EINSTEIN'S EQUATION IN THE VECTOR MODEL FOR GRAVITATIONAL FIELD

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ABSTRACT: *In this paper, based on the vector model for gravitational field we deduce a equation to determinate the metric of space- time. This equation is similar to Einstein's equation. The metric of space – time outside a static spherical symmetric body is also determined. It gives a small supplementation to the Schwarzschild metric in the General Theory of Relativity but no singular sphere exists.*

1. INTRODUCTION

From the assumption of the Lorentz invariance of gravitational mass, we used the vector model to describe gravitational field in the non- relativistic case and the relativistic one [1]. In these descriptions, space- time is flat yet because we did not consider to the influence of gravitational field upon the metric of space- time yet. From the previous paper [2], we have known that the field of inertial forces is just the field of gravitational force and moreover space- time is curvature with the present of inertial forces [3]. Therefore space – time also becomes the curvature one with the present of gravitational field.

In this paper we shall deduce a equation to describe the relation between gravitational field, a vector field, with the metric of space- time. This equation is similar to Einstein's equation. We say it as *Einstein's equation in the vector model for gravitational field.*

This equation is deduced from a Lagrangian which is similar to the Lagrangians in the vector – tensor models for gravitational field [4,5,6,7]. Nevertheless in those models the vector field takes only a supplemental role beside the gravitational field which is a tensor field. The tensor field is just the metric tensor of space- time. Those authors wanted to homogenize the vector field with the electromagnetic field . In this model the gravitational field is the vector field and its resource is gravitational mass of bodies. This vector field and the energy- momentum tensor of gravitational matter determine the metric of space – time. The second part is a Einstein's essential idea and it is required so that this model has the classical limit.

In this paper we also deduce a solution of this equation for a static spherical symmetric body. The obtained metric is different to the Schwarzschild metric with a small supplementation of high degree and no singular sphere exists.

2. LAGRANGIAN AND FIELD EQUATION

We choose the following action

$$S = S_E + S_{Mg} + S_g \qquad (1)$$

with $S_{H-E} = \int \sqrt{-g}(R+\Lambda)d^4x$ is the classical Hilbert –Einstein action .

S_{Mg} is the gravitational matter action.

$S_g = \omega \int \frac{1}{16\pi} \sqrt{-g}(E_{g\mu\nu} E_g^{\mu\nu}) d^4x$ is the gravitational action.

Where $E_{g\mu\nu}$ is tensor of strength of gravitational field.

Variation of the action (1) with respect to the metric tensor leads to the following modified Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda = -\frac{8\pi G}{c^4} T_{Mg.\mu\nu} + \omega T_{g.\mu\nu} \qquad (2)$$

Note that

- Variation of the Hilbert – Einstein action leads to the left- hand side of equation (2) as in the General Theory of Relativity.

- Variation of the gravitational matter action S_{Mg} leads to the energy- momentum tensor of the gravitational matter $T_{Mg.\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_{Mg}}{\delta g^{\mu\nu}}$

- Variation of the gravitational action S_g leads to the energy- momentum tensor of gravitational field $T_{g.\mu\nu} \equiv -\frac{2}{\omega\sqrt{-g}} \frac{\delta S_g}{\delta g^{\mu\nu}}$

Let us discuss particularly to two tensors in the right – hand side of equation (2).

We recall that the original Einstein's equation is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda = -\frac{8\pi G}{c^4} T_{\mu\nu} \qquad (3)$$

Where $T_{\mu\nu}$ is the energy- momentum tensor of the matter. For example, for a fluid matter of non- interacting particles with the proper inertial mass density $\rho_0(x)$, with a field of 4- velocity $u^\mu(x)$ and a field of pressure $p(x)$, the energy- momentum tensor of the matter is [8,9]

$$T^{\mu\nu} = \rho_0 c^2 u^\mu u^\nu + p(u^\mu u^\nu - g^{\mu\nu}) \qquad (4)$$

If we say ρ_{g0} as the gravitational mass density of this fluid matter, the energy- momentum tensor of the gravitational matter is

$$T_{Mg}^{\mu\nu} = \rho_{g0} c^2 u^\mu u^\nu + p(u^\mu u^\nu - g^{\mu\nu}) \qquad (5)$$

