compaction profile measured in the cross section that intersecting cut through the center of bowl compaction at the end of numerical simulation taking into account influence of Poisson's ratio on compaction. (Case1 with E=86500psi, ν=0.21, Case2 with E=86500psi, ν=0.29).

Table 3. Sensitivity results of numerical reservoir simulation.

Young's modulus (psi)	Poisson's ratio	Max compaction at well location (ft)	Min compaction at boundary (ft)
68000	0.21	3.27	3.18
68000	0.29	2.86	2.64
68000	0.4	1.99	1.82
86500	0.21	2.74	2.40
86500	0.29	2.39	2.21
100000	0.21	2.46	2.32
210000	0.3	1.21	1.13
210000	0.21	1.41	1.33
210000	0.4	0.79	0.72

2.1.3 Monte Carlo Simulation Results

The results of the Monte Carlo simulation are presented in comparison with results from reservoir simulation. For experiment-1 with no uncertain value, the compaction result is 3.27ft and subsidence is 0.91ft. The results of compaction lie exactly in accordance with results provided by the Eclipse 300 simulation in base case (First case in table 3). This shows that Geertsma's equations (Equations 1 & 2) can be used as a good approximation for complicated model such as Eclipse 300.

In experiment-2, Young's modulus data collected from two wells of the deep-water field were fitted with a distribution. The results show that the exponential distribution is the best fit with Chi-square measure. The mean of Young's modulus is 86,508.81psi and a standard deviation is 41,17psi. (Figure 3)

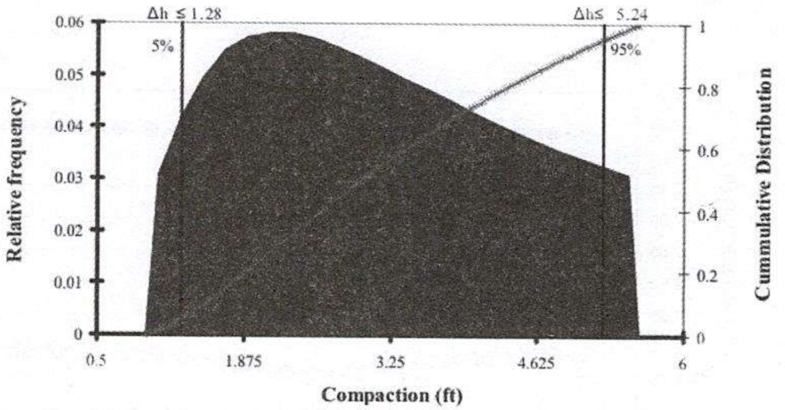


Figure 3: Compaction distribution for experiment-2. The mean of Young's modulus used in the experiment-2 is 86,508.81psi and a standard deviation is 41,17psi. The constant value of Poisson's ratio is 0.21.

Once the exponential distribution was determined, it replaced the Young's modulus single value. Monte Carlo simulation approach was performed for 10,000 iterations. The results present that the uncertainty in Young's modulus results in a compaction distribution that has a mean of 3.11ft and a standard deviation of 1.24ft for both the probability and cumulative distribution functions.

In addition, there is a 90% confidence interval where compaction falls between 1.28ft – 5.24ft. The distribution also indicates that due to the existence of uncertainty in Young's modulus, there is a 50% chance that the compaction is greater than 3.1ft. As a result, this should help a decision maker to collect more data and try to reduce the range of uncertainty and the possibility of greater compaction happening during the field life. These estimates should be accounted for during the field development.

Furthermore, Monte Carlo simulation results yield subsidence values with the mean of 0.87ft and a standard deviation of 0.34ft. These results show that because of the uncertainty in Young's modulus, the subsidence impact could range with a 90% confidence interval from 0.36ft to 1.46ft (Figure 4).

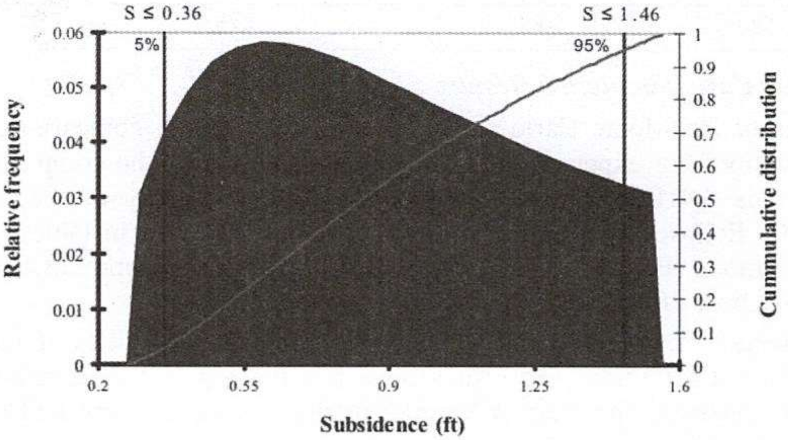


Figure 4: Subsidence distribution for experiment-2. The mean of Young's modulus used in the experiment-2 is 86,508.81psi and a standard deviation is 41,17psi. The constant value of Poisson's ratio is 0.21.

