DETERMINING THE CHARACTERISTICS OF ELEMENT OR SUBSTRUCTURE USING THE VIBRATION MEASUREMENT

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(Received on March 24th, 2000)

ABSTRACT: ***This paper presents some properties of the dynamic stiffness matrix (DSM) and the dynamic flexural matrix (DFM) (matrix of transfer function) of structure. These properties may be used to identify the characteristics of structure.***

1. INTRODUCTION

In this paper we consider two problems.

In the first problem we determine the natural frequencies of the structure, the natural frequencies of the elements of the structure and the ones of the structure with new constraints by using the DSM or DFM of the structure.

The second problem shows the possibility to determine the characteristics of the elements of the structure (or the cracks in the elements of the structure) by using the measured data in field of these elements.

2. Problem 1:

Consider the frame in Fig 1.1 with the following data $E = 3.10^7 kN/m^2$, $\rho = 2.4 t/m^3$, $k_i^* = 0.03$ (structure damping).

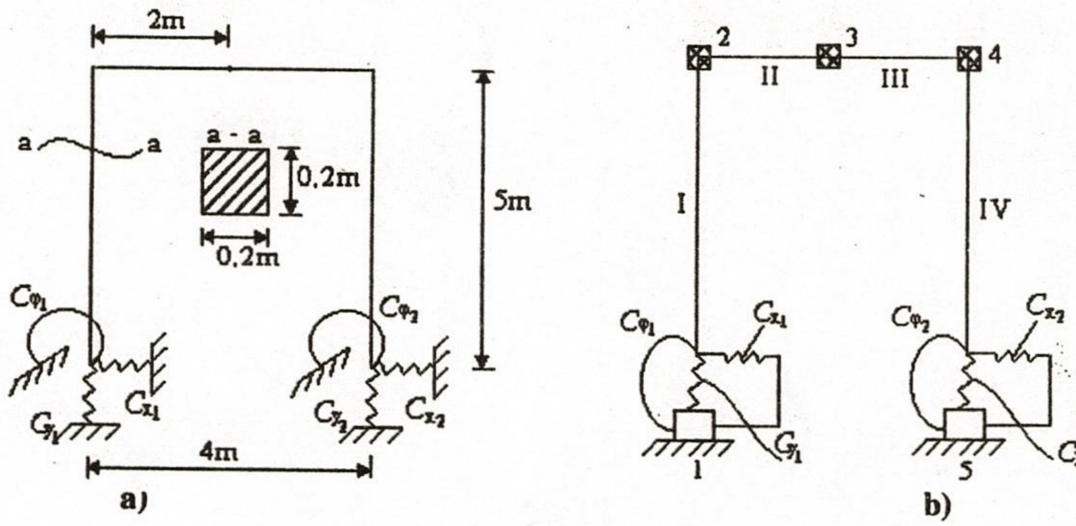


Fig 1.1

The frame may be consider as the structure in Fig 1.1b, it comprises 5 nodes and 4 elements. Element I has three internal springs at the first end $C_{x_1}, C_{\varphi_1}, C_{y_1}$ and element IV has three internal springs at the second end $C_{x_2}, C_{\varphi_2}, C_{y_2}$.

The DSM K of the free frame (without constraints) are calculated from eqs (3.7.14), (3.7.15) (see [1]), it has the order 5 x 3 = 15. Because nodes 1 and 5 are build - in, the dynamic stiffness matrix K^c of the constrained structure is obtained by deleting the rows and columns 1, 2, 3, 13, 14, 15 of matrix K. Therefore, its order remains 15 – 6 = 9.

By including the damping $k_i^* = 0.03$, the coefficients k_{ij} of matrix K and the Det(K) are complex numbers.

In Fig 1.2b, c the value of |Det(K)| = abs[Det(K)] are plotted against ω(in s^{-1}) for the case $C_{\varphi_1} = 0, C_{x_1} = C_{y_1} = C_{\varphi_2} = C_{x_2} = C_{y_2} = \infty$, in this case the frame is shown in Fig 1.2a.

It is seen that the minimum points of the curves are corresponding to the natural frequencies of the frame ω_k (k = 1, 2, 3, .., n) and the maximum points of the curves are corresponding to the natural frequencies of the elements of frame $\overline{\omega}_k^{(i)}$ (k = 1, 2, ... ; i = I, II, III, IV).

