LANDAU-POMERANCHUK-MIGDAL EFFECT FOR FINITE TARGETS. A FEYNMAN PATH INTEGRAL APPROACH

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Abstract: In this paper a straighforward Feynman path integral formalism is discussed and applied to the problem of the suppression of bremsstrahlung radiation of long-wavelength photons, the Landau-Pomeranchuk-Migdal (LPM) effect, in a target of finite thickness with multiple scattering. Taking into account the side effect yields corrections to the Migdal theory. The method is also applied to the treatment of the standard LPM effect in the presence of an external field. In brief, it is shown that preliminary numerical results are in very good agreeing with experimental data.

HIỆU ỨNG LANDAU-PEMRANCHUK-MIGDAL CHO CÁC BẢN TINH THỂ HỮU HẠN. PHƯƠNG PHÁP TÍCH PHÂN LỘ TRÌNH FEYNMAN

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Tóm tắt: Trong công trình này hình thức luận tích phân lộ trình Feynman được bàn đến và áp dụng cho bài toán sụt giảm cường độ bức xạ hãm của các photon sóng dài, còn được gọi là hiệu ứng Landau-Pomeranchuk-Migdal (LPM), trong các bản hữu hạn có tính đến hiệu ứng tán xạ nhiều lần. Việc tính đến các hiệu ứng biên cho phép chúng tôi hiệu chỉnh lý thuyết của Migdal. Phương pháp của chúng tôi cũng được áp dụng để lý giải hiệu ứng LPM trong sự hiện diện của trường ngoài. Các kết quả tính toán số rất phù hợp với các kết quả thực nghiệm

1 Introduction

While passing through amorphous medium, high energy particles undergo scattering on atoms randomly in the formation length. These uncorrelated acts cause a suppression of the emission of long wave-length photons in comparison with that predicted by Bethe-Heitler bremsstrahlung theory for isolated atoms [1]. Landau and Pomeranchuk [2] first described the effect, using the classical radiation theory with taking into account multiple scattering of particle inside the formation length in a qualitative manner. Subsequently, a quantum theory of this effect was developed by Migdal [3], treating multiple scattering in a dynamical manner via the kinetic equation for the position and velocity distribution function of particle in the medium. A good report of the problem was given in [4]. In his theory Migdal treated medium as infinitely thick, for which it works quite well in agreeing with the measurements of the LPM effect at SLAC [5,6]. However, as the target thickness decreases the boundary effect becomes more important and the Migdal formulas fail to account for the boundary of the targets [5,6].

The interest in the LPM effect now is increased. Blankenbeckler and Drell presented the eikonal approach to the LPM effect [7]. Baier, Dokshitzer, Peigne, Schiff [8] and Levin [9] made a generation of the LPM effect to QCD for nuclear matter medium.

Here we have developed a method, using the Feynman path integral formalism, applicable for performing the statistical averages that treat the LPM suppression effect. It is worth noting that one can easily treat the standard LPM effect in the presence of an uniform electric field by changing paths in path integrals. Our treatment includes as limiting cases Bethe-Heitler bremsstrahlung relevant for very thin target, and the LPM effect for infinitely thick medium. For the later case our results go accurately over into the ones of Baryshevskii and Tikhomirov for the standard LPM in an external field [10] and of Migdal for the standard LPM [3]. Taking into account the boundary effect allows us to make corrections to the well-known Migdal formulas for thick target.