The results of Monte Carlo simulation provide the decision maker with possible scenario that might happen and the right response to each subsidence outcome.

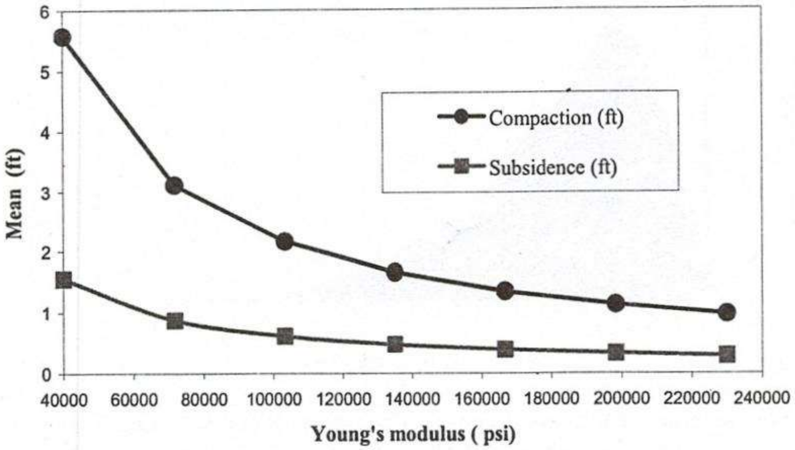


Figure 5. The impact of Young's modulus on compaction and subsidence

As shown in Figure 5, it is interesting to note values of E ranging approximately from 40,000 to 140,000psi impact the compaction and subsidence more than compared to E values lying beyond 140,000psi where the impact is really small. This shows that uncertainty beyond 140,000psi is insignificant to values of compaction and subsidence.

In experiment-3, data for Poisson's ratio from two wells were fitted with a normal distribution as the best fit based on the Chi-square measure.

In this experiment, Poisson's ratio distribution has a mean of 0.29 and a standard deviation of 0.09 and it is truncated leaving a range of 0.02 – 0.5 as shown in Figure 1b. The impact of introducing uncertainty in both Young's modulus and Poisson's ratio has resulted in a compaction mean of 2.43ft and standard deviation of 1.24ft and a 90% confidence interval of 0.72 – 4.72ft (Figure 6).

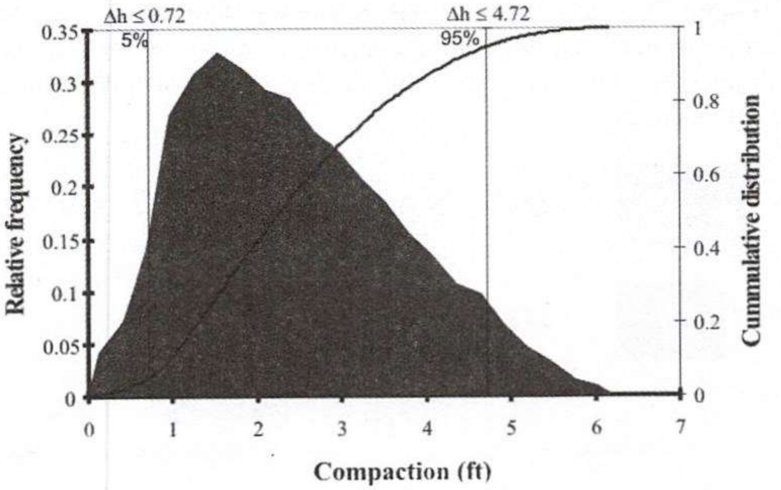


Figure 6. Compaction uncertainty for experiment-3. The mean of Young's modulus used in the experiment-3 is 86,508.81psi and a standard deviation is 41,17psi. Poisson's ratio distribution has a mean of 0.29 and a standard deviation of 0.09.