This is relative to the properties of DSM as shown in [1]:

- The determinant of DSM is equal to zero by all frequencies of free vibration of complete structure: Det $[K^c(\omega_j)] = 0$

- For the case without damping the determinant of DSM becomes infinity by the frequencies of free vibration of each element of structure:

$$\text{Det}\,[K^c(\overline{\omega}_j)] \rightarrow \infty$$

In Fig 1.2b the first minimum points are corresponding to the first frequencies of the frame:

$\omega_1 = 21.2746 s^{-1}$; $\omega_2 = 101.1122 s^{-1}$; $\omega_3 = 156.508 s^{-1}$; $\omega_4 = 214.4725 s^{-1}$

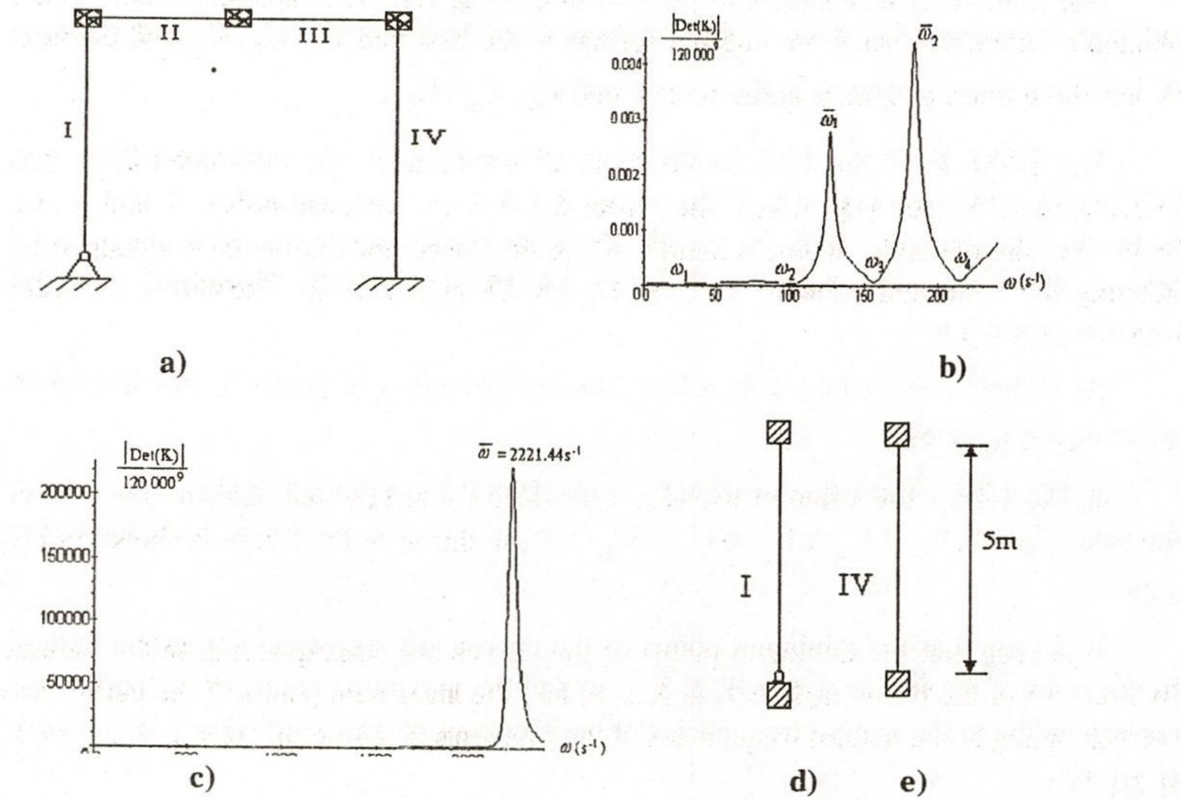


Fig 1.2

The first maximum point $\overline{\omega}_1 = 125.89s^{-1}$ is the first frequency of lateral vibration of the element I (Fig 1.2d) and the second maximum point $\overline{\omega}_2 = 182.68s^{-1}$ is the first frequency of the element II (Fig 1.2e).