2 Radiation cross–section as functional of particle trajectories

For this purpose we adopt the Schwinger operator method [11], its generalization was developed by Baier and Katkov [2]. Thus we have the radiation cross–section summed over the polarizations of the photon and of the final electron, and averaged over the polarizations of the initial electron

$$\begin{aligned}
\frac{dW}{d\omega d\Omega} &= \frac{e^2\omega}{4\pi^2}\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{\infty}dt_2\Big[\left(\vec{A}_1\vec{A}_2\right)-\left(\vec{A}_1\vec{n}\right)\left(\vec{A}_2\vec{n}\right)+\left(\vec{B}_1\vec{B}_2\right)+\left(\vec{B}_1\vec{n}\right)\left(\vec{B}_2\vec{n}\right)\Big]\times \\
&\times\ \exp\left\{i\frac{\varepsilon}{\varepsilon'}\left[\vec{k}\left(\vec{r}_2-\vec{r}_1\right)-\omega\tau\right]\right\}, \qquad (1) \\
\vec{A} &= \frac{1}{2\sqrt{\varepsilon\varepsilon'}}\left(\frac{\sqrt{\varepsilon+m}}{\sqrt{\varepsilon'+m}}\vec{p}\,'+\frac{\sqrt{\varepsilon'+m}}{\sqrt{\varepsilon+m}}\vec{p}\right), \\
\vec{B} &= \frac{1}{2\sqrt{\varepsilon\varepsilon'}}\left(\frac{\sqrt{\varepsilon'+m}}{\sqrt{\varepsilon+m}}\vec{p}-\frac{\sqrt{\varepsilon+m}}{\sqrt{\varepsilon'+m}}\vec{p}\,'\right), \\
\varepsilon' &= \varepsilon-\omega,\ \vec{p}\,'=\vec{p}-\vec{k},\ \vec{n}=\vec{k}/\omega,\ \tau=t_2-t_1,
\end{aligned}$$

where $\vec{k}$, ω are the momentum and energy of the photon, $\vec{p}$ and ε - of the initial electron. Indices 1 and 2 correspond to the moments of time t_1 and t_2 respectively.[1]

Let the electron, having initial velocity $\vec{v}_0$, enter a target at the moment $t=0$ and go out of it at the later moment $T \approx L$ (L is the target thickness). The Z axis is directed along the direction of velocity $\vec{v}_0$. Since typical values of scattering angles of

[1] Relativistic units $\hbar = c = 1$ are being used throughout.

ultra-relativistic particles by atoms of medium are small, one can use the small-angle approximation and represent the particle velocity as

$$\vec{v}(t) = \vec{v}_0\left(1-\frac{\vec{\vartheta}^2(t)}{2}\right)+\vec{\vartheta}(t), \qquad \vec{v}_0\vec{\vartheta}(t)=0. \tag{2}$$

$$\vec{\vartheta}(t) = \begin{cases} 0 & t\leq 0 \\ \vec{\vartheta}(t) & 0<t<T \\ \vec{\vartheta}_{\mathrm{T}} & t\geq T. \end{cases}$$

By using the small-angle approximation (2) and integrating (1) with respect to the directions of the photon momentum, we can express the probability of emission in the form

$$\frac{dW}{d\omega} = -\frac{e^2}{\pi}\,\mathrm{Im}\int_{-\infty}^{\infty}dt_1\int_{t_1}^{\infty}dt_2\frac{\exp(-ia\tau)}{\tau}\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\frac{\partial}{\partial\mu}\right]_{\mu=0}\times$$
$$\prod_{i=x,y}\exp\left\{-ia\gamma^2\int_{t_1}^{t_2}\vartheta_i^2(t)dt+\frac{ia\gamma^2}{\tau}\left[\int_{t_1}^{t_2}\vartheta_i(t)dt\right]^2+\mu\left(\vartheta_{2i}-\vartheta_{1i}\right)^2\right\} \tag{3}$$

where $a=\omega\varepsilon/2\varepsilon'\gamma^2$. We recall that in an amorphous medium the formation length is defined as

$$l_{\mathrm{f}}=\frac{1}{a}=\frac{2\varepsilon'\gamma^2}{\varepsilon\omega}, \tag{4}$$

and the mean square multiple scattering angle per unit length is given by

$$\left\langle\vartheta_{\mathrm{s}}^2\right\rangle=\left(\frac{E_{\mathrm{s}}}{\varepsilon}\right)^2\frac{1}{X_0}. \tag{5}$$