The impact on subsidence as a result of allowing for both E and ν has resulted in a mean of 0.60ft and standard deviation of 0.37ft. A 90% confidence interval was estimated to range from 0.12 – 1.33ft (Figure 7).

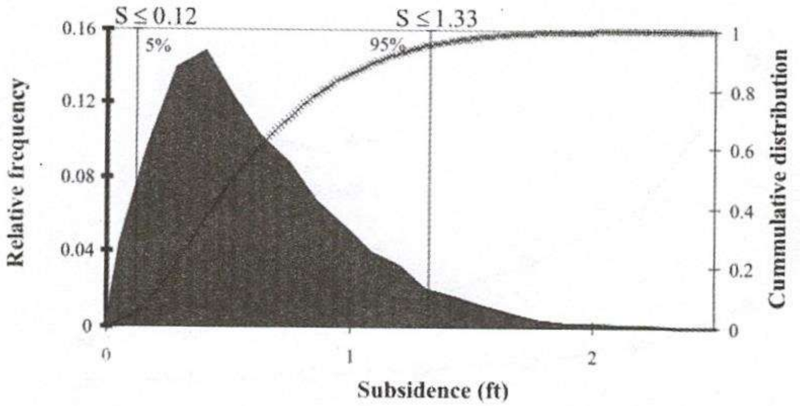
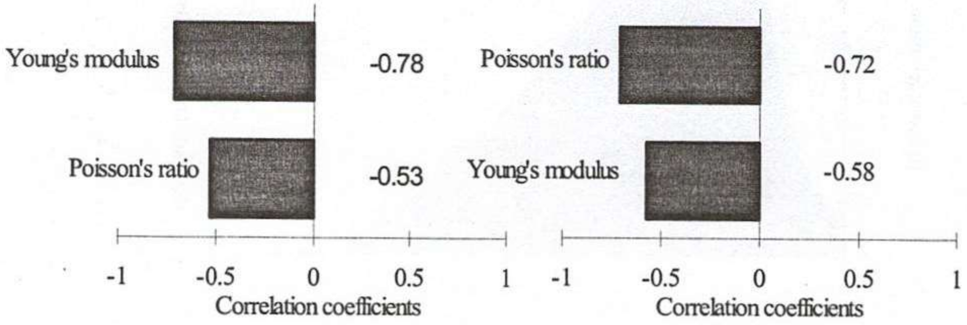


Figure 7: Subsidence uncertainty for experiment-3. The mean of Young's modulus used in the experiment-3 is 86,508.81psi and a standard deviation is 41,17psi. Poisson's ratio distribution has a mean of 0.29 and a standard deviation of 0.09

It is important to emphasis that the difference between experiment-2 and experiment-3 is treating Poisson's ratio as uncertain. In the later case, compaction results with addition of Poisson's ratio as uncertain variable has reduced the mean but the standard deviation is the same. Furthermore, the addition of Poisson's ratio on subsidence has resulted in a decrease in the mean, with approximately the same value for standard deviation. The mean values are consistent with results found using numerical simulation methods. The advantage of Monte Carlo simulation is that it has the ability to investigate the impact of variation of both E and ν simultaneously, compared to numerical simulation where each variable is changed while others are held constant.

A sensitivity analysis was performed to assess the impact of Young's modulus and Poisson's ratio on compaction. The Tornado plot for compaction (Figure 8a) shows that Young's modulus has a greater impact than Poisson's ratio implying that more effort should be directed toward estimating Young's modulus than estimating Poisson's ratio. Similar sensitivity analysis was done for subsidence (Figure 8b). Here we expected Young's modulus to have a bigger impact, however it was interesting to note that correlation coefficient for Poisson's ratio are larger than for Young's modulus indicating that more effort should be directed toward estimating Poisson's ratio when estimating subsidence.



(a) (b)

Figure 8. Tornado plot for (a) compaction, (b) subsidence.

When all the experiments were combined for the case of compaction (Figure 9), it is clear that as we add the uncertainty of Young's modulus, the compaction mean was reduced. In experiment-3 when the Poisson's ratio uncertainty was introduced, the mean compaction was reduced which is reflected in the left shift of the cumulative distribution function. As we add the uncertainty of pore fluid pressure reduction, compaction mean has increased again and the standard deviation has increased due to the addition of uncertain parameter. This clearly shows that pore fluid pressure reduction increases the compaction mean, because it has positive impact on compaction while both Young's modulus and Poisson's ratio have negative impacts.