In Fig 1.2c the maximum point $\overline{\omega} = 2221.44s^{-1}$ is the first frequency of longitudinal vibration of the bar in Fig 1.2d.

Let us consider the matrix of transfer function or the dynamic flexural matrix (*DFM*) $H = K^{-1}$. The component $h_{ij}(\omega)$ of H is the complex amplitude of the response $\hat{v}_i$ to the force $F_j = e^{j\omega t}$.

It is $h_{ij} = h_{v_iF_j}(\omega)$; $h_{\dot{v}_iF_j} = j\omega h_{v_iF_j}(\omega)$; $h_{\ddot{v}_iF_j} = -\omega^2 h_{v_iF_j}(\omega)$

The matrix H is obtained by inverting the matrix K.

According to the properties of DFM as shown in [1] :

- The determinant of DFM H(ω) becomes infinity by the frequencies of complete structure and equals zero by the frequencies of each element of structure:

$$\text{Det}\,[H(\omega_k)] \rightarrow \infty$$

$$\text{Det}\,[H(\varpi_k)] = 0 \quad (\text{see 6.2.21: [1]})$$

- Zero points of function $h_{ii}(\omega) = 0$ are the frequencies of free vibration of structure with constrained coordinate q_i(q_i=0).

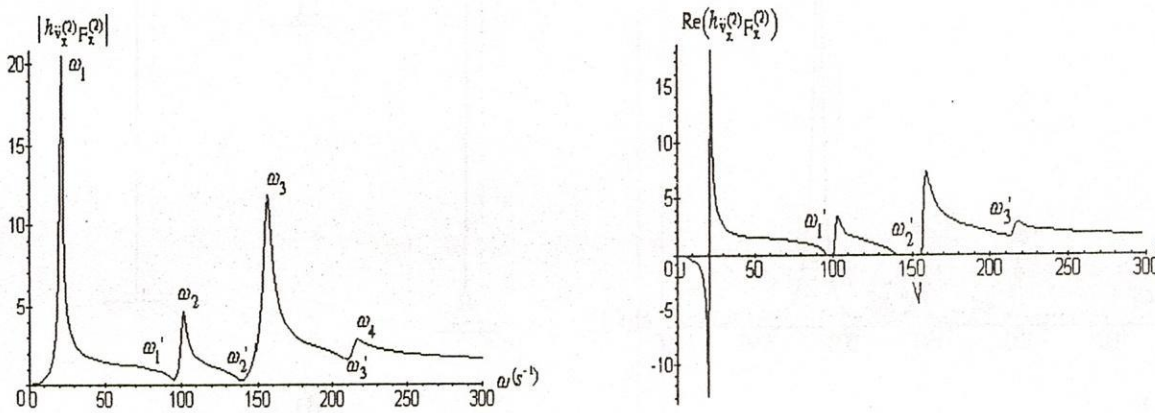
It is seen that the peaks of the curve of absolute value $\left|h_{\ddot{v}_i F_j}(\omega)\right|$ or imaginary value $\mathrm{Im}\left(h_{\ddot{v}_i F_j}\right)$ (plotted against ω) are corresponding to the natural frequencies of the frame and the minimum points of $\left|h_{\ddot{v}_i F_j}(\omega)\right|$, $\mathrm{Im}\left(h_{\ddot{v}_i F_j}\right)$ or zero points of real part $\mathrm{Re}\left(h_{\ddot{v}_i F_j}\right)$ are corresponding to the frequencies of the frame with the new constraint $v_i = 0$. That may be seen in Figs 1.3, 1.4 and 1.5.

Figs 1.3a, b, c show the curves of $\left|h_{\ddot{v}_x^{(2)} F_x^{(2)}}\right|$, $\mathrm{Re}\left(h_{\ddot{v}_x^{(2)} F_x^{(2)}}\right)$, $\mathrm{Im}\left(h_{\ddot{v}_x^{(2)} F_x^{(2)}}\right)$ plotted against ω for the frame in Fig 1.2a.

The maximum points in Figs 1.3a, 1.3c are corresponding to the natural frequencies of the frame in Fig 1.2a: ω_1, ω_2, ω_3, ω_4.