Here X_0 is the radiation length and $E_{\mathrm{s}}=m\sqrt{4\pi/\alpha}\approx 21$ MeV. The probability of photon emission is defined by ordinary Bethe-Heitler bremsstrahlung theory as

$$\frac{dW_{\mathrm{BH}}}{d\omega}=\frac{e^2\gamma^2\left\langle\vartheta_{\mathrm{s}}^2\right\rangle T}{12\pi\omega}\left[\frac{2\left(\varepsilon^2+\varepsilon'^2\right)}{\varepsilon^2}+\frac{\omega^2}{\varepsilon^2}\right]. \tag{6}$$

The physics of the LPM effect is that bremsstrahlung is suppressed when the mean square multiple scattering angle over the formation length l_{f} : $\langle\vartheta_{\mathrm{f}}^2\rangle=\langle\vartheta_{\mathrm{s}}^2\rangle\, l_{\mathrm{f}}$ is greater than or equal to the square characteristic angle between the incident electron and the produced photon: $\vartheta_\gamma^2\sim 1/\gamma^2$.

3 Feynman path integral formalism and multiple scattering problem

To treat the LPM effect for a target of arbitrary thickness one should average (3) over all possible trajectories of the electron. For this purpose it is necessary to divide the integral over t's into parts corresponding to different regions of the particle trajectory

$$\int_{-\infty}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 = \int_{-\infty}^{0} dt_1 \int_{t_1}^{0} dt_2 + \int_{0}^{T} dt_1 \int_{t_1}^{T} dt_2 + \int_{T}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 + $$
$$+ \int_{-\infty}^{0} dt_1 \int_{0}^{T} dt_2 + \int_{-\infty}^{0} dt_1 \int_{T}^{\infty} dt_2 + \int_{0}^{T} dt_1 \int_{T}^{\infty} dt_2 \,. \quad (7)$$

Thus, the probability of photon bremsstrahlung will consist of six terms

$$\frac{dW}{d\omega} = \sum_{\alpha \leq \beta} \frac{dW^{\alpha\beta}}{d\omega}, \quad (\alpha, \beta = 1, 2, 3) \,. \quad (8)$$

In formula (8) the terms $W^{\alpha\alpha}$ describe the probability of photon emission, which is formed by part of particle trajectory in vacuum before it enters the target ($\alpha = 1$), inside the target ($\alpha = 2$), and after emerging from the target ($\alpha = 3$). The terms $W^{\alpha\beta}$ ($\alpha < \beta$) - interference between them. By using the following transformation

$$\exp\left\{\frac{ia\gamma^2}{\tau}\left[\int_{t_1}^{t_2} \vartheta_i(t)dt\right]^2\right\} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(-q^2\right) \exp\left[q2\gamma\sqrt{\frac{ia}{\tau}} \int_{t_1}^{t_2} \vartheta_i(t)dt\right] dq, \quad (9)$$

one can find that in this instance, as in many others, the general expression for the probability of a radiation process, accompanying relativistic charged particle passage through medium, involves some kind of a particle trajectory functional of the quadratic form

$$G\left[\vartheta_i(t)\right]_{t_1}^{t_2} = \exp\left\{\int_{t_1}^{t_2} \left[-ia\vartheta_i^2(t) + b_i\vartheta_i(t)\right] dt\right\} . \quad (10)$$

Here we denote a and b_i some coefficients. The averaging of the radiation cross-section over all possible particle trajectories leads to the calculation of the continual integral of this functional in accordance with the conditional Wiener measure, which corresponds to the probability that a particle trajectory was in the "corridor" $\vartheta_i(t) + d\vartheta_i(t)$ in the space of all possible trajectories with a fixed end.