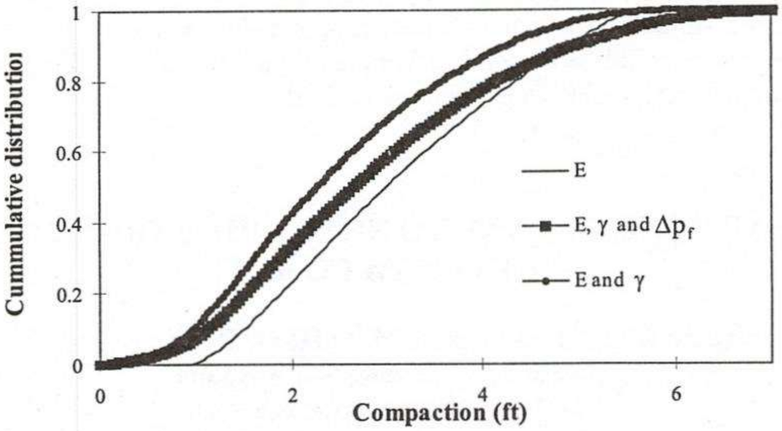


Figure 9. Compaction as uncertainty variables (E, ν and Δpf) are added.

3. DISCUSSION

Equation 1 enables us to recognize four parameters influencing reservoir compaction behavior: (1) Poisson's ratio, (2) Young's modulus, (3) reduction of pore fluid pressure and (4) thickness of reservoir. The numerical results showed that high Poisson's ratio and high Young's modulus reservoir cause a much lower compaction. However, these parameters were measured at ambient conditions, which can cause some skew error compared to measured values at in-situ condition. So, the tests in overburden conditions should be conducted on measurement on Young's modulus and Poisson's ratio to get more adequate data.

The stochastic results also showed that the reduction of pore fluid pressure has smaller impact on compaction. In most cases, the drop in pore fluid pressure in gas field from beginning production period to abandonment is small [17]. So, the consideration of reduction of pore fluid pressure may be neglected in gas field. However, in other oil and gas fields, particularly for fields with solution gas drive [17], the drop in pore fluid pressure should be considerable for a compaction investigation, even if this field is a hard rock reservoir where Young's modulus is larger than 140,000psi.

The stochastic analysis is based on the fitted distribution of input data, which is chosen automatically by computer. Different distribution could lead to big difference in standard deviation results in both compaction and subsidence. To get the best results, validation process should make on real subsidence and compaction data that is not easy to obtain in field.

4.CONCLUSIONS

The stochastic-based simulation of compaction and subsidence highlighted the following key issues that are not generally mentioned in numerical simulation methods.

Young's modulus has more impact on compaction than Poisson's ratio. Values of Young's modulus in this deep-water field in Gulf of Mexico ranging beyond 140,000psi have an insignificant effect on compaction and subsidence. This value could be used as a quantity for prediction of other compaction reservoirs instead of soft rock definition.

The influence of Poisson's ratio on subsidence is more important than the effect of Young's modulus. So, it is better to use Poisson's ratio when estimating subsidence in cases when there are inadequate Young's modulus data. Additionally, the reduction of pore fluid pressure has less impact on subsidence and compaction although it is the main reason for increasing effective stress when using the simplified equation for Monte Carlo simulation.

Numerical simulation with deterministic parameters is still valid in purpose of comparison with stochastic simulation. Although the type of reservoir model that has been built is simple, the study shows that all compaction results of sensitive analysis are in the range of 50% confident interval of stochastic simulation in which compaction problem could be happened in reservoir. So, stochastic simulation could be a useful technique to quickly evaluate the compaction without any complicated numerical simulation in deep-water field.

ĐÁNH GIÁ TÁC ĐỘNG CỦA YẾU TỐ NGẪU NHIÊN TRONG VIỆC DỰ BÁO SỤT LÚN VÀ CỐ KẾT

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TÓM TẮT: *Nội dung bài báo trình bày phương pháp stochastic sử dụng mô phỏng Monte Carlo ứng dụng trong dự báo sụt lún và cố kết tại mỏ dầu khí nước sâu tại vịnh MeXiCo. Kết quả cho thấy ảnh hưởng trong việc xử dụng các phân bố ngẫu nhiên cho dữ liệu ban đầu như Young modulus, tỉ lệ Poisson và sụt giảm áp suất lỗ rỗng trong việc dự báo sụt lún và cố kết. Dự báo từ mô hình tính toán ngẫu nhiên sụt lún và cố kết được so sánh với kết quả mô phỏng vỉa xử dụng phần mềm Eclipe 300. Kết quả tính toán dựa trên phương pháp stochastic kết luận: Các giá trị xác định của mô phỏng vỉa nằm trong khoảng hợp lý của mô hình ngẫu nhiên. Các phân tích cũng chỉ ra Young modulus tác động nhiều nhất đến các tính toán cố kết so với tỉ số Poisson. Hơn nữa đối với các thành hệ có giá trị Young modulus lớn hơn 140.000 psi tại vịnh Mexico, hiện tượng sụt lún và cố kết sẽ ít xảy ra. Cuối cùng, dựa vào các kết quả tính toán với mô hình stochastic, các vỉa có khả năng cố kết và sụt lún sẽ được dự báo trên toàn vùng mỏ.*