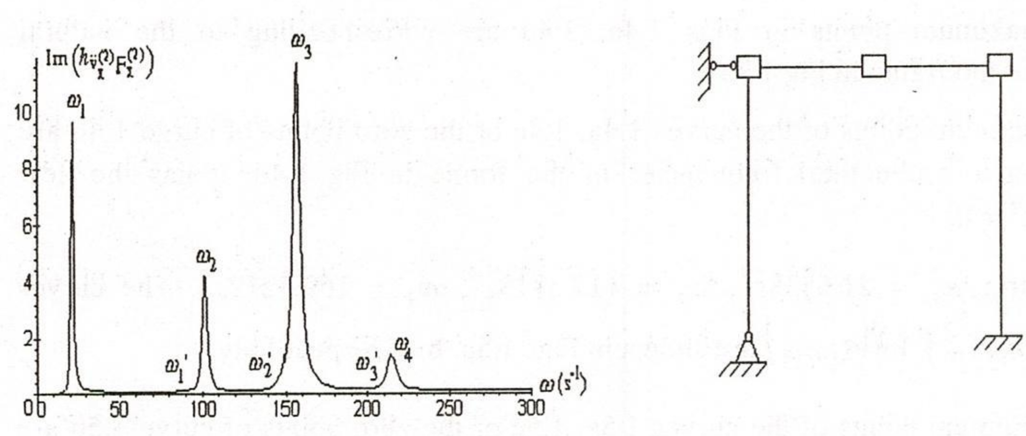
The minimum points of the curve in Figs 1.3a, 1.3c or the zero points of the curve in Fig 1.3b are corresponding to the natural frequencies of the frame in Fig 1.3d. This frame has the new constraint $v_x^{(2)}=0$.

They are: $\omega_1^{'} = 95.7044s^{-1}$, $\omega_2^{'} = 141.0383s^{-1}$, $\omega_3^{'} = 212.4768s^{-1}$.



a) b)

Fig 1.3



c) d)