For the fluid matter of electrically charged particles with the gravitational mass ρ_{g0}, a field of 4- velocity $u^\mu(x)$, and a the electrical charge density $\sigma_0(x)$, the energy – momentum tensor of the gravitational matter is

$$T^{\mu\nu}_{Mg} = \rho_{g0}c^2u^\mu u^\nu + \frac{1}{4\pi}(-F^\mu_\alpha F^{\alpha\nu} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta})_g \quad (6)$$

The word "g" in the second term group indicates that we choose the density of gravitational mass which is equivalent to the energy density of the electromagnetic field. Where $F_{\alpha\beta}$ is the electromagnetic field tensor.

Note that because of the close equality between the inertial mass and the gravitational mass, the tensor $T_{\mu\nu}$ is closely equivalent to the tensor $T_{Mg.\mu\nu}$. The only distinct character is that the inertial mass depends on inertial frame of reference while the gravitational mass does not depend one. However the value of ρ_0 in the equation (4) is just the proper density of inertial mass, therefore it also does not depend on inertial frame of reference. Thus, the modified Einstein's equation (2) is principally different with the original Einstein's equation (3) in the present of the gravitational energy- momentum tensor in the right-hand side.

From the above gravitational action, the gravitational energy-momentum tensor is

$$T_{g.\mu\nu} \equiv -\frac{2}{\omega\sqrt{-g}}\frac{\delta S_g}{\delta g^{\mu\nu}} = \frac{1}{4\pi}(E^{\alpha}_{g.\mu}E_{g.\nu\alpha} - \frac{1}{4}g_{\mu\nu}E^{\alpha\beta}_g E_{g.\alpha\beta}) \quad (7)$$

Where $E_{g.\alpha\beta}$ is the tensor of strength of gravitational field [1]. The expression of (7) is obtained in the same way with the energy- momentum tensor of electromagnetic field.

Let us now consider the equation (2) for the space- time outside a body with the gravitational mass M_g (this case is similar to the case of the original Einstein's equation for the empty space). However in this case, the space is not empty although it is outside the field resource, the gravitational field exists everywhere. We always have the present of the gravitational energy-momentum tensor in the right-hand side of the equation (2) .When we reject the cosmological constant Λ, the equation (2) leads to the following form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \omega T_{g.\mu\nu} \quad (8)$$

or :

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \omega\frac{1}{4\pi}(E^{\alpha}_{g.\mu}E_{g.\alpha\nu} - \frac{1}{4}g_{\mu\nu}E^{\alpha\beta}_g E_{g.\alpha\beta}) \quad (9)$$

3.THE EQUATIONS OF GRAVITATIONAL FIELD IN CURVATURE SPACE- TIME

We have known the equations of gravitational field in flat space- time [1]

$$\partial_k E_{g.mn} + \partial_m E_{g.nk} + \partial_n E_{g.km} = 0 \quad (10)$$

and

$$\partial_i D^{ik}_g = J^k_g \quad (11)$$

The metric tensor is flat in these equations.

When the gravitational field exists, because of its influence to the metric tensor of space-time, we shall replace the ordinary derivative by the covariant derivative. The above equations become

$$E_{g.mn;k} + E_{g.nk;m} + E_{g.km;n} = 0 \qquad (12)$$

$$\frac{1}{\sqrt{-g}} \partial_i (\sqrt{-g} D_g^{ik}) = J_g^k \qquad (13)$$

4. THE METRIC TENSOR OF SPACE-TIME OUTSIDE A STATIC SPHERICAL SYMMETRICAL BODY

We resolve the equations (9), (12) ,(13) outside the resource to find the metric tensor of space- time. Thus we have the following equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \omega \frac{1}{4\pi} (E^{\alpha}_{g.\mu} E_{g.\alpha\nu} - \frac{1}{4} g_{\mu\nu} E^{\alpha\beta}_{g} E_{g.\alpha\beta}) \qquad (14)$$

