TEMPERATURE DEPENDENCE OF MAGNETIC SUSCEPTIBILITY OF THE TWO-SUBLATTICE FRUSTRATED ISING SYSTEMS

Hoang Zung
College of Natural Sciences, VNU HCM
Mai Suan Li
Institute of Physics, Warsaw, Poland
(*Received February 15, 1999*)

Abstract: The temperature dependence of the susceptibility of the frustrated two-sublattice Ising system is studied in the mean field approximation. The susceptibility of the frustrated and disordered antiferromagnet is found to have a minimum in the ergodic phase. The field cooled susceptibility of the strongly frustrated ferrimagnet may depend on the temperature even in the nonergodic phase. Our findings agree qualitatively with the experimental data.

PACS numbers: 71.28.+d, 71.27.+a

1 Introduction

There has been a considerable interest in the study of magnetic systems with multi-sublattice structure where the spin glass phase may appear due to the competition between ferro- and antiferromagnetic interactions [1, 2]. In particular, the behavior of magnetic susceptibility of the two-sublattice systems has been intensively investigated both experimentally [2-6] and theoretically [7, 8]. It is well known that the field cooled susceptibility, χ_{FC}, of frustrated ferro- and antiferromagnets is temperature independent below the freezing point T_g. On the other hand, experiment shows that FC susceptibility of frustrated ferrimagnets depends on temperature in the nonergodic phase ($T < T_g$) [2, 5, 6]. Another interesting feature of multi-sublattice systems is unusual behavior of the static susceptibility of frustrated antiferromagnets even in the ergodic phase ($T > T_g$) [4]. A qualitative explanation of such temperature dependence of susceptibility is the goal of our report. We will relay on the two-sublattice frustrated Ising model, proposed in [8].

2 Basic equations

We will consider a frustrated Ising system consisting of two spin sublattices and model it by the Sherrington-Kirkpatrick - type Hamiltonian [8]

$$\mathcal{H} = \sum_{i,j} J_{ij} S_{1i} S_{2j} - \sum_{p} \sum_{i,j} I_{ij}^{(p)} S_{pi} S_{pj} - H \sum_{p} g_p \sum_{i=1}^{N_p} S_{pi} \quad (1)$$

Here the subscript p labels the spin sublattices ($p = 1, 2$), S_{pi} are Ising spins in nature ($S_{pi} = \pm S_p$), H is the applied magnetic field, N_p and g_p are the number of spins and Lande factor of the p^{th} sublattice respectively. The inter- and intra-sublattice exchange interactions J_{ij} and $I_{ij}^{(p)}$ are supposed to be Gaussian distributed with the mean values and variences given by:

$$< J_{ij} >= J_0, \quad < (J_{ij} - J_0)^2 >^{1/2} = J,$$
$$< I_{ij}^{(p)} = I_{0p}, \quad < (I_{ij}^{(p)} - I_{0p})^2 >^{1/2} = I_p \tag{2}$$

Using the Parisi ansatz [9], in [8] we derived the self-consistent system of equations for our model (1). The equations for the sublattice magnetizations $\tilde{m}_p$ and the Parisi functions $\tilde{q}_p$ are

$$\tilde{m}_p = \int P_p(0, y)\tilde{m}_p(0, y)dy, \quad \tilde{q}_p = \int P_p(x, y)\tilde{m}_p^2(x, y)dy \tag{3}$$

where the functions $\tilde{m}_p(x, y)$ and $P_p(x, y)$ satisfy the equations

$$\dot{\tilde{m}}_p = -\frac{\dot{Q}_p(x)}{2}\left[\tilde{m}_p''(x, y) + \frac{2x}{T}\tilde{m}_p(x, y)\tilde{m}_p'(x, y)\right] \tag{4}$$

and

$$\dot{P}_p(x, y) = \frac{\dot{Q}_p(x)}{2}\left[P_p''(x, y) - \frac{2x}{T}\left(P_p\tilde{m}_p\right)'\right] \tag{5}$$

with the boundary conditions

$$\tilde{m}_p(1, y) = \tanh(y/T), \quad P_p(0, y) = \frac{1}{\sqrt{2\pi Q_p(0)}}\exp\left[-\frac{(y - h_p)^2}{2Q_p(0)}\right] \tag{6}$$

