Science & Technology Development Journal, 24(3):2019-2030

Open Access Full Text Article

Research Article

An algorithms for planning S-curve motion profile considering arbitrary short path length

Thanh Luan Bui[1], Truong Giang Le[2], Phuc Thinh Doan[2,*]

Use your smartphone to scan this QR code and download this article

ABSTRACT

Motion control is a key technology in controlling machine tools, manipulator robots, automation machines, etc. In motion controllers, velocity and position are controlled by closed-loop feedback to create desired motions. The main target in motion control is fast, accurate, small oscillations, no overshoot in both position and velocity. However, the traditional velocity planning methods cannot satisfy the requirements. One of the most popular control methods is using S-curve motion profile. Many papers and researchers have reported on this method. If the path length is long enough, the S-curve motion profile is easy to achieve through the 3^{rd} order polynomial model consisting of seven stages. The challenge becomes hard when the path length is small. It should be noted that the path length is arbitrary in the usual numerical control (NC) program. This paper proposes a new algorithm for planning S-curve motion profile in considering arbitrary short path length. There are two steps to perform a S-curve motion profile algorithm: type judgment and parameter calculation. Base on condition constraints, a categorization of 8 types using 3^{rd} order polynomial S-curve model, is introduced. The proposed algorithm ensures that the system always has the solution. The simulation results demonstrate the effectiveness of the proposed algorithm.

Key words: Motion Control, S-Curve, Motion Profile, Short Path Length, Point-to-Point Motion Control

[1]*Industrial Maintenance Training Center, HCMUT, VNU-HCM*

[2]*Nguyen Tat Thanh University, Ho Chi Minh City, Vietnam*

Correspondence

Phuc Thinh Doan, Nguyen Tat Thanh University, Ho Chi Minh City, Vietnam

Email: dpthinh@ntt.edu.vn

History

- Received: 2021-06-12
- Accepted: 2021-07-15
- Published: 2021-08-10

DOI : 10.32508/stdj.v24i3.2588

INTRODUCTION

Nowadays, the demand for robots and automation machines is increasing rapidly. Motion control is a key technology in controlling machine tools, manipulator robots, and automation machines in the manufacturing industry. The main target in motion control is fast, accurate, small oscillations, no overshoot in both position and velocity. Unfortunately, the traditional motion control methods cannot satisfy the requirements of high-accuracy machining[1–3].

The common motor drivers on the market only support the simple trapezoidal velocity profiles. The trapezoidal Acc/Dec velocity profiles have non-smooth transition, discontinuous acceleration, and jumping phenomenon. These properties induce the vibration of machine tools during the movement[4,5]. As a result, trapezoidal velocity profiles tend to cause overshoots and excite residual vibrations that require time for the machine to reach the final position with desired precision. This could be a potential problem for the precision system. Therefore, to improve productivity and reduce cost and execution time, it is necessary to develop a new motion planning method. As a result, S-curve motion profile has been proposed to reduce the tendency to excite the vibration of the system[6].

Nguyen et al. investigate a generalization of the polynomials S-curve model and make the comparison of 3^{rd}, 4^{th}, and 5^{th} order S-curve motions[7]. He also introduces a new trigonometric jerk model and comparison it with the 5^{th} order polynomial model. Though the trigonometric model is as simple as the 3^{rd} order polynomial, it produces as good performance as the 5th order polynomial model. However, Nguyen's method is too complex and time-consuming to be used in practice. In general, the computation load increases with the increase in the polynomial order. Therefore, higher-degree polynomial profiles demand more computational efforts.

Chen et al. proposed an algorithm that considering a short path length that can be applied to NC program. According to the travel length, the motion is divided into seven types[8]. However, the method did not provide a clear solution after type classification.

Huang et al. provided a categorization of 12 types using the 3^{rd} order polynomial S-curve model[9]. However, this research did not consider a short path length. Therefore, when applying Huang's algorithm, the system becomes non-solution in some situations. This paper proposes a new algorithm for planning S-curve motion profile in considering arbitrary short

Cite this article : Bui T L, Le T G, Doan P T. **An algorithms for planning S-curve motion profile considering arbitrary short path length**. *Sci. Tech. Dev. J.;* 24(3):2019-2030.

path length to overcome the above-mentioned shortcomings. The algorithm is required to generate a motion profile with a confined jerk, acceleration, and speed. However, in the case of path length is very small, it is impossible for all system parameter inputs for motion control to be satisfied. The most important is moving path length must be satisfied. Therefore, the proposed algorithm consists of a feed rate modification and an end velocity modification. There are two steps to perform a S-curve motion profile algorithm: S-curve type judgment and parameters calculation. Base on condition constrains, a categorization of 8 types using the 3^{rd} order polynomial S-curve model is introduced. After type judgment, time parameters for each segment of the S-curve are calculated.

Furthermore, if S-curve belongs to the type that has a small path length, feed rate modification and end velocity modification are also processed. Thus, the proposed algorithm ensures that the system always has a solution even considering arbitrary short path length. The simulation results demonstrate the effectiveness of the proposed algorithm.

The outline of this paper is organized as follows: Firstly, a brief introduction on the motion profiles and math expression for 3^{rd} order polynomial S-curve are given. Secondly, the proposed algorithm for S-curve classification and parameter calculation is introduced. Thirdly, the algorithm simulation results and discussion are described. Finally, the conclusions are summarized.

3^{rd} ORDER POLYNOMIAL S-CURVE MATH EXPRESSION

In this section, the profiles and math expression of 3^{rd} order polynomial S-curve are presented[9]. The S-curve profiles consist of seven stages: increase acceleration, constant acceleration, decrease acceleration, constant velocity, increase deceleration, constant deceleration, and decrease deceleration. The 3^{rd} order polynomial S-curve profile is shown in Figure 1.

The math expression is expressed as follows:

First stage: Increase acceleration

$$\forall t \in [t_0, t_1]\quad \begin{cases} j_i(t) = j_{max} \\ a_i(t) = a_0 + j_{max}\tau_1 \\ v_i(t) = v_0 + a_0\tau_1 + \dfrac{1}{2} j_{max}\tau_1^2 \\ s_i(t) = s_0 + v_0\tau_1 + \dfrac{1}{2} a_0\tau_1^2 + \dfrac{1}{6} j_{max}\tau_1^3 \end{cases} \tag{1}$$

Second stage: Constant acceleration

$$\forall t \in [t_1, t_2]\quad \begin{cases} j_i(t) = 0 \\ a_i(t) = a_{max} = j_{max} T_1 \\ v_i(t) = v_i(t_1) + a_{max}\tau_2 \\ s_i(t) = s_i(t_1) + v_i(t_1)\tau_2 + \dfrac{1}{6} a_{max}\tau_2^2 \end{cases} \tag{2}$$