Fig 1.3

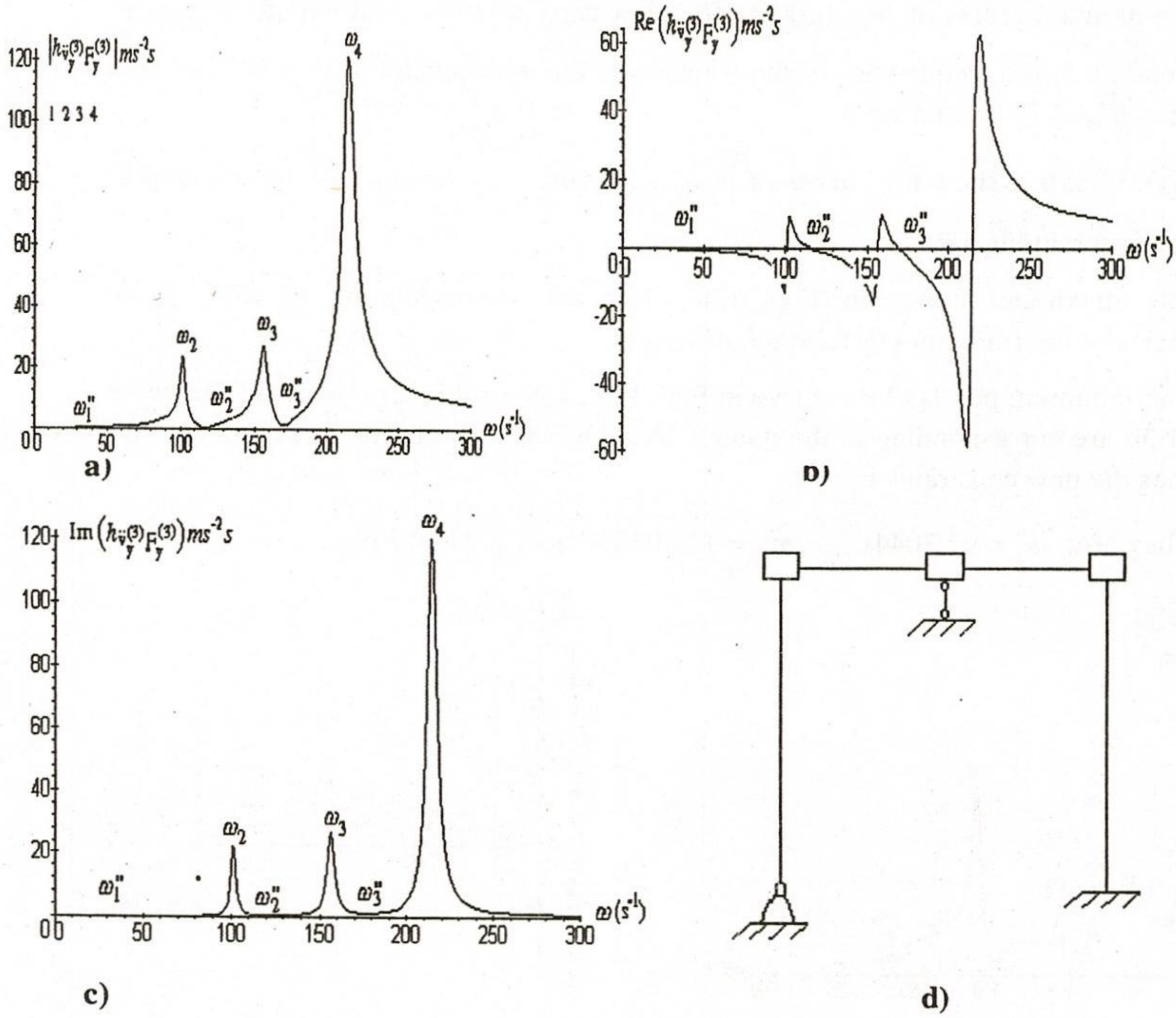


Fig 1.4

The curves $\left|h_{\ddot{v}_y^{(3)}F_y^{(3)}}\right|$, $\mathrm{Re}\left(h_{\ddot{v}_y^{(3)}F_y^{(3)}}\right)$, $\mathrm{Im}\left(h_{\ddot{v}_y^{(3)}F_y^{(3)}}\right)$ are shown in Figs 1.4a, b, c, respectively.

The maximum points in Figs 1.4a, 1.4c are corresponding to the natural frequencies of the frame in Fig 1.2a.

The minimum points of the curves 1.4a, 1.4c or the zero points of curve 1.4b are corresponding to the natural frequencies of the frame in Fig 1.4d, it has the new constraint $v_y^{(3)} = 0$.

They are: $\omega_1^{"} = 21.6538s^{-1}$, $\omega_2^{"} = 117.3713s^{-1}$, $\omega_3^{"} = 169.7219s^{-1}$. The curves $\left|h_{\ddot{v}_y^{(2)}F_y^{(2)}}\right|$, $\mathrm{Re}\left(h_{\ddot{v}_y^{(2)}F_y^{(2)}}\right)$, $\mathrm{Im}\left(h_{\ddot{v}_y^{(2)}F_y^{(2)}}\right)$ are shown in Figs 1.5a, b, c, respectively.

The minimum points of the curves 1.5a, 1.5c or the zero points of curve 1.5b are corresponding to the natural frequencies of the frame in Fig 1.5d, it has the new constraint $v_y^{(2)} = 0$.

In these frequencies there are the natural frequencies of longitudinal vibration of the element I:

$\overline{\omega}_{l_1} = 2221.44 s^{-1}$, $\omega_{l_K} = k\overline{\omega}_{l_1}$ ($k = 2, 3, 4, ..$).