In the Eqs.(3)-(6) we have introduced the following notations

$$\tilde{m}_p = m_p/S_p, \quad \tilde{q}_p(x) = q_p(x)/S_p^2, \quad \tilde{g}_p = S_p g_p,$$
$$\tilde{J} = S_1 S_2 J, \quad \tilde{J}_0 = S_1 S_2 J_0, \quad \tilde{I}_p = S_p^2 I_p, \quad \tilde{I}_{0p} = S_p^2 I_{0p} \tag{7}$$
$$n_p = N_p/N, \quad \alpha_p = \sqrt{n_p/n_{p'}} \quad (p \neq p')$$

and

$$h_p = \tilde{g}_p H + \tilde{I}_{0p}\tilde{m}_p - \alpha_{p'}\tilde{J}_0\tilde{m}_{p'}, \quad Q_p(x) = \tilde{I}_p^2\tilde{q}_p(x) + \alpha_{p'}\tilde{J}^2\tilde{q}_{p'},$$
$$\dot{f}(x, y) = \partial f/\partial x, \quad f'(x, y) = \partial f/\partial y \tag{8}$$

The Parisi marginal stability condition for our model is given by

$$\left(\tilde{J}^4 - \tilde{I}_1^2\tilde{I}_2^2\right)A_1(x)A_2(x) + \sum_p \tilde{I}_p^2 A_p(x) = 1 \tag{9}$$

where

$$A_p(x) = \int P_p(x,y)\left(\tilde{m}_p'(x,y)\right)^2 dy \tag{10}$$

In the ergodic phase $(T > T_g)$ the functions $\tilde{m}_p(x,y)$ and $P_p(x,y)$ do not depend on x and may be replaced by $\tilde{m}_p(1,y)$ and $P_p(0,y)$ respectively. Then from Eqs.(3)-(6) we get the state equations for our system in the ergodic phase

$$\tilde{m}_p = < \tanh E_p(z) >_c, \quad \tilde{q}_p = < \tanh^2 E_p(z) >_c \tag{11}$$

where

$$E_p(z) = T^{-1}\left(\tilde{g}_p H + \tilde{I}_{0p}\tilde{m}_p - \alpha_{p'}\tilde{J}_0 m_{p'} + z\sqrt{\tilde{I}_p^2 q_p + \alpha_{p'}\tilde{J}^2\tilde{q}_{p'}}\right),$$
$$< A(z) >_c = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-z^2/2} A(z)dz \tag{12}$$

The Parisi marginal stability condition (9) transforms into the equation of the De Almeida Thouless line

$$\frac{\tilde{J}^4 - \tilde{I}_1^2\tilde{I}_2^2}{T^4} < \cosh^{-4} E_1(z) >_c < \cosh^{-4} E_2(Z) >_c + \sum_p \frac{\tilde{I}_p^2}{T^2} < \cosh^{-4} E_p(z) >_c = 1 \tag{13}$$

Together with Eqs.(11), last equation allows us to determine the transition temperature T_g.

The Eqs.(3)-(13) had been derived in our previous papers [8] and will be used in the next sections for studying the static susceptibility of the two-sublattice frustrated Ising systems.