$$E_{g.mn;k} + E_{g.nk;m} + E_{g.km;n} = 0 \qquad (15)$$

$$\partial_i (\sqrt{-g} E_g^{ik}) = 0 \qquad (16)$$

Because the resource is static spherical symmetrical body, we also have the metric tensor in the Schwarzschild form as follows [8]

$$g_{\mu\alpha} = \begin{pmatrix} e^{\nu} & & & \\ & -e^{\lambda} & & \\ & & -r^2 & \\ & & & -r^2 \sin^2\theta \end{pmatrix} \qquad (17)$$

and

$$g^{\mu\alpha} = \begin{pmatrix} e^{-\nu} & & & \\ & -e^{-\lambda} & & \\ & & -r^{-2} & \\ & & & -\frac{1}{r^2 \sin^2\theta} \end{pmatrix} \qquad (18)$$

The left-hand side of (14) is the Einstein 's tensor , it has only the non-zero components as follows [8,9,10]

$$R_{00} - \frac{1}{2} g_{00} R = e^{\nu-\lambda} (-\frac{\lambda'}{r} + \frac{1}{r^2}) - \frac{1}{r^2} e^{\nu} \qquad (19)$$

$$R_{11}-\frac{1}{2}g_{11}R=-\frac{\nu'}{r}-\frac{1}{r^2}+\frac{1}{r^2}e^{\lambda} \tag{20}$$

$$R_{22}-\frac{1}{2}g_{22}R=e^{-\lambda}[\frac{r^2}{4}\nu'\lambda'-\frac{r^2}{4}(\nu')^2-\frac{r^2}{2}\nu''-\frac{r}{2}(\nu'-\lambda')] \tag{21}$$

$$R_{33}-\frac{1}{2}g_{33}R=(R_{22}-\frac{1}{2}g_{22}R)\sin^2\theta \tag{22}$$

$R_{\mu\nu}=0$, $g_{\mu\nu}=0$ with $\mu\neq\nu$.

The tensor of strength of gravitational field $E_{g.\mu\nu}$ when it was corrected the metric tensor needs corresponding to a static spherical symmetrical gravitational field $E_g(r)$. From the form of $E_{g.\mu\nu}$ in flat space- time [1]

$$E_{g.\mu\nu}=\begin{pmatrix} 0 & -E_{gx}/c & -E_{gy}/c & -E_{gz}/c \\ E_{gx}/c & 0 & H_{gz} & -H_{gy} \\ E_{gy}/c & -H_{gz} & 0 & H_{gx} \\ E_{gz}/c & H_{gy} & -H_{gx} & 0 \end{pmatrix} \tag{23}$$

For static spherical symmetrical gravitational field, the magneto–gravitational components $\vec{H}_g=0$. We consider only in the X- direction, therefore the components $E_{gy},E_{gz}=0$. We find a solution of $E_{g.\mu\nu}$ in the following form

$$E_{g.\mu\nu}=\frac{1}{c}E_g(r)\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{24}$$

Note that because $E_{g.\mu\nu}$ is a function of only r, it satisfies the equation (15) regardless of function $E_g(r)$.The function $E_g(r)$ is found at the same time with μ and và ν from the equations (14) and (16). Raising indices in (24) with $g^{\alpha\beta}$ in (18), we obtain

$$E_g^{\mu\alpha}=\frac{1}{c}e^{-(\nu+\lambda)}E_g(r)\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{25}$$

and

$$\sqrt{-g}E_g^{\mu\alpha} = \frac{1}{c}e^{-(\nu+\lambda)/2}r^2E_g\sin\theta\begin{pmatrix}0 & 1 & 0 & 0\\ -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \tag{26}$$