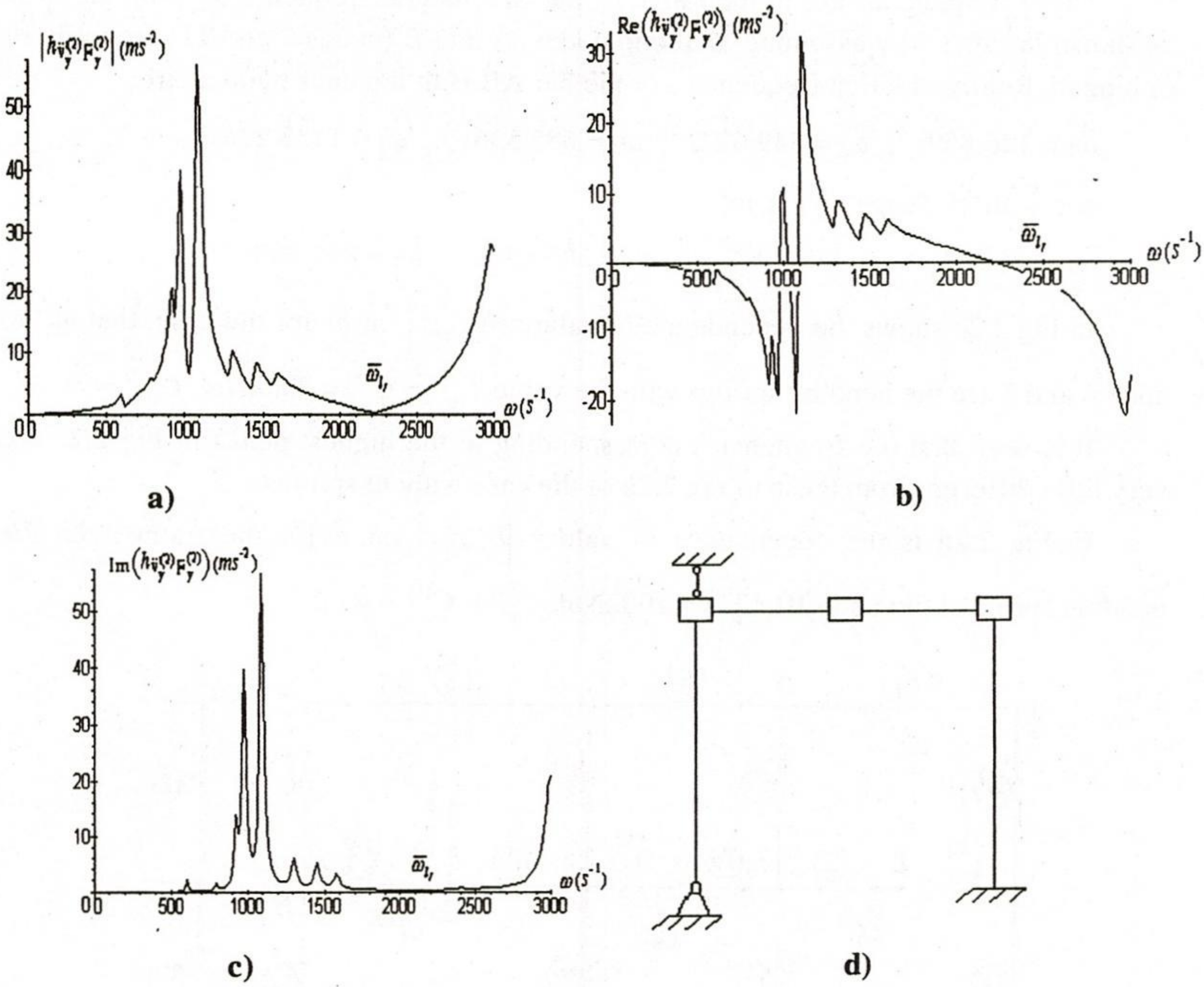


Fig 1.5

3. Problem 2:

Consider the frame in Fig 2.1. It comprises 15 nodes, 17 elements and 1 class with the data:

$E = 3\text{x}10^7 \text{kN}/m^2$; $A = 0.04m^2$;

$J = 0.000133m^4$; $k_i^* = 0.03$

Nodes 6, 8, 10 are hinged with bending springs. If the stiffness of these springs is infinity, these nodes become build - in, in this case the first natural frequencies of the frame are:

$\omega_1 = 16.432s^{-1}$; $\omega_2 = 52.206s^{-1}$; $\omega_3 = 86.436s^{-1}$; $\omega_4 = 102.313s^{-1}$.

Assume that the vertical force acts on the node 10, its value is expressed by equation (2.5.29) (see [1]) with $t_1 = 0.566\text{x}10^{-3}s$.

Fig 2.2a, b show the dependence of amplitude of transfer function $\left|h_{\hat{v}_y^{(10)}F_y^{(10)}}\right|$ in $\left(\dfrac{ms^{-2}}{kN}\right)$ and complex amplitude $\left|\hat{v}_y^{(10)}\right|(ms^{-1})$ on frequency ω (in s^{-1}), respectively.