3 Magnetic susceptibility in ergodic phase

The total magnetic susceptibility of a two-sublattice system is given by

$$\chi = n_1\tilde{g}_1\chi_1 + n_2\tilde{g}_2\chi_2 \tag{14}$$

where χ_p is the susceptibility of the p^{th} sublattice

$$\chi_p = \partial\tilde{m}_p/\partial H|_{H=0} \tag{15}$$

Differentiating Eqs.(11) with respect to H we derived the equations for χ_p

$$\left[1 - \frac{\tilde{I}_{01}}{T}(1-\tilde{q}_1)\right]\chi_1 + \alpha_2\frac{\tilde{J}_0}{T}(1-\tilde{q}_1)\chi_2 + \frac{\tilde{I}_1^2 U_1}{T^2}\lambda_1 + \alpha_2\frac{\tilde{J}^2 U_1}{T^2}\lambda_2 = \frac{1-\tilde{q}_1}{T},$$
$$\alpha_1\frac{\tilde{J}_0}{T}(1-\tilde{q}_2)\chi_1 + \left[1 - \frac{\tilde{I}_{02}}{T}(1-\tilde{q}_2)\right]\chi_2 + \alpha_1\frac{\tilde{J}^2 U_2}{T^2}\lambda_1 + \frac{\tilde{I}_1^2 U_2}{T^2}\lambda_2 = \frac{1-\tilde{q}_2}{T},$$
$$-2\frac{\tilde{I}_{01}U_1}{T}\chi_1 + 2\alpha_2\frac{\tilde{J}_0 U_2}{T}\chi_2 + \left(1 - \frac{\tilde{I}_1^2 V_1}{T^2}\right)\lambda_1 - \alpha_2\frac{\tilde{J}^2 V_1}{T^2}\lambda_2 = 2U_1/T,$$
$$2\alpha_1\frac{\tilde{J}_0 U_1}{T}\chi_1 - 2\frac{\tilde{I}_{02}U_2}{T}\chi_2 - \alpha_1\frac{\tilde{J}^2 V_2}{T^2}\lambda_1 + + \left(1 - \frac{I_2^2 V_2}{T^2}\right)\lambda_2 = 2U_2/T \tag{16}$$

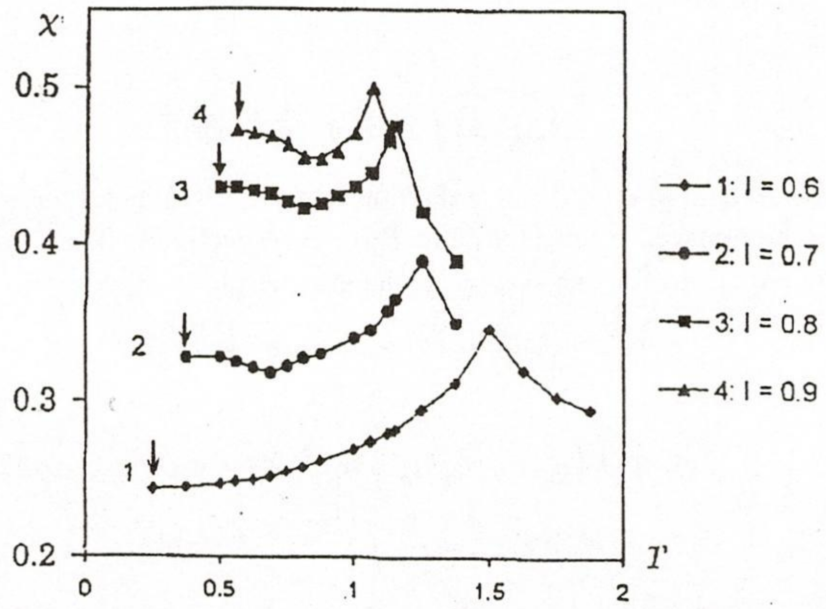

Figure 1: The temperature and intra-sublattice frustration dependence of the static susceptibility of the two-equivalent sublattice magnet in the ergodic phase. Arrows in the figure indicate the freezing point T_g determined by Eq.(13). The mean value of the exchange interactions is chosen to be equal $I_0 = J_0 = 1$. The varience of the intra-sublattice interaction $I = 0.6, 0.7, 0.8$ and 0.9.

where

$$\lambda_p \equiv \partial\tilde{q}_p/\partial H,\ U_p = \tilde{m}_p - < \tanh^3 E_p >_c,\ V_p = 1 - 4\tilde{q}_p + 3 < \tanh^4 E_p >_c \tag{17}$$

In Fig.1 the result of numerical interation of Eqs.(16) for the case of two equivalent sublattice system ($\alpha_1 = \alpha_2 = 1, I_{01} = I_{02} \equiv I, I_1 = I_2 \equiv I, S_1 = S_2 = 1$) is presented. We choose the mean value of inter- and intra-sublattice interactions $J_0 = I_0 = 1$ and their variences $J = 0$ and $I = 0.6, 0.7, 0.8, 0.9$. Obviously, in this case the intra-sublattice frustration is dominant ($J/I \simeq 0$). As can be seen in Fig.1, $\chi(T)$ decreases monotically with reducing temperature for $I = 0.6$. However, as the intra-sublattice frustration is increased ($I = 0.7, 0.8$ and 0.9) a *minimum* of $\chi(T)$ occurs and it sharps up. Such a interesting behavior of $\chi(T)$ had been observed experimentally by Ito, Aruga et al [4] for the Ising antiferromagnetic compound $Fe_xMn_{1-x}TiO_3$.