Third stage: Decrease acceleration

$$\forall t \in [t_2, t_3]\quad \begin{cases} j_i(t) = -j_{max} \\ a_i(t) = a_i(t_2) - j_{max}\tau_3 \\ v_i(t) = v_i(t_2) + a_i(t_2)\tau_3 - \frac{1}{2} j_{max}\tau_3^2 \\ s_i(t) = s_i(t_2) + v_i(t_2)\tau_3 + \frac{1}{2} a_i(t_2)\tau_3^2 - \frac{1}{6} j_{max}\tau_3^3 \end{cases} \tag{3}$$

Fourth stage: Constant velocity

$$\forall t \in [t_3, t_4]\quad \begin{cases} j_i(t) = 0 \\ a_i(t) = 0 \\ v_i(t) = v_i(t_3) \\ s_i(t) = s_i(t_3) + v_i(t_3)\tau_4 \end{cases} \tag{4}$$

Fifth stage: Increase deceleration

$$\forall t \in [t_4, t_5]\quad \begin{cases} j_i(t) = -j_{max} \\ a_i(t) = a_i(t_4) - j_{max}\tau_5 \\ v_i(t) = v_i(t_4) + a_i(t_4)\tau_5 - \frac{1}{2} j_{max}\tau_5^2 \\ s_i(t) = s_i(t_4) + v_i(t_4)\tau_5 + \frac{1}{2} a_i(t_4)\tau_5^2 - \frac{1}{6} j_{max}\tau_5^3 \end{cases} \tag{5}$$

Sixth stage: Constant deceleration

$$\forall t \in [t_5, t_6]\quad \begin{cases} j_i(t) = 0 \\ a_i(t) = -a_{max} \\ v_i(t) = v_i(t_5) - a_{max}\tau_6 \\ s_i(t) = s_i(t_5) + v_i(t_5)\tau_6 - \dfrac{1}{2} a_{max}\tau_6^2 \end{cases} \tag{6}$$

Seventh stage: Decrease deceleration

$$\forall t \in [t_6, t_7]\quad \begin{cases} j_i(t) = j_{max} \\ a_i(t) = a_i(t_6) + j_{max}\tau_7 \\ v_i(t) = v_i(t_6) + a_i(t_6)\tau_7 - \frac{1}{2} j_{max}\tau_7^2 \\ s_i(t) = s_i(t_6) + v_i(t_6)\tau_7 + \frac{1}{2} a_i(t_6)\tau_7^2 - \frac{1}{6} j_{max}\tau_7^3 \end{cases} \tag{7}$$

BUILDING AN ALGORITHM FOR PLANNING S-CURVE MOTION PROFILE CONSIDERING SHORT PATH LENGTH

The S-curve motion profile classification

The system parameters cover maximum acceleration a_{max}; maximum jerk j_{max}. The trajectory parameters

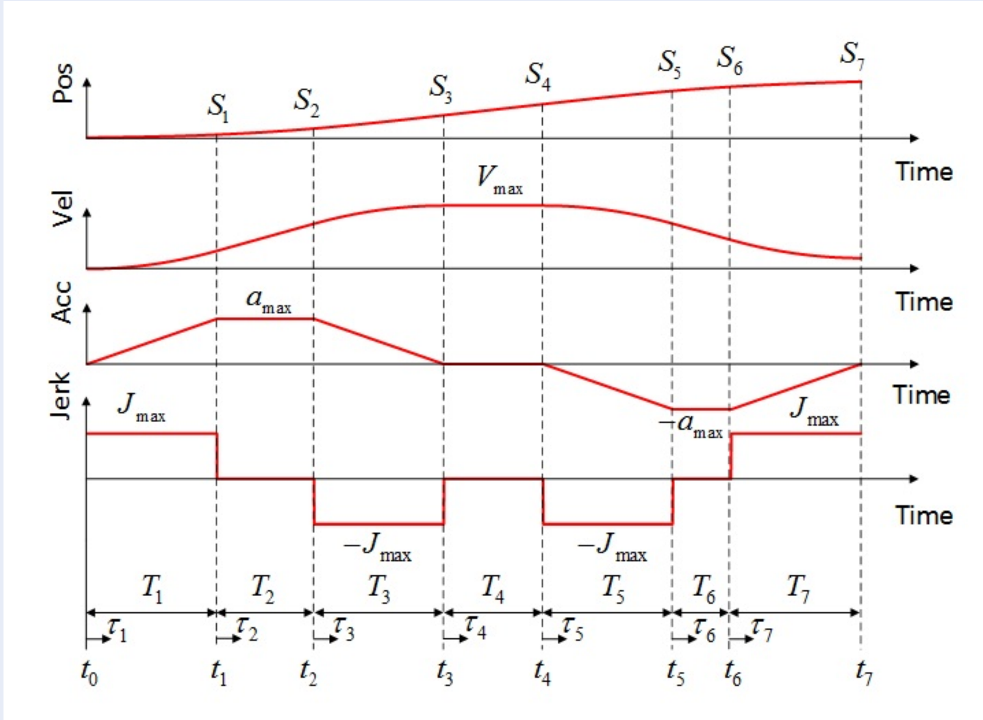

Figure 1: The 3^{rd} order polynomial S-curve motion profile.

include path length S_i; start velocity v_s; end velocity v_e; and feed rate v_{cf}.

According to different trajectory parameters, the S-curve motion profile can be classified as 8 types, including 4 basic types and 4 extended types, as shown in Figure 2.

Four basic types have acceleration speed zone, constant-speed zone, and deceleration-speed zone. Four extended types do not have a constant-speed zone. According to the path length, the S-curve motion profile is divided into 8 types.

In four basic types, v_{cf} is reachable. Type 1~ Type 4 is as follows:

- In Type 1, a_{max} is reachable and $-a_{max}$ is reachable.
- In Type 2, a_{max} is reachable and $-a_{max}$ is unreachable.
- In Type 3, a_{max} is unreachable and $-a_{max}$ is reachable.
- In Type 4, a_{max} is unreachable and $-a_{max}$ is unreachable.

In four extended types, v_{cf} is unreachable. Type 5~ Type 8 is as follows:

- In Type 5, a_{max} is reachable and $-a_{max}$ is reachable.
- In Type 6, a_{max} is reachable and $-a_{max}$ is unreachable.
- In Type 7, a_{max} is unreachable and $-a_{max}$ is reachable.
- In Type 8, a_{max} is unreachable and $-a_{max}$ is unreachable.

Path length calculation

This section presents the convenient formulas to calculate the path length, as the acceleration profile is ladder or triangle shape, are presented[9]. The path length is only calculated when the system acceleration or deceleration. Therefore, it does not include the path length in the constant velocity section.