Four highest peaks of these curves are corresponding to the following natural frequencies of the frame:

$$\omega = 102.313s^{-1};\quad \omega = 496.657s^{-1};\quad \omega = 593.980s^{-1};\quad \omega = 772.397s^{-1}.$$

These frequencies are in the range of the first natural frequencies of the bar A B as shown in Fig 2.1 by assuming that end nodes A and B (nodes 9 and 11) are build in or hinged. Really, the first frequencies of the bar AB with the ends build in are:

$$\omega_1 = 126.859s^{-1};\ \omega_2 = 349.692s^{-1};\ \omega_3 = 685.536s^{-1};\ \omega_4 = 1133.226s^{-1}.$$

and with the hinged ends are:

$$\omega_1 = 55.962s^{-1};\ \omega_2 = 223.847s^{-1};\ \omega_3 = 503.656s^{-1};\ \omega_4 = 895.389s^{-1}.$$

In Fig 2.2c shows the dependence of values $\left|h_{\ddot{v}_y^{(10)}F_y^{(10)}}\right|$ on ω for the case, that at the node 6 and 8 are the bending springs with the value $C_\varphi^{(6)} = C_\varphi^{(8)} = 200\text{kN}m,\ C_\varphi^{(10)} = \infty$.

It is seen that the frequencies corresponding to the highest peaks in Fig 2.2c are very little different from these in Fig 2.2a of the case without springs.

In Fig 2.2d is the dependence of values $\left|h_{\ddot{v}_y^{(10)}F_y^{(10)}}\right|$ on ω for the frame with the bending spring at the node 10 $C_\varphi^{(10)} = 200\text{kN}m,\ C_\varphi^{(6)} = C_\varphi^{(8)} = \infty$.

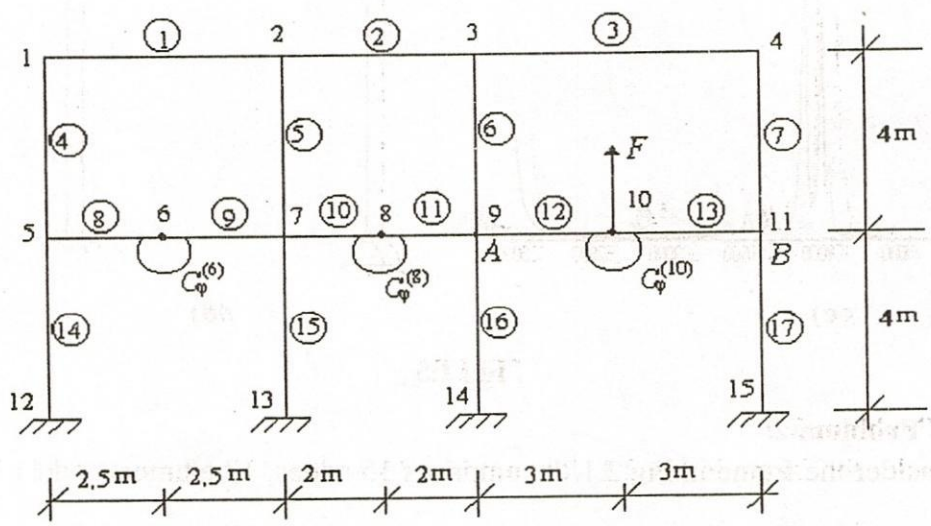
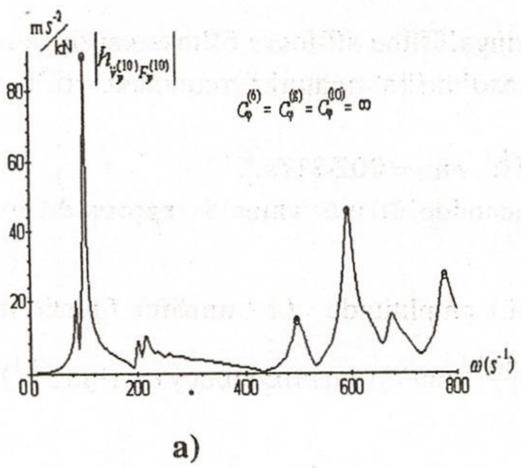
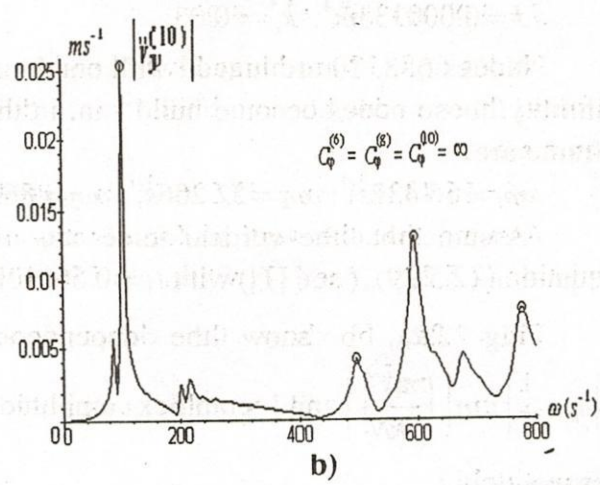