Substituting (26) into (16), we obtain an only nontrivial equation

$$\left[e^{-(\nu+\lambda)/2}r^2E_g\sin\theta\right]' = 0 \tag{27}$$

We obtain a solution of (27)

$$e^{-(\nu+\lambda)/2}r^2E_g\sin\theta = cons\tan t$$

or

$$E_g = e^{(\nu+\lambda)/2}.\frac{cons\tan t}{r^2} \tag{28}$$

We require that space- time is Euclidian one at infinity, it leads to that both ν and $\lambda \to 0$ when $r \to \infty$, therefore the solution (28) has the normal classical form when r is larger, i.e.

$$E_g \to -\frac{GM_g}{r^2}$$

Therefore $$cons\tan t \equiv -GM_g \tag{29}$$

To solve the equation (14), we have to calculate the energy- momentum tensor in the right-hand side of it. We use (28) to rewrite the tensor of strength of gravitational field in three forms as follows

$$E_{g.\mu\alpha} = \frac{1}{c}e^{(\nu+\lambda)/2}(-\frac{GM_g}{r^2})\begin{pmatrix}0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \tag{30}$$

$$E_g^{\mu\alpha} = \frac{1}{c}e^{-(\nu+\lambda)/2}(-\frac{GM_g}{r^2})\begin{pmatrix}0 & 1 & 0 & 0\\ -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \tag{31}$$

$$E^{\alpha}_{g.\mu} = \frac{1}{c}(-\frac{GM_g}{r^2})\begin{pmatrix} 0 & e^{(\nu-\lambda)/2} & 0 & 0 \\ e^{(\lambda-\nu)/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (32)$$

we obtain the following result

$$T_{g.\mu\alpha} = \frac{1}{4\pi}[E_{g.\mu\beta}E^{\beta}_{g.\alpha} - \frac{1}{4}g_{\mu\alpha}E_{g.kl}E^{kl}_{g}]$$

$$= \frac{G^2M_g^2}{8\pi c^2 r^4}\begin{pmatrix} e^{\nu} & 0 & 0 & 0 \\ 0 & -e^{\lambda} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} \quad (33)$$

From the equations (14),(19),(20),(21),(22) and(33), we have the following equations

$$e^{\nu-\lambda}(-\frac{\lambda'}{r} + \frac{1}{r^2}) - \frac{1}{r^2}e^{\nu} = \omega\frac{G^2M_g^2}{8\pi c^2 r^4}e^{\nu} \quad (34)$$

$$-\frac{\nu'}{r} - \frac{1}{r^2} + \frac{1}{r^2}e^{\lambda} = -\omega\frac{G^2M_g^2}{8\pi c^2 r^4}e^{\lambda} \quad (35)$$

$$e^{-\lambda}[\frac{r^2}{4}\nu'\lambda' - \frac{r^2}{4}(\nu')^2 - \frac{r^2}{2}\nu'' - \frac{r}{2}(\nu' - \lambda')] = \omega\frac{G^2M_g^2}{8\pi c^2 r^4}r^2 \quad (36)$$

Multiplying two members of (34) with $e^{-(\nu-\lambda)}$ then add it with (35), we obtain

$$\nu' + \lambda' = 0 \Rightarrow \nu + \lambda = constant \quad (37)$$

Because both ν and λ lead to zero at infinity, the constant in (37) has to be zero.