When the acceleration profiles are ladder shape on both the acceleration and deceleration stages, the path length is S_{lald} calculated as follows:

$$S_{lald}\left(V_{cf}\right) = \frac{j_{max}\left(V_{cf}^2 - V_s^2\right) + a_{max}^2\left(V_{cf}+V_s\right)}{2a_{max}j_{max}} + \frac{j_{max}\left(V_{cf}^2 - V_e^2\right) + a_{max}^2\left(V_{cf}+V_e\right)}{2a_{max}j_{max}} \quad (8)$$

When the acceleration profile is ladder shape on the acceleration stage, but triangle shape on the deceler-

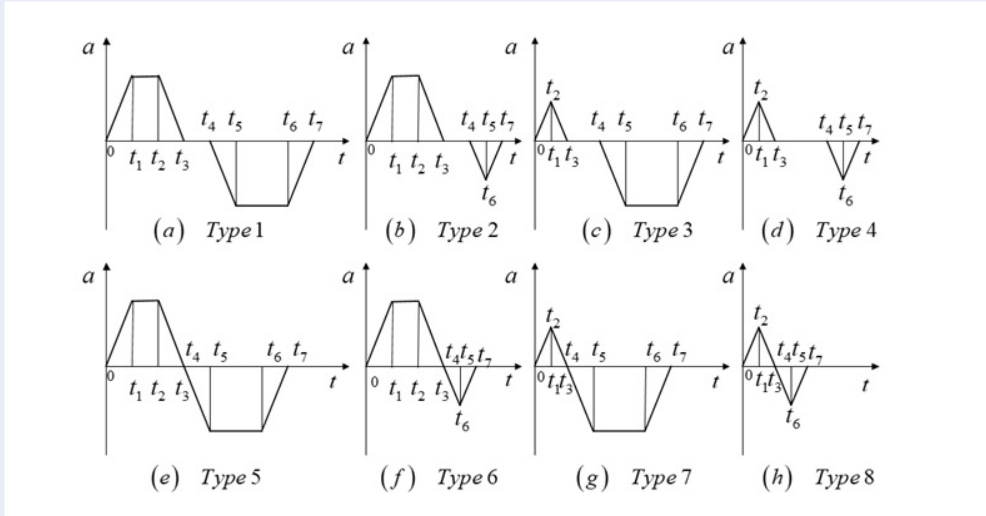

Figure 2: Eight types of S-curve are used in planning S-curve motion profile.

ation stage, the path length is S_{lald} calculated as follows:

$$S_{lald}\left(V_{cf}\right) = \frac{j_{max}\left(V_{cf}^2 - V_s^2\right) + a_{max}^2\left(V_{cf}+V_s\right)}{2a_{max}j_{max}} + \left(V_{cf}+V_e\right)\sqrt{\frac{V_{cf}-V_e}{j_{max}}} \tag{9}$$

When the acceleration profile is triangle shape on the acceleration stage, but ladder shape on the deceleration stage, the path length is S_{lald} calculated as follows:

$$S_{tald}\left(V_{cf}\right) = \left(V_{cf}+V_s\right)\sqrt{\frac{V_{cf}-V_s}{j_{max}}} + \frac{j_{max}\left(V_{cf}^2 - V_e^2\right) + a_{max}^2\left(V_{cf}+V_e\right)}{2a_{max}j_{max}} \tag{10}$$

When the acceleration profiles are triangle shape on both the acceleration and deceleration stages, the path length is S_{lald} calculated as follows:

$$S_{tald}\left(V_{cf}\right) = \left(V_{cf}+V_s\right)\sqrt{\frac{V_{cf}-V_s}{j_{max}}} + \left(V_{cf}+V_e\right)\sqrt{\frac{V_{cf}-V_e}{j_{max}}} \tag{11}$$

The Acc/Dec algorithm in considering of short path length

There are two steps to perform a S-curve motion profile algorithm: type judgment and parameters calculation.

Step 1: Type judgment

Considering the velocity in type 1 ~ type 4:

- a_{max} is reachable, then condition $(C1)$ $v_{cf} - v_s \geq a_{max}^2/j_{max}$ satisfied.
- a_{max} is unreachable, then condition $(C2)$ $v_{cf} - v_s < a_{max}^2/j_{max}$ satisfied.
- $-a_{max}$ is reachable, then condition $(C3)$ $v_{cf} - v_e \geq a_{max}^2/j_{max}$ satisfied.
- $-a_{max}$ is unreachable, then condition $(C4)$ $v_{cf} - v_e < a_{max}^2/j_{max}$ satisfied.

Considering the path length in type 1 ~ type 4 where v_{cf} is reachable, and :

- Type 1, the acceleration profiles are ladder shape on both the acceleration and deceleration stages then the condition is as follows: $(C5)$ $s_i > s_{lald}\left(v_{cf}\right)$.
- Type 2, the acceleration profile is ladder shape on the acceleration stage, but triangle shape on the deceleration stage then condition is as follows: $(C6)$ $s_i > s_{lald}\left(v_{cf}\right)$.
- Type 3, the acceleration profile is triangle shape on the acceleration stage, but ladder shape on the deceleration stage then condition is as follows: $(C7)$ $s_i > s_{tald}\left(v_{cf}\right)$.
- Type 4, the acceleration profiles are triangle shape on both the acceleration and deceleration stages then condition is as follows: $(C8)$ $s_i > s_{tald}\left(v_{cf}\right)$.

From the above consideration, 4 basic types are classified as follows:

If conditions $(C1)$, $(C3)$ and $(C5)$ are satisfied, it belongs to type 1.

If conditions $(C1), (C4)$ and $(C6)$ are satisfied, it belongs to type 2.
If conditions $(C2), (C3)$ and $(C7)$ are satisfied, it belongs to type 3.
If conditions $(C2), (C4)$ and $(C8)$ are satisfied, it belongs to type 4.
However, if the path length is short, and conditions (C5), (C6), (C7), (C8) in type 1~ type 4 respectively are not satisfied, then 4 basic types cannot be implemented. In this case, the feed rate v_{cf} is adjusted to a new value v_{max}, and the path length is achieved. For that case, 4 proposed extended types consist of type 5~ type 8 can be used.
To classify the type of S-curve, there are 4 proposed groups in which group 1 is the group that satisfies conditions (C1) and (C3), group 2 is the group that satisfies conditions (C1) and (C4), group 3 is the group that satisfies conditions (C2) and (C3) and group 4 is the group that satisfies conditions (C2) and (C4).
Table 1 shows the conditions for S-curve type classification.
Figure 3 is the condition constraints for S-curve type classification for 4 groups. Blue color indicates a satisfied condition, and red color indicates the unsatisfied condition.