4 FC susceptibility in nonergodic phase

As mentioned above, the field cooled susceptibility χ_{FC} of frustrated ferro- and antiferromagnets does not depend on temperature below the freezing point T_g. An issue to be addressed is whether the χ_{FC} of frustrated ferrimagnets depends on temperature. To study this problem we have to deal with the equations of state below T_g [Eqs. (3)-(9)]. For simplicity we neglect the intra-sublattice interaction ($I_{ij}^{(p)} = 0$) and suppose that the inter-sublattice frustration J is strong enough so that there is not long range magnetic order in the absence of a magnetic field ($J_0 = 0$).

Differentiating Eq.(3) with respect to H and using Eq.(6) we obtain

$$\chi_{FC}^{(p)} = \tilde{g}_p \lim_{h\to 0}\left(\int P_p(0,y)\tilde{m}_p'(0,y)dy\right) = \tilde{g}_p\tilde{m}_p'(0,0) \tag{18}$$

Using the last equation and the Parisi marginal stability condition (9) at $x = 0$ we get

$$\chi_{FC}^{(1)}.\chi_{FC}^{(2)} = \tilde{g}_1.\tilde{g}_2.\tilde{J}^{-2} \tag{19}$$

For frustrated ferro- and antiferromagnets we have $\chi_{FC}^{(1)} = \chi_{FC}^{(2)}$ and obviously the relation (19) leads to temperature independence of their FC susceptibility. But for frustrated ferrimagnets $\chi_{FC}^{(1)} \neq \chi_{FC}^{(2)}$ and the situation must be quite different.

In order to show the temperature dependence of $\chi_{FC}(T)$ of frustrated ferrimagnet we derive an analitical expression for $\chi_{FC}^{(p)}$ to the first order in $\tau \equiv (T_g - T)/T_g$. As in the Parisi theory [9] the functions $\tilde{q}_p(x)$ vary on the interval $0 \leq x \leq x_1$ and are equal to some constant $\tilde{q}_p(x_1)$ above the point x_1. Analyzing Eqs.(3) and (9) at $x = 0$ and $x = x_1$ we find to the lowest order in τ

$$\tilde{q}_p(x) = \begin{cases} (1+\alpha_p)x/2(1+\alpha_p^2) & 0 \leq x \leq x_1 \\ 2\tau/(1+\alpha_p) & x_1 < x < 1 \end{cases} \tag{20}$$

Substituting (20) into

$$\chi_{FC}^{(p)} = \tilde{g}_p\left(1 - \int_0^1 \tilde{q}_p(x)dx\right) \tag{21}$$

we obtain the FC sublattice susceptibilities of our model

$$\chi_{FC}^{(p)}(T) = \tilde{g}_p\tilde{J}^{-1}\left[1 + \left(1 - \frac{2}{1+\alpha_p}\right)\tau\right] + 0(\tau^2) \tag{22}$$

It is easy to see that $\chi_{FC}^{(p)}$ given by the last equation satistifies Eq.(19)

Finally, from Eqs.(14) and (22) we get the total FC susceptibility of the two-sublattice Ising system in the vicinity of T_g

$$\begin{aligned} \chi_{FC} &= \sum_{p=1,2} n_p\tilde{g}_p\chi_{FC}^{(p)}(T) \\ &= \sum_p n_p\tilde{g}_p^2\tilde{J}^{-1}\left[1 + \left(1 - \frac{2}{1+\alpha_p}\right)\tau\right] + 0(\tau^2) \end{aligned} \tag{23}$$

For the two-equivalent sublattice system, such as antiferromagnet, the spin concentrations are equal ($\alpha_1 = \alpha_2 = 1$) and consequently its FC susceptibility is temperature independent. But for the two non-equivalent sublattice system, such as ferrimagnet, the spin concentrations are different ($\alpha_1 \neq \alpha_2 \neq 1$). Therefore the FC susceptibility of this system must be temperature dependent. Moreover, it is easy to show from Eq.(23) that if $\tilde{g}_2 > \tilde{g}_1$ and the spin concentrations n_1 and n_2 satisfy the condition