Therefore, we have $\nu = -\lambda$ (38)

Using (37), we rewrite (36) as follows

$$e^{\nu}[-\frac{r^2}{4}(\nu')^2 - \frac{r^2}{4}(\nu')^2 - \frac{r^2}{2}\nu'' - \frac{r}{2}(\nu' + \nu')] = \omega\frac{G^2M_g^2}{8\pi c^2 r^4}r^2$$

or

$$e^{\nu}[(\nu')^2 + \nu'' + \frac{2}{r}\nu'] = -\omega\frac{G^2M_g^2}{4\pi c^2 r^4} \quad (39)$$

$$e^{\nu}[(\nu')^2 + \nu''] + \frac{2}{r}\nu' e^{\nu} = -\omega\frac{G^2M_g^2}{4\pi c^2 r^4} \quad (40)$$

We rewrite (40) in the following form

$$(e^{\nu}\nu')' + \frac{2}{r}\nu' e^{\nu} = -\omega \frac{G^2 M_g^2}{4\pi c^2 r^4} \tag{41}$$

Put $y = e^{\nu}\nu'$, (41) becomes

$$y' + \frac{2}{r}y = -\omega \frac{G^2 M_g^2}{4\pi c^2 r^4} \tag{42}$$

The differential equation (42) has the standard form as follows

$$y' + p(r)y = q(r) \tag{43}$$

The solution $y(r)$ is as follows [11]

Put: $$\mu(r) = e^{\int p(r)dr} = e^{\int \frac{2}{r}dr} = e^{2\ln r} = r^2 \tag{44}$$

We have

$$y(r) = \frac{1}{\mu(r)}\left(\int q(r).\mu(r)dr + A\right)$$

$$= \frac{1}{r^2}\left[\int\left(-\omega\frac{G^2 M_g^2}{4\pi c^2 r^4}\right)r^2 dr + A\right]$$

$$= \frac{1}{r^2}\left[\omega\frac{G^2 M_g^2}{4\pi c^2 r} + A\right]$$

$$= \omega\frac{G^2 M_g^2}{4\pi c^2 r^3} + \frac{A}{r^2} \tag{45}$$

Where A is the integral constant.

From the above definition of $y = e^{\nu}\nu'$, we have

$$e^{\nu}\nu' = (e^{\nu})' = \omega\frac{G^2 M_g^2}{4\pi c^2 r^3} + \frac{A}{r^2}$$

or

$$e^{\nu} = \int\left(\omega\frac{G^2 M_g^2}{4\pi c^2 r^3} + \frac{A}{r^2}\right)dr$$

$$= -\omega\frac{G^2 M_g^2}{8\pi c^2 r^2} - \frac{A}{r} + B \tag{46}$$

Where B is a new integral constant.

We shall determine the constants A,B from the non-relativistic limit. We know that the Lagrangian describing the motion of a particle in gravitational field with the potential φ_g has the form [10]

$$L = -mc^2 + \frac{mv^2}{2} - m\varphi_g$$

The corresponding action is

$$S = \int L dt = -mc\int (c - \frac{v^2}{2c} + \frac{\varphi_g}{c})dt = -mc\int ds$$

we have $$ds = (c - \frac{v^2}{2c} + \frac{\varphi_g}{c})dt$$

that is $$ds^2 = (c^2 + \frac{v^4}{4c^2} + \frac{\varphi_g^2}{c^2} - v^2 + 2\varphi_g - \frac{v^2\varphi_g}{c^2})dt^2$$

$$= (c^2 + 2\varphi_g)dt^2 - v^2 dt^2 + ...$$

$$= c^2(1 + 2\frac{\varphi_g}{c^2})dt^2 - dr^2 + ... \qquad (47)$$

Where we reject the terms which lead to zero when c approaches to infinity.

Comparing (47) with the our line element (we reject the terms in the coefficient of dr^2)

$$ds^2 = e^{\nu} c^2 dt^2 - dr^2 \qquad (48)$$

we get

$$-\frac{A}{r} + B \equiv 2\frac{\varphi_g}{c^2} + 1$$

$$\equiv -2\frac{GM_g}{c^2 r} + 1 \qquad (49)$$

From (49) we have

$$A = 2\frac{GM_g}{c^2} \qquad (50)$$

$$B = 1$$

The constant ω does not obtain in the non relativistic limit because it is in high accurate terms, we shall determine it later.