In group 1:
If conditions (C1), (C3), and (C5) are satisfied, it belongs to type 1
If conditions (C1), (C3), (C9) are satisfied, and (C5) are unsatisfied, it belongs to type 5
If conditions (C1), (C3), (C10), (C13) are satisfied, and (C5), (C9) are unsatisfied, it belongs to type 6
If conditions (C1), (C3), (C10) are satisfied, and (C5), (C9), (C13) are unsatisfied, it belongs to type 7
If conditions (C1), (C3) are satisfied, and (C5), (C9), (C10) are unsatisfied, it belongs to type 8

In group 2
If conditions (C1), (C4), and (C6) are satisfied, it belongs to type 2
If conditions (C1), (C4), (C11) are satisfied, and (C6) is unsatisfied, it belongs to type 6
If conditions (C1), (C4) are satisfied, and (C6), (C11) are unsatisfied, it belongs to type 8

In group 3
If conditions (C2), (C3), and (C7) are satisfied, it belongs to type 3
If conditions (C2), (C3), (C11) are satisfied, and (C7) is unsatisfied, it belongs to type 7
If conditions (C2), (C3) are satisfied, and (C7), (C11) are unsatisfied, it belongs to type 8

In group 4
If conditions (C2), (C4), and (C8) are satisfied, it belongs to type 4
If conditions (C2), (C4) are satisfied, and (C8) is unsatisfied, it belongs to type 8
From Figure 3, all cases of system parameters and trajectory parameters can be classified by proposed 8 types that is easy to recognize with note that (C1) is negative of (C2) and (C3) is negative of (C4). In type judgment all conditions are checked and its negative also are checked. Therefore, the system always has solution. However, the system parameters and trajectory parameter will be selected with some conditions as follows:

- $a_{max}, j_{max}, v_e, v_s, v_{cf}, s_i \geq 0$ all parameter is positive.
- $|v_s - v_e| \leq a_{max}^2 / j_{max}$ if system belongs to type 6, type7, type 8.

In the case condition $|v_s - v_e| \leq a_{max}^2 / j_{max}$ is not satisfied, the end velocity should be modified to acceptable value $v_e' = v_s + a_{max}^2 / j_{max}$..

Step 2: parameters calculation
After type judgment, time parameters for each segment of S-curve are calculated. Furthermore, if S-curve belongs to type 5 ~ type 8, feed rate modification and end velocity modification also are processed.

Type 1: v_{cf} reachable, a_{max} reachable, $-a_{max}$ reachable
When conditions (C1), (C3), and (C5) are satisfied, it is belonged to type 1
The time parameters are calculated by

$$\begin{cases} T_1 = T_3 = T_5 = T_7 = \dfrac{a_{max}}{j_{max}} \\ T_2 = \dfrac{v_{cf} - v_s}{a_{max}} - T_1,\ T_6 = \dfrac{v_{cf} - v_e}{a_{max}} - T_5 \\ T_4 = \dfrac{s_i - s_{lald}\left(v_{cf}\right)}{v_{cf}} \end{cases} \tag{12}$$

Type 2: v_{cf} reachable, a_{max} reachable, and $-a_{max}$ unreachable.
When conditions (C1), (C4), and (C6) are satisfied, it is belonged to type 2
The time parameters are calculated by

$$\begin{cases} T_1 = T_3 = \dfrac{a_{max}}{j_{max}}, T_2 = \dfrac{v_{cf} - v_s}{a_{max}} - T_1 \\ T_5 = T_7 = \sqrt{\dfrac{v_{cf} - v_s}{a_{max}}},\ T_6 = 0 \\ T_4 = \dfrac{s_i - s_{lald}\left(v_{cf}\right)}{v_{cf}} \end{cases} \tag{13}$$

Type 3: v_{cf} reachable, a_{max} unreachable, and $-a_{max}$ reachable.
When conditions (C2), (C3), and (C7) are satisfied, it is belonged to type 3

Table 1: List of condition constraints for type classification

$(C1)\ v_{cf} - v_s \geq a_{max}^2 / j_{max}$	$(C5)\ s_i > s_{lald}\left(v_{cf}\right).$	$(C9)\ s_i > s_{lald}\left(max\left\{v_s, v_e\right\} + a_{max}^2/j_{max}\right)$
$(C2)\ v_{cf} - v_s < a_{max}^2 / j_{max}$	$(C6)\ s_i > s_{lald}\left(v_{cf}\right).$	$(C10)\ s_i \geq s_{lald}\left(min\left\{v_s, v_e\right\} + a_{max}^2/j_{max}\right)$
$(C3)\ v_{cf} - v_e \geq a_{max}^2 / j_{max}$	$(C7)\ s_i > s_{tald}\left(v_{cf}\right)$	$(C11)\ s_i \geq s_{tald}\left(min\left\{v_s, v_e\right\} + a_{max}^2/j_{max}\right)$
$(C4)\ v_{cf} - v_e < a_{max}^2 / j_{max}$	$(C8)\ s_i > s_{tald}\left(v_{cf}\right).$	$(C12)\ s_i > s_{tald}\left(max\left\{v_s, v_e\right\}\right)$
		$(C13)\, V_s \leq V_e$

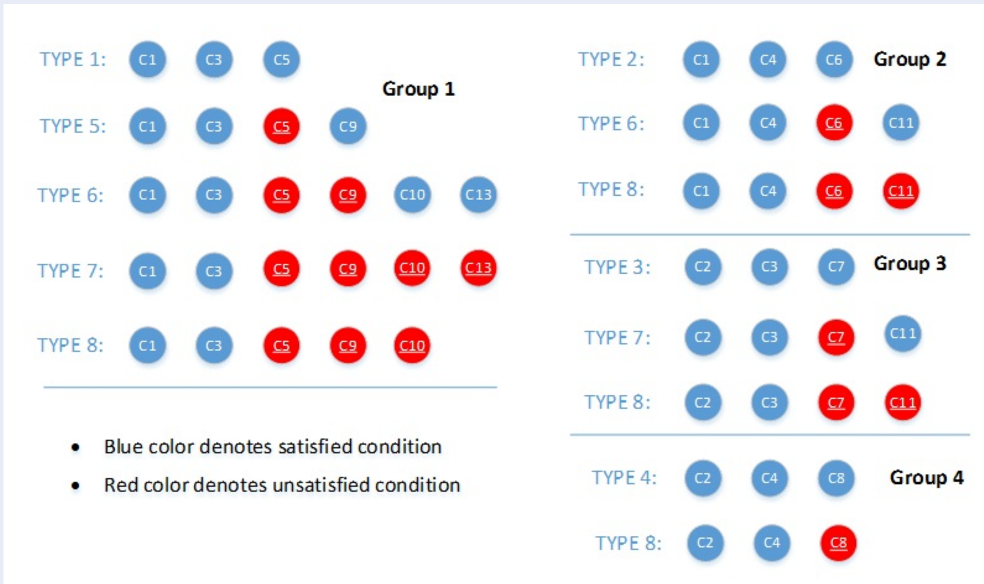

Figure 3: Summarizing condition constraints for S-curve type judgment.

The time parameters are calculated by

$$\begin{cases} T_1 = T_3 = \sqrt{\dfrac{v_{cf} - v_s}{a_{max}}}, T_2 = 0 \\ T_5 = T_7 = \dfrac{a_{max}}{j_{max}},\ T_6 = \dfrac{v_{cf} - v_s}{a_{max}} - T_5 \\ T_4 = \dfrac{s_i - s_{tald}\left(v_{cf}\right)}{v_{cf}} \end{cases} \tag{14}$$

Type 4:v_{cf} reachable, a_{max} unreachable, and $-a_{max}$ unreachable.