REFERENCES

[1]. Behrenbruch, P., M. van den Boer, H. van Engen and J. van Lookeren. Bachaquero field study- Part 6: Basic reservoir and well data. *The Hague, the Netherlands, Shell Internationale Petroleum Maatschappij B.V,* (1979).

[2]. DSH (1996). Unaxial Compaction Measurement Summary. *Louisiana, Shell Offshore Inc.*

[3]. Terratek Inc. TR96-27 BP Exploration Complressibility measurements-Europa MC 934. *Salt lake City, Utah, University Research park: 1-1,* (1996)

[4]. Jim, U., W. Bailey, F.-A. Reem and C. Otto (2006). Offshore Aquifer Update, *Onshore fault Seal Analysis, and Preliminary Numerical Simulation of Coastal Subsidence Risk. Western Australia, CSIRO.*

[5]. Ta, Q. D. and S. P. Hunt (2006). Stress Variability Around Large Structural Features and Its Impact on Permeability for Coupled Modelling Simulations. *4th Asian Rock Mechanics Symposium (ARMS), Singapore.*

[6]. Ta, Q. D. and S. P. Hunt (2005). Applying fully coupled geomechanics and fluid flow model theory to petroleum wells. *The 40th U.S. Symposium on Rock Mechanics, Alaska.*

[7]. Gambolati, G., F. Massimiliano, T. Pietro, D. Roberto and L. Giuditta. "Finite element analysis of land subsidence above depleted reservoirs with pore pressure gradient and

total stress formulations." *International Journal for Numerical and Analytical Methods in Geomechanics* 25: 307-327. (2001)

[8]. Biot, M. A.. "General theory of three Dimenssional consolidation." *Journal of applied physics*: 155-164, (1940)

[9]. Gutierrez, M. Fully Coupled Analysis of Reservoir Compaction and Subsidence. *European Petroleum Conference, London, United Kingdom,* (1994).

[10]. Terry, S., B. Garfield, P. Panos and F. John. Fully Coupled Geomechanics in a Commercial Reservoir Simulator. SPE 65107 paper, *presented at SPE European Petroleum Conference, Paris, France.*(2000)

[11]. Massimiliano, F., G. Giuseppe, T. Pietro and B. Domenico. "Stochastic poromechanical modeling of anthropogenic land subsidence." *International Journal of Solids and Structures* 43: 3324–3336. (2006)

[12]. Collin, F., C. Schroeder, V. De Gennaro and A. Bolle. "A deterministic/stochastic model to predict the variation in bulk modulus of chalk." *Geotechnique* 55(2): 135–141. (2005)

[13]. Diego, G. F., A. M. Marcio and P. Felipe. "Stochastic computational modelling of highly heterogeneous poroelastic media with long-range correlations." *International Journal for Numerical and Analytical Methods in Geomechanics* 28: 1-32. (2004)

[14]. Al-Harthy, M. H., A. K. Khurana, S. Begg and R. B. Bratvold. "Sequential and system approach for evaluating inverstment decision-influence of functional dependencies and interactions." *APPEA journal*: 511-522. (2006)

[15]. Murtha, J. Decisions Involving Uncertainty, *A @Risk Tutorial for the Petroleum Industry, Palisade Corporation.* (2000).

[16]. Eclipse. Simulation Software Manuals 2005A, *Schlumberger.* (2005).

[17]. Geertsma, J. "Land subsidence above compaction oil and gas reservoir." SPE 3730 - *Journal of Petroleum Technolgy*: 734-744. (1973).

[18]. Holt, R. M. "Permeability Reduction Induced by a Nonhydrostatic Stress Field." *SPE Formation Evaluation* Volume 5 (Number 4, December): 444-448. (1990).

[19]. Wan, J. Stabilized finite element methods for coupled geomechanics and multiphase flow. *Petroleum engineering department,* Stanford University. (2003).