We should determine first this measure. Motion of relativistic charged particles in a transverse plane is described by a Langeven equation

$$\varepsilon\ddot{\vec{\rho}}(t) - \vec{F}(t) = \vec{f}(t) \,, \quad (11)$$

where $\vec{\rho} = (x, y)$, $\vec{F}(t)$ is the regular force, acting upon a particle, $\vec{f}(t)$ is the Gaussian δ–correlation random force with a zero mean value

$$\langle f_i(t')f_k(t)\rangle = \varepsilon^2\sigma_i\delta_{ik}\delta(t'-t), \quad i, k = x, y, \quad (12)$$
$$\sigma_i = d\left\langle\vartheta_i^2(t)\right\rangle/dt, \quad t \approx z. \quad (13)$$

We stress that this model reflects the random step nature of multiple scattering, which can be characterized by the formal mean square scattering angle per unit length (13). It is smaller by a factor of 2 than the value given in (5)

$$\sigma_x + \sigma_y = \sigma \equiv \left\langle\vartheta_s^2\right\rangle/2 \,. \quad (14)$$

This is justified in the framework of the small–angle approximation, which is adopted in our derivation when the particles approach to the scattering centres at not very short distances. A more accurate description of the particle scattering on the Coulomb potential has recently been given in [8]. The continual description of the multiple scattering behaviour, as it is represented in (12-13), is physically due to the fact that at high energies, owing to a large value of the formation length, relativistic particles suffer on this length a large number of successive scattering acts by small angles by atoms of medium. Since the particle mean free path in amorphous and crystalline media is large compared to the range of the potential, we can regard the successive scatterings as independent (12). Let us define the solution of equation (11) in the absence of the random force in term of $\vec{\rho}_0(t)$. Then we have

$$E\dot{\vec{\vartheta}}_s(t) = \vec{f}(t) \tag{15}$$

where
$$\vec{\vartheta}_s(t) = \vec{\vartheta}(t) - \vec{\vartheta}_0(t), \quad \vec{\vartheta}_0(t) = \dot{\vec{\rho}}_0(t).$$

Solution of equation (15) with the Gaussian δ–random force (12) is the Wiener measure [13,14]

$$d_W\vec{\vartheta}_s(t) = \prod_{i=x,y} d_W\vartheta_{si}(t), \tag{16}$$

$$d_W\vartheta_{si}(t) = \frac{1}{N_i}\exp\left[-\frac{1}{2\sigma_i}\int_{t_1}^{t_2}\dot{\vartheta}_{si}^2(t)dt\right]\prod_{t_1}^{t_2} d\vartheta_{si}(t),$$

where N_i is the normalizing factor, chosen from the condition $\int d_W\vartheta_{si}(t) = 1$. The conditional Wiener measure can be written in the Feynman notations [14]

$$d_W(t_1, \vartheta_{s1i}; t_2, \vartheta_{s2i}) = \exp\left[-\frac{1}{2\sigma_i}\int_{t_1}^{t_2}\dot{\vartheta}_{si}^2(t)dt\right]\mathrm{D}\vartheta_{si}(t), \tag{17}$$

with the help of which we shall average the functional $G\left[\vartheta_i(t)\right]_{t_1}^{t_2}$ over all trajectories with fixed ends $\vartheta_{i1,2} = \vartheta_i(t_{1,2})$

$$\left\langle G\left[\vartheta_i(t)\right]_{t_1}^{t_2}\right\rangle_* = \int G\left[\vartheta_i(t)\right]_{t_1}^{t_2}\exp\left[-\frac{1}{2\sigma_i}\int_{t_1}^{t_2}\dot{\vartheta}_{si}^2(t)dt\right]\mathrm{D}\vartheta_{si}(t). \tag{18}$$