When conditions (C2), (C4), and (C8) are satisfied, it is belonged to type 4

The time parameters are calculated by

$$\begin{cases} T_1 = T_3 = \sqrt{\dfrac{v_{cf} - v_s}{a_{max}}}, T_2 = 0 \\ T_5 = T_7 = \sqrt{\dfrac{v_{cf} - v_s}{a_{max}}},\ T_6 = 0 \\ T_4 = \dfrac{s_i - s_{tald}\left(v_{cf}\right)}{v_{cf}} \end{cases} \tag{15}$$

Type 5:v_{cf} unreachable, a_{max} reachable, and $-a_{max}$ reachable.

When conditions (C1), (C3), (C9) are satisfied, and (C5) are unsatisfied, it is belonged to type 5

The reachable maximum velocity is calculated as follows:

$s_i = s_{lald}\left(v_{max}\right)$ where $v_{max} \in \left[max\left\{v_s, v_e\right\} + a_{max}^2/j_{max}, v_{cf}\right]$

The time parameters are calculated by

$$\begin{cases} T_1 = T_3 = T_5 = T_7 = \dfrac{a_{max}}{j_{max}} \\ T_2 = \dfrac{v_{max} - v_s}{a_{max}} - T_1,\ T_6 = \dfrac{v_{max} - v_e}{a_{max}} - T_5 \\ T_4 = 0 \end{cases} \tag{16}$$

Type 6:v_{cf} unreachable, a_{max} reachable, and $-a_{max}$ unreachable.

(C6A): (C1), (C3), (C10), (C13) are satisfied, and (C5), (C9) are unsatisfied

(C6B): (C1), (C4), (C11) are satisfied, and (C6) is unsatisfied

Either condition (C6A) or (C6B) is satisfied; it is belonged to type 6

The reachable maximum velocity is calculated as follows:

$s_i = s_{lald}(v_{max})$ where $v_{max} \in \left[min\{v_s, v_e\} + a_{max}^2/j_{max}, v_{cf}\right]$

The time parameters are calculated by

$$\begin{cases} T_1 = T_3 = \dfrac{a_{max}}{j_{max}}, T_2 = \dfrac{v_{max} - v_s}{a_{max}} - T_1 \\ T_5 = T_7 = \sqrt{\dfrac{v_{max} - v_e}{j_{max}}},\ T_6 = 0 \\ T_4 = 0 \end{cases} \quad (17)$$

Type 7: v_{cf} unreachable, a_{max} unreachable, and $-a_{max}$ reachable.

(C7A): (C1), (C3), (C10) are satisfied, and (C5), (C9), (C13) are unsatisfied

(C7B): (C2), (C3), (C11) are satisfied, and (C7) is unsatisfied

Either condition (C7A) or (C7B) is satisfied; it is belonged to type 7

The reachable maximum velocity is calculated as follows:

$s_i = s_{tald}(v_{max})$ where $v_{max} \in \left[min\{v_s, v_e\} + a_{max}^2/j_{max}, v_{cf}\right]$

The time parameters are calculated by

$$\begin{cases} T_1 = T_3 = \sqrt{\dfrac{v_{max} - v_s}{j_{max}}}, T_2 = 0 \\ T_5 = T_7 = \dfrac{v_{max}}{j_{max}},\ T_6 = \dfrac{v_{max} - v_e}{j_{max}} - T_5 \\ T_4 = 0 \end{cases} \quad (18)$$

Type 8: v_{cf} unreachable, a_{max} unreachable, and $-a_{max}$ unreachable.

(C8A): (C1), (C3) are satisfied, and (C5), (C9), (C10) are unsatisfied

(C8B): (C1), (C4) are satisfied, and (C6), (C11) are unsatisfied

(C8C): (C2), (C3) are satisfied, and (C7), (C11) are unsatisfied

(C8D): (C2), (C4) are satisfied, and (C8)

When any condition of (C8A), (C8B), (C8C), and (C8D) is satisfied, it is belonged to type 8

if the condition (C12) is not satisfied, then the equation $s_i = s_{tatd}(v_{max})$ is no solution. To solve this case, adjust the value of end velocity as follows $v_e^{'} = v_s$.

The reachable maximum velocity is calculated as follows:

If condition (C12) is satisfied (type 8A)

$s_i = s_{tald}(v_{max})$ where $v_{max} \in \left[max(v_s, v_e), v_{cf}\right]$

If condition (C12) unsatisfied (type 8B)

$v_e^{'} = v_s$ and $s_i = s_{tatd}^{'}(v_{max})$ where $v_{max} \in \left[v_s, v_{cf}\right]$

The time parameters are calculated by

$$\begin{cases} T_1 = T_3 = \sqrt{\dfrac{v_{max} - v_s}{j_{max}}}, T_2 = 0 \\ T_5 = T_6 = \sqrt{\dfrac{v_{max} - v_e}{j_{max}}},\ T_6 = 0 \\ T_4 = 0 \end{cases} \quad (19)$$

RESULTS

In this section, simulation has been done for 4 groups with various different path lengths using Matlab.

Table 2 shows trajectory parameters and simulation results for four groups to verify the proposed algorithm.

The maximum acceleration a_{max} is $2m/s^2$, the maximum jerk j_{max} is $100m/s^3$.

In the simulation for group 1, $v_{cf} = 0.008$, $v_s = 0.01$ and $v_e = 0.022$ therefore conditions (C1) and (C3) are satisfied.

In type 7, start velocity and end velocity is converted to each other.

with trajectory parameters are selected as group 1, the convenient path length is calculated as follows:

$s_{lald}(v_{cf}) = 0.005$; $s_{lald}\left(max\{v_s, v_e\} + a_{max}^2/j_{max}\right) = 0.0033$;

$s_{lald}\left(min\{v_s, v_e\} + a_{max}^2/j_{max}\right) = 0.0024$; $s_{tatd}(max\{v_s, v_e\}) = 0.00035$

The path length is selected in different values $s_i = \{0.0082; 0.0042; 0.0028; 0.0028; 0.0018; 0.0003\}$ to verify the proposed algorithm in type 1, type 5, type 6, type 7, type 8A, type 8B, respectively.

Figure 4 shows the simulation results for group 1. First of all, the final position is the same as the given position for all types and for all groups that are the most important key point. Second, except for type 1, the maximum velocity is always smaller than the feed rate velocity that is suitable. Third, except for type 8b, the start velocity and end velocity are satisfied. In type 8B, because the path length is too small, the end velocity is not satisfied, and it is converted to the same start velocity that is also acceptable.