Thus, we get the following line element

$$ds^2 = c^2(1 - 2\frac{GM_g}{c^2 r} - \omega\frac{G^2M_g^2}{8\pi c^2 r^2})dt^2 - (1 - 2\frac{GM_g}{c^2 r} - \omega\frac{G^2M_g^2}{8\pi c^2 r^2})^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (51)$$

we put $\frac{\omega}{8\pi} = \frac{\omega'}{c^2}$ and rewrite the line element (51)

$$ds^2 = c^2(1 - 2\frac{GM_g}{c^2 r} - \omega'\frac{G^2M_g^2}{c^4 r^2})dt^2 - (1 - 2\frac{GM_g}{c^2 r} - \omega'\frac{G^2M_g^2}{c^4 r^2})^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (52)$$

We determine the parameter ω' from the experiments in the Solar system. We use the Robertson – Eddington expansion [9] for the metric tensor in the following form

$$ds^2 = c^2\left(1-2\alpha\frac{GM_g}{c^2 r}+2(\beta-\alpha\gamma)\frac{G^2M_g^2}{c^4r^2}+\ldots\right)dt^2-\left(1+2\gamma\frac{GM_g}{c^2r}+\ldots\right)dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2) \quad (53)$$

When comparing (52) with (53), we have

$$\alpha=\gamma=1 \quad (54)$$

and
$$\omega'=2(1-\beta) \quad (55)$$

The predictions of the Einstein field equations can be neatly summarized as

$$\alpha=\beta=\gamma=1 \quad (56)$$

From the experimental data in the Solar system, people [43] obtained

$$\left(\frac{2-\beta+2\gamma}{3}\right)=1.00\pm0.01 \quad (57)$$

With $\gamma=1$ in this model, we have

$$\omega'=2(1-\beta)=0.00\pm0.06 \quad (58)$$

Thus
$$|\omega'|\le 0.06 \quad \text{hence} \quad |\omega|\le 0.48\pi c^{-2} \quad (59)$$

The line element (52) gives a very small supplementation to the Schwarzschild line element.

It is interesting to note that the function $e^{\nu}=1-2\frac{GM_g}{c^2r}-\omega'\frac{G^2M_g^2}{c^4r^2}$ takes the form shown in Fig.1

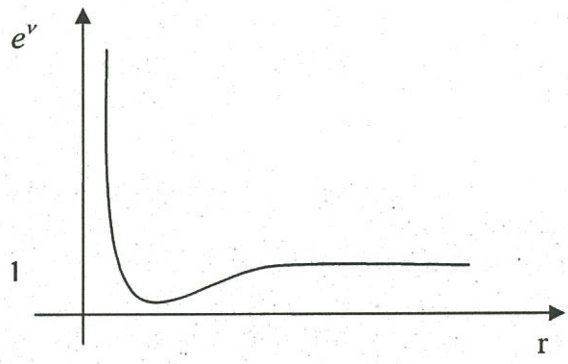

Fig.1. the graphic of function e^{ν}

In particular, we see that no singular sphere exists in the line element (52), unlike the case of the ordinary Schwarzschild line element which possesses a singular sphere at $r=2\frac{GM_g}{c^2}$.

5. CONCLUSION

In conclusion, based on the vector model for gravitational field we deduce a modified Einstein's equation. This equation gives a small supplementation to the results of General Theory of Relativity and in particular no singular sphere exists. Some different effects of GTR will be investigated later.

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TÓM TẮT: *Trong bài báo này, dựa trên mô hình véctơ cho trường hấp dẫn chúng tôi rút ra một phương trình để xác định mêtric của không – thời gian. Phương trình này là tương tự với phương trình Einstein. mêtric của không – thời gian bên ngoài một vật đối xứng cầu, dừng cũng được xác định. Nó cho một bổ chính nhỏ vào phần tử đường Schwarzchild của Thuyết Tương Đối Tổng Quát nhưng không có cầu kì dị trong nghiệm này.*

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