In order to make the general treatment of the standard LPM effect in the presence of an external field, which is assumed to be uniform in the transverse plane, one can change path in path integral (18)

$$\vartheta_{si}(t) \to \vartheta_i(t) - \vartheta_{0i}(t). \tag{19}$$

This procedure has been described in [15]. The functional method was used in [16] to treat the standard LPM effect in classical limit for an infinite medium. However, in this work continual integral was calculated directly by its definition, that deals with difference equations and matrix of infinite dimension. These calculations are very cumbersome. This work has been extended by Baryshevskii and Tikhomirov [10]. The application of Feynman path integral method, which we describe here, leads to a simpler and more straightforward derivation of the LPM effect in the most general case. The interested reader can find that path integral (18) is easily calculated by the Feynman method. For this purpose we should take factor out of the path integrand, which contains the whole dependence of integral on trajectory ends. Thus obtained path integral over trajectories is calculated by means of a spectral representation [14]. The case with an uniform external electric field is the simplest one. We write out here for further reference the result of calculation for this case

$$\left\langle \exp\left\{\int_{t_1}^{t_2}\left[-ia\vartheta_i^2(t)+b_i\vartheta_i(t)\right]dt\right\}\right\rangle_* = \sqrt{\frac{\eta_i}{2\sigma_i\pi\sinh\eta_i\tau}}\times$$

$$\exp\left(\left[\tfrac{1}{\sigma_i}\left(\vartheta_{i2}-\vartheta_{i1}\right)w_i-\tfrac{w_i^2}{2\sigma_i}\tau\right]+\tfrac{1}{2\sigma_i\sinh\eta_i\tau}\left\{-\eta_i\cosh\eta_i\tau\left(\vartheta_{i1}^2+\vartheta_{i2}^2\right)+2\eta_i\vartheta_{i1}\vartheta_{i2}+\right.\right.$$

$$\left.\left.\tfrac{2\sigma_i b_i}{\eta_i}\left(\cosh\eta_i\tau-1\right)\left(\vartheta_{i1}+\vartheta_{i2}\right)+\tfrac{2\sigma_i^2 b_i^2}{\eta_i^2}\left[\tfrac{1}{2}\tau\sinh\eta_i\tau+\tfrac{1}{\eta_i}\left(1-\cosh\eta_i\tau\right)\right]\right\}\right), \qquad (20)$$

where $\eta_i = \sqrt{2i\sigma_i a}$, $\tau = t_2 - t_1$, $\vec{w} = e\vec{E}/\varepsilon$ is the transverse acceleration of the electron. We will carry out the averaging of the probability of photon emission (3) with the help of one or two quadratures, that depends on the correlative link between different parts of particle trajectory

$$\left\langle h\left(\vartheta_{2i}\right)G\left[\vartheta_i(t)\right]_0^{t_2}\right\rangle = \int_{-\infty}^{\infty} h\left(\vartheta_{2i}\right)\left\langle G\left[\vartheta_i(t)\right]_0^{t_2}\right\rangle_* d\vartheta_{2i}, \qquad (21)$$

$$\left\langle h\left(\vartheta_{1i},\vartheta_{2i}\right)G\left[\vartheta_i(t)\right]_0^{t_2}\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h\left(\vartheta_{1i},\vartheta_{2i}\right)\left\langle G\left[\vartheta_i(t)\right]_0^{t_1}\right\rangle_*\left\langle G\left[\vartheta_i(t)\right]_{t_1}^{t_2}\right\rangle_* d\vartheta_{1i}d\vartheta_{2i} \qquad (22)$$

where $0 \le t_1 \le t_2 \le T$. Thus, one should use (21) and (22) to compute $\langle dW^{12}/d\omega\rangle$ and $\langle dW^{22}/d\omega\rangle$ respectively. It is the same for $\langle dW^{13}/d\omega\rangle$ and $\langle dW^{23}/d\omega\rangle$, just with the substitution $t_2 \to T$. The quantities $\langle dW^{11}/d\omega\rangle$ and $\langle dW^{33}/d\omega\rangle$ vanish as we use a subtraction procedure which should lead to no emission in the limit of no scattering and zero field (see below).