In the simulation for group 2, $v_{cf} = 0.008$, $v_s = 0.02$ and $v_e = 0.005$ therefore conditions (C1) and (C4) are satisfied.

with trajectory parameters are selected as group 2, the convenient path length is calculated as follows:

$s_{latd}(v_{cf}) = 0.0048$; $s_{tatd}\left(min\{v_s, v_e\} + a_{max}^2/j_{max}\right) = 0.0027$; $s_{tatd} = (max\{v_s, v_e\}) = 0.0012$

The path length is selected in different values $s_i = \{0.0082; 0.0047; 0.0026; 0.0011\}$ to verify the proposed algorithm in type 2, type 6, type 8A, type 8B, respectively.

Table 2: Trajectory parameter and simulation results for the proposed algorithms.

Group	Type	Start Velocity (m/s)	End Velocity (m/s)	Feedrate (m/s)	Given Path Length (m)	Final Path Length (m)	Maximum velocity (m/s)	Total time (s)
1	1	0.01	0.022	0.08	0.0082	0.0082	0.08	0.1443
	5	0.01	0.022	0.08	0.0042	0.0042	0.0719	0.0959
	6	0.01	0.022	0.08	0.0028	0.0028	0.0553	0.0791
	7	0.022	0.01	0.08	0.0028	0.0028	0.0553	0.0791
	8A	0.01	0.022	0.08	0.0018	0.0018	0.0415	0.0634
	8B	0.01	0.022	0.08	0.0003	0.0003	0.0139	0.0251
2	2	0.02	0.005	0.08	0.0082	0.0082	0.08	0.1277
	6	0.02	0.005	0.08	0.0047	0.0047	0.0795	0.0841
	8A	0.02	0.005	0.08	0.0026	0.0026	0.0591	0.0586
	8B	0.02	0.005	0.08	0.0011	0.0011	0.0314	0.0428
3	3	0.005	0.02	0.08	0.0082	0.0082	0.08	0.1277
	7	0.005	0.02	0.08	0.0047	0.0047	0.0795	0.0841
	8A	0.005	0.02	0.08	0.0026	0.0026	0.0591	0.0586
	8B	0.005	0.02	0.08	0.0011	0.0011	0.0529	0.0214
4	4	0.05	0.065	0.08	0.0082	0.0082	0.08	0.1113
	8A	0.05	0.065	0.08	0.0029	0.0029	0.0716	0.0457
	8B	0.05	0.065	0.08	0.0013	0.0013	0.0539	0.025
	8B	0.05	0.065	0.08	0.00001	0.00001	0.05	0.0002

Figure 5 shows the simulation results for group 2. First of all, the final position is the same as the given position for all types and for all groups; that is the most important key point. Second, except for type 2, the maximum velocity is always smaller than the feed rate velocity that is suitable. Third, except for type 8B, the start velocity and end velocity are satisfied. In type 8B, because the path length is too small, the end velocity is not satisfied, and it is converted to the same start velocity that is also acceptable.

In the simulation for group 3, $v_{cf} = 0.008$, $v_s = 0.005$ and $v_e = 0.02$ therefore conditions (C2) and (C3) are satisfied.

with trajectory parameters are selected as group 3, the convenient path length is calculated as follows:

$s_{lald}\left(v_{cf}\right) = 0.0048$; $s_{tatd}\left(min\left\{v_s, v_e\right\} + a_{max}^2/j_{max}\right) = 0.0027$;$s_{tatd} = \left(max\left\{v_s, v_e\right\}\right) = 0.0012$

The path length is selected in different values $s_i = \{0.0082; 0.0047; 0.0026; 0.0011\}$ to verify the proposed algorithm in type 3, type 7, type 8A, type 8B, respectively.

Figure 6 shows the simulation results for group 3. First of all, the final position is the same as the given position for all types and for all groups. That is the most important key point. Second, except for type 3, the maximum velocity is always smaller than the feed rate velocity that is suitable. Third, except for type 8b, the start velocity and end velocity are satisfied. In type 8B, because the path length is too small, the end velocity is not satisfied, and it is converted to the same start velocity that is also acceptable.

In the simulation for group 4, $v_{cf} = 0.008$, $v_s = 0.05$ and $v_e = 0.065$ therefore conditions (C2) and (C4) are satisfied.

with trajectory parameters are selected as group 4, the convenient path length is calculated as follows:

$s_{tatd}\left(v_{cf}\right) = 0.004$; $s_{tatd}\left(max\left\{v_s, v_e\right\}\right) = 0.0014$

The path length is selected in different values $s_i = \{0.0082; 0.0029; 0.0013; 0.0001\}$ to verify the proposed algorithm in type 4, type 8A, type 8B, respectively.

Type 8B is performed twice with a path length of normal value and a very small value.