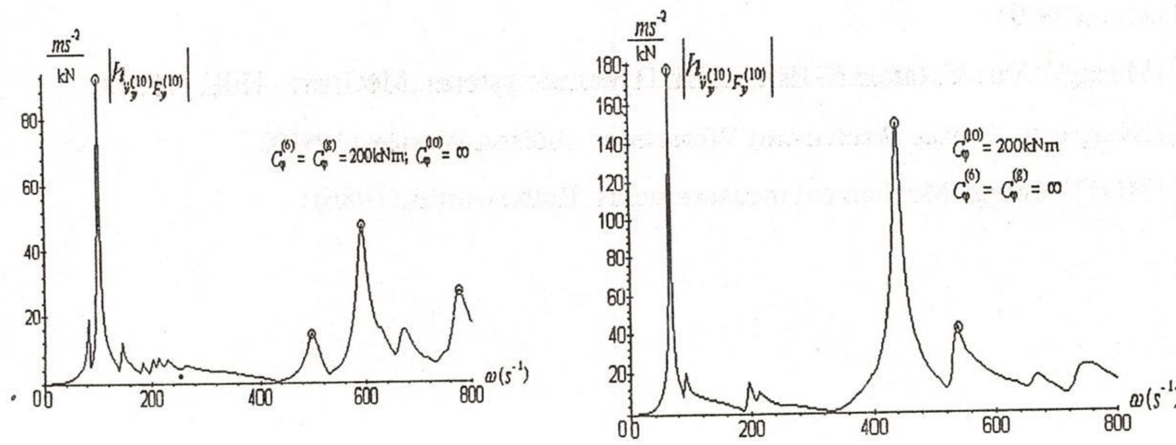
Fig 2.1



a)



b)



c) d)

Fig 2.2

The highest peaks of the curve in Fig 2.2d are corresponding to frequencies:

$\omega = 66.803 s^{-1}$; $\omega = 438.837 s^{-1}$; $\omega = 534.505 s^{-1}$

They are much different from these in Fig 2.2a.

That means the frequencies corresponding to the highest peaks of the curves $\left|h_{\ddot{v}_y^{(10)} F_y^{(10)}}(\omega)\right|$ may be used to identify the value of $C_\varphi^{(10)}$.

To evaluate the quality of the element or the substructure, it is better if the positions of the applied force and the measured points should be put on the element or the substructure.

4. CONCLUSION:

Through the above results we obtain the method to determine the characteristics of the structure or the elements of the structure by using the properties of DSM or DFM. Especially, the method is used to identify the cracks in the elements of the structure.

XÁC ĐỊNH CÁC ĐẶC TRƯNG CỦA PHẦN TỬ KẾT CẤU BẰNG PHƯƠNG PHÁP ĐO DAO ĐỘNG

Văn Hữu Thịnh - Nguyễn Xuân Hùng

TÓM TẮT: *Trong bài báo này, chúng tôi trình bày một số tính chất của Phương pháp ma trận độ cứng động lực và ma trận độ mềm động lực của kết cấu. Những tính chất đó có thể sử dụng để nhận dạng các đặc trưng của kết cấu.*

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