$$1 < n_1/n_2 < \tilde{g}_1^2/\tilde{g}_2^2 \tag{24}$$

then χ_{FC} decreases with decreasing T. For $\bar{g}_2 < \bar{g}_1$ an opposite situation will take place. Thus we can conclude that the FC susceptibility of the strongly frustrated Ising ferrimagnets may not only decrease but also increase with reducing temperature. Such a temperature dependence of χ_{FC} had been observed in [2, 5, 6]. The static susceptibility measurements show that FC susceptibility of the spinels $FeAlO_4$ and $FeIn_2S_4$ decreases with reducing T while it increases in the spinel $FeGa_2O_4$. The authors of Ref.[2, 5] proposed that these differences should reflect a different extent of magnetic correlation in the three spinels. Our calculations seem to support this point of view.

In summary, we have studied the temperature dependence of the two-sublattice frustrated Ising systems in the framework of the mean field theory. We have shown that the FC susceptibility of the strongly frustrated Ising ferrimagnets depends on temperature in the nonergodic phase while the susceptibility of the frustrated antiferromagnets may have a minimum in the ergodic phase. Such a temperature dependence turns out to be in qualitative agreement with experimental data.

This work was partly supposed by the Vietnam National Basic Research Program on the Natural Sciences KT/04.

Sự phụ thuộc nhiệt độ của độ cảm từ trong các hệ Ising hai phân mạng mất trật tự

Hoàng Dũng - Mai Xuân Lí

Tóm tắt: Khảo sát sự phụ thuộc vào nhiệt độ của độ cảm từ trong các hệ từ Ising hai phân mạng mất trật tự nhờ gần đúng trường trung bình. Chứng tỏ độ cảm từ của hệ phản sắt từ mất trật tự có cực tiểu trong pha ergodic. Khác với các hệ sắt từ và phản sắt từ mất trật tự, trong hệ ferri từ mất trật tự độ cảm từ FC có khả năng phụ thuộc vào nhiệt độ ở nhiệt độ dưới T_g (pha phi ergodic). Các kết quả tìm được phù hợp định tính với thực nghiệm.

References

[1] See, for example, K. Binder and A.P. Young, Rev. Mod. Phys. **58**, 801 (1986); I.Ya. Korenblit and E.F. Shender, Usp. Fiz. Nauk **157**, 267 (1989).

[2] E. Agostinelli, D. Fiorani and A.M.Tesla, in *Fundamental and Applicative Aspects of Disordered Magnetism*, Eds by P. Allia, D. Fiorani and L. Lanotte (World Scientific,1989) p.30 and references therein.

[3] J. Gelard, F. Bensamka, D. Bertrand, A.F. Fert, J.P. Redoules, S. Legrand, J. Phys. **C16**, L939 (1983); P. Z. Wong , S. von Molnar, T. T. M. Palstra, J. A. Mydosh, H. Yoshizawa, S. M. Shapiro, A. Ito. Phys. Rev. Lett. **55**, 2043 (1985).

[4] A. Ito, H. Aruga, M. Kikuchi, Y. Syono and H. Takei. Solid State Commun. **66**, 475 (1988); H. Aruga, A. Ito, H. Wakabayashi, T. Goto. J. Phys. Soc. Jpn. **57**, 2636 (1988).

[5] J.L. Soubeyroux, D.Fiorani, E. Agostinelli, S.C. Bahargava, J.L. Dorman, Journal de Physique **C8**, 1117 (1988).

[6] J.L. Dorman, M. Nogues, J. Phys.: Condensed Matter **2**, 1223 (1980); J.K. Srivastava, J. Phys. Chem. Solid **49**, 115 (1988).

[7] I. Ya. Korenblit, Ya.V. Fyodorov, E.F. Shender. Zh. Eksp. Teor. Fiz. **92**, 710 (1987); Ya.V. Fyodorov, I.Ya. Korenblit, E.F. Shender, Europhys. Lett.**4** , 827 (1987); H. Takayama, Prog. Theor. Phys. **80**, 827 (1988)

[8] M.S. Li, L.Q. Nguyen, A.V. Vedyaev and H. Zung, J. Magn. Magn. Mater. **96**, 1975 (1991); I.Ya. Korenblit, Ya.V. Feodorov, H. Zung. Fiz. Tverd. Tela **32**, 1441 (1990).

[9] G. Parisi, Phys. Lett. **77A**, 203 (1979); J. Phys **A13**, L115 (1980).