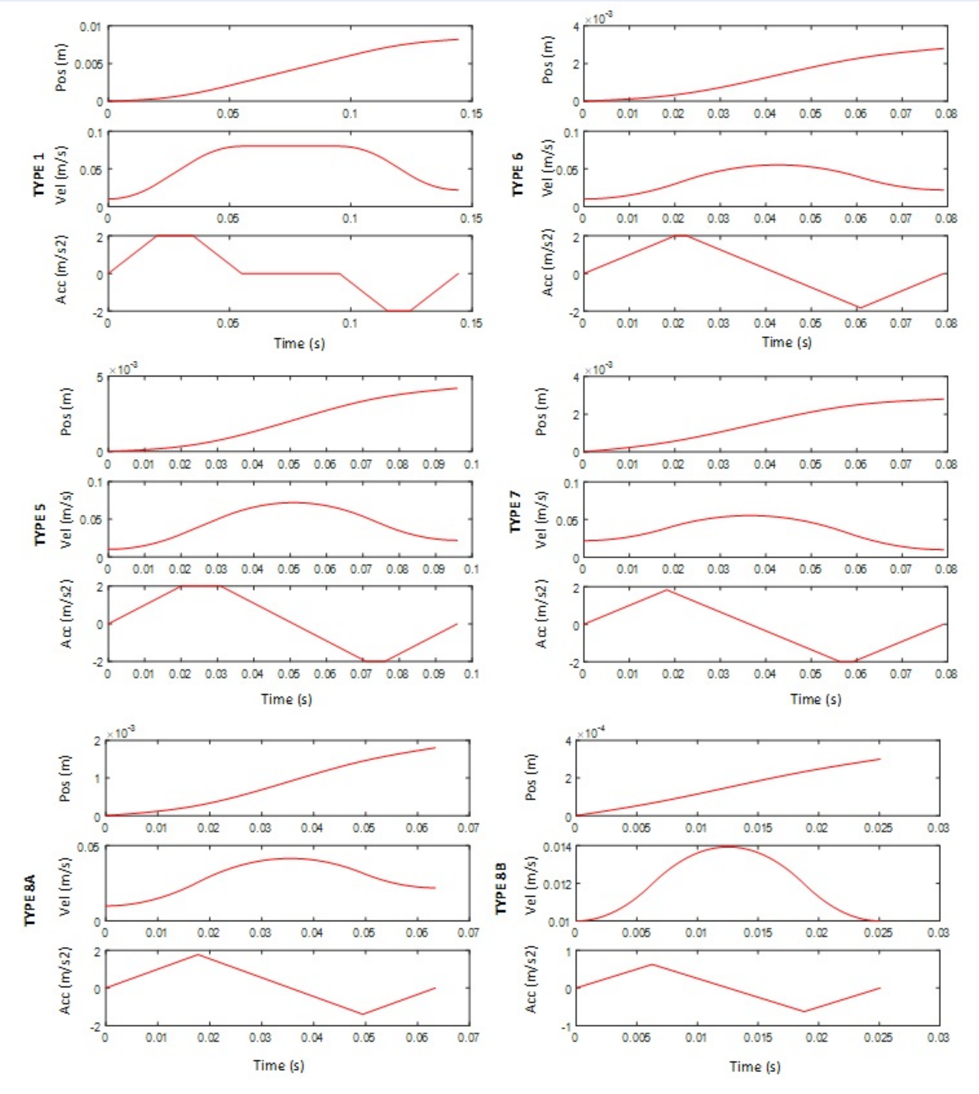

Figure 4: Simulation results for group 1 in planning S-curve motion profile.

Figure 7 shows the simulation results for group 4. First of all, the final position is the same as the given position for all types and for all groups. That is the most important key point. Second, except for type 4, the maximum velocity is always smaller than the feed rate velocity that is suitable. Third, except for type 8B, the start velocity and end velocity are satisfied. In type 8B, because the path length is too small, the end velocity is not satisfied, and it is converted to the same start velocity that is also acceptable.

DISCUSSION

It is obvious that the results in Fig.4 ~ Fig. 7 are consistent with trajectories shown in Fig. 2.

The simulation results are also shown in Table 2. It can be seen that the final position is the same as the given position for all types and for all groups that prove the effectiveness of the proposed algorithm.

The simulation results in group 1, group 2, group 3, and group 4, shows that the proposed algorithm can be applied to any value of the given path length, even very small path length.

Type 1 ~ type 4, the maximum velocity is the same as feed rate velocity.

Type 5 ~ type 8, the maximum velocity is always smaller than feed rate velocity.

In type 8B, because the path length is too small, the end velocity is not satisfied and it is converted to the

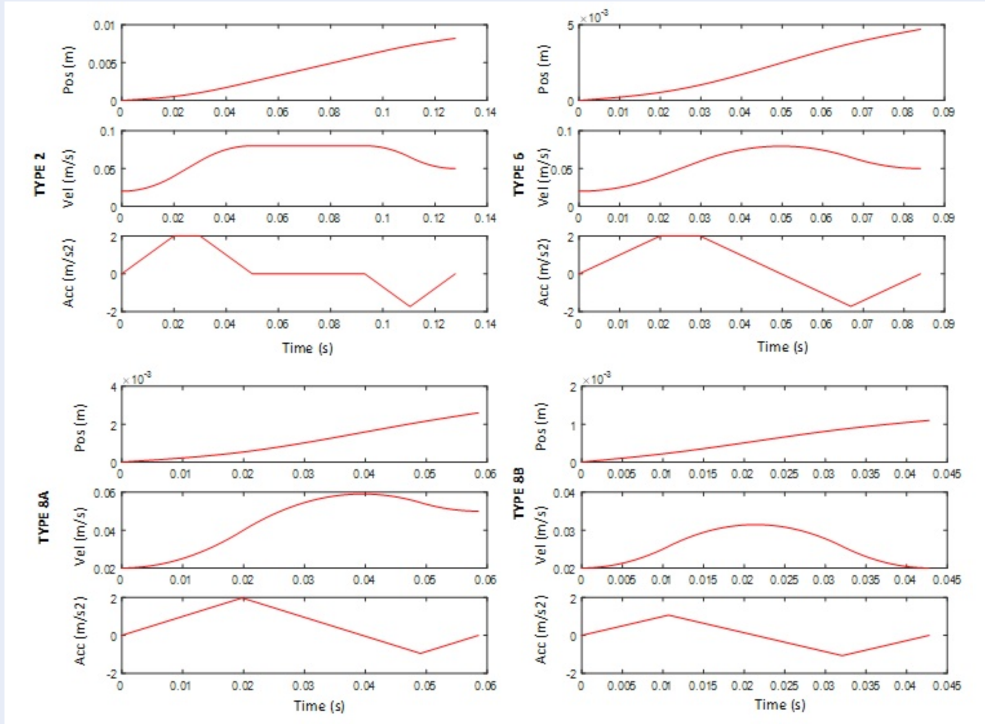

Figure 5: Simulation results for group 2 in planning S-curve motion profile.

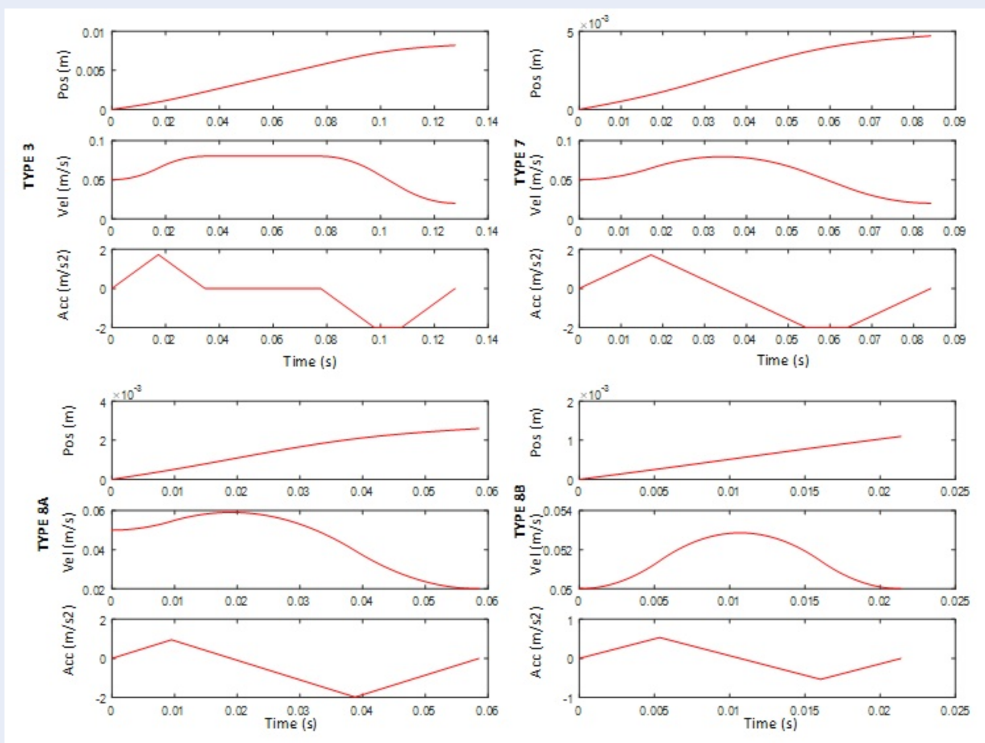

Figure 6: Simulation results for group 3 in planning S-curve motion profile.

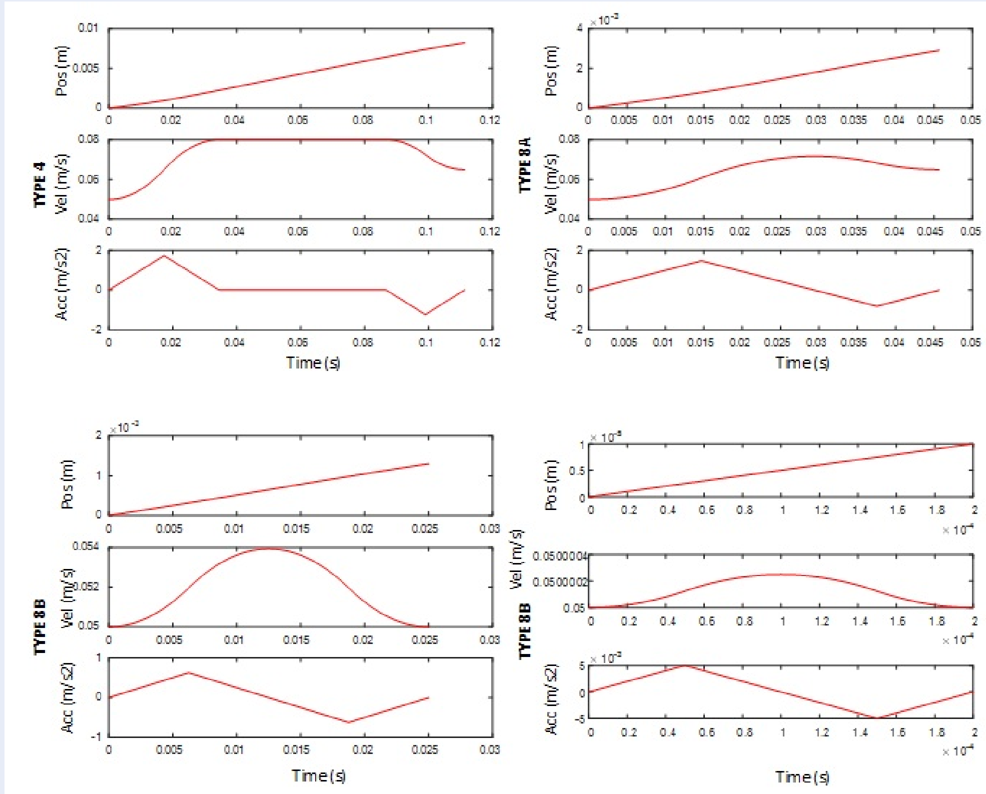

Figure 7: Simulation results for group 4 in planning S-curve motion profile.

same start velocity. But this situation is acceptable.

In types 6 ~ type 8, v_s and v_e should be selected so that $|v_s - v_e| \leq a_{max}^2 / j_{max}$ must be satisfied otherwise v_e will be converted to the same as v_s.

CONCLUSIONS

The proposed algorithm has the ability to generate S-curve trajectory even very small path lengths. The algorithm also ensures that the actuator effort limit is not exceeded and the motion is fast, accurate, with small oscillations, no overshoot in both position and velocity. It is strongly believed that this works can be widely applied in the manufacturing industry, robotics, tool machines. Motion controllers can effectively implement them. The simulation result demonstrates the efficiency and smoothness of the proposed algorithm.

However, there is still plenty of room for further research on this work. We have planned to implement the algorithms into the interpolation technique to optimize the transition velocity.

ACKNOWLEDGMENTS

This research is funded by Nguyen Tat Thanh University. We acknowledge the support of time and facilities from Nguyen Tat Thanh University for this study.

NOMENCLATURE

Pos: Position
Vel: Velocity
Acc: Acceleration
Dec: Deceleration
a_{max}: maximum acceleration
j_{max}: maximum jerk
v_s: start velocity
v_e: end velocity
$v_e^{'}$: end velocity after modification
v_{cf}: feed rate
v_{max}: feed rate after modification
s_i: path length
$t_0 \sim t_7$: point of time, t_{i-1} is start point time and t_i is end point time of section ith.
$\tau_0 \sim \tau_7$: internal time of each section. It starts from zero until T_i for section ith.
$T_0 \sim T_7$: time period of each section

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest.

AUTHORS CONTRIBUTIONS

All authors conceived the study, participated in its design and coordination and helped to draft the manuscript. The authors read and approved the final-manuscript.

REFERENCES

1. Li D, et al. The implementation and experimental research on an S-curve acceleration and deceleration control algorithm with the characteristics of end-point and target speed modification on the fly. Int J Adv Manuf Technol . 2017;91:1145–1169. Available from: https://doi.org/10.1007/s00170-016-9715-9.
2. Yang Q, et al. Characteristics of serrated chip formation in high-speed machining of metallic materials. Int J Adv Manuf Technol. 2016;86:1201–1206. Available from: https://doi.org/10.1007/s00170-015-8265-x.
3. Luo FY, et al. A universal velocity profile generation approach for high-speed machining of small line segments with look-ahead. Int J Adv Manuf Technol. 2007;35:505–518. Available from: https://doi.org/10.1007/s00170-006-0735-8.
4. Der-Min Tsay and Cheng-Feng Lin: Asymmetrical inputs for minimizing residual response. IEEE International Conference on Mechatronics. 2005;p. 235–240. Available from: https://doi.org/10.1109/ICMECH.2005.1529259.
5. Wang L, Cao J. A look-ahead and adaptive speed control algorithm for high-speed CNC equipment. Int J Adv Manuf Technol . 2012;63:705–717. Available from: https://doi.org/10.1007/s00170-012-3924-7.
6. Fang Y, et al. Smooth and time-optimal S-curve trajectory planning for automated robots and machines. Mechanism and Machine Theory. 2019;137:127–153. Available from: http://dx.doi.org/10.1016/j.mechmachtheory.2019.03.019.
7. Kim DN. On Algorithms for Planning S-Curve Motion Profiles. Int. Journal of Advanced Robotic Systems. 2008;5(1):99–106. Available from: https://doi.org/10.5772/5652.
8. Youdong C. Look-Ahead Algorithm with Whole S-Curve Acceleration and Deceleration. Advances in Mechanical Engineering. 2013;5:974152. Available from: http://dx.doi.org/10.1155/2013/974152.
9. Huang H. An adjustable look-ahead acceleration/deceleration hybrid interpolation technique with variable maximum feedrate. Int J Adv Manuf Technol. 2018;95:1521–1538. Available from: https://doi.org/10.1007/s00170-017-1277-y.