4 Target of arbitrary thickness in the presence of an external field

Consider the LPM effect with symmetric scattering in the presence of uniform electric field, directed along the x-axis

$$\sigma_x = \sigma_y = \frac{\sigma}{2}, \quad w_y = 0, \quad w_x = w. \qquad (23)$$

After a simple calculation with the Gaussian integrals we can obtain the explicit results in the form

$$\left\langle\frac{dW^{12}}{d\omega}\right\rangle = -\frac{e^2}{\pi}\,\mathrm{Im}\int_{-\infty}^{0}dt_1\int_0^T dt_2\frac{\eta}{\cosh\eta t_2\left(\tanh\eta t_2-\eta t_1\right)}\times$$

$$\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\left(\frac{\sigma}{\eta}P_0+\frac{w^2}{\eta^2}P_0^2\right)\right]\exp\left(-ia\tau-\frac{w^2}{\sigma}t_2+\frac{w^2}{\sigma\eta}P_0\right), \tag{24}$$

$$\left\langle\frac{dW^{22}}{d\omega}\right\rangle=-\frac{e^2}{\pi}\operatorname{Im}\int_0^T dt_1\int_{t_1}^T dt_2\frac{\eta}{\sinh\eta\tau}\times$$
$$\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{2\varepsilon\varepsilon'}\left(\frac{\sigma}{\eta}\tanh\frac{\eta\tau}{2}+\frac{2w^2}{\eta^2}\tanh^2\frac{\eta\tau}{2}\right)\right]\exp\left(-ia\tau-\frac{w^2}{\sigma}\tau+\frac{2w^2}{\sigma\eta}\tanh\frac{\eta\tau}{2}\right) \tag{25}$$

$$\left\langle\frac{dW^{13}}{d\omega}\right\rangle=-\frac{e^2}{\pi}\operatorname{Im}\int_{-\infty}^0 dt_1\int_T^\infty dt_2\frac{\eta}{\cosh\eta T\left\{\eta(\tau-T)+\left[1-\eta^2t_1(t_2-T)\right]\tanh\eta T\right\}}\times$$
$$\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\left(\frac{\sigma}{\eta}P+\frac{w^2}{\eta^2}P^2\right)\right]\exp\left(-ia\tau-\frac{w^2}{\sigma}T+\frac{w^2}{\sigma\eta}P\right), \tag{26}$$

$$\left\langle\frac{dW^{23}}{d\omega}\right\rangle=-\frac{e^2}{\pi}\operatorname{Im}\int_0^T dt_1\int_T^\infty dt_2\frac{\eta}{\cosh\eta(T-t_1)\left[\eta(t_2-T)+\tanh\eta(T-t_1)\right]}\times$$
$$\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\left(\frac{\sigma}{\eta}P_1+\frac{w^2}{\eta^2}P_1^2\right)\right]\exp\left(-ia\tau-\frac{w^2}{\sigma}(T-t_1)+\frac{w^2}{\sigma\eta}P_1\right), \tag{27}$$

where

$$\begin{aligned}
\tau &= t_2-t_1,\quad \eta=\sqrt{i\sigma\gamma^2a}\,,\\
P_0 &= \frac{\tanh\eta t_2\left(2\tanh\frac{\eta t_2}{2}-\eta t_1\right)}{(\tanh\eta t_2-\eta t_1)},\\
P &= \frac{\tanh\eta T\left[2\tanh\frac{\eta T}{2}+\eta(\tau-T)\right]}{\left[1-\eta^2t_1(t_2-T)\right]\tanh\eta T+\eta(\tau-T)},\\
P_1 &= \frac{\tanh\eta(T-t_1)\left[2\tanh\frac{\eta(T-t_1)}{2}+\eta(t_2-T)\right]}{\left[\tanh\eta(T-t_1)+\eta(t_2-T)\right]}.
\end{aligned} \tag{28}$$

Changing variables in (27) to $t_1=T-t_2,\ t_2=T-t_1$, and interchanging orders of integrals, one can find

$$\left\langle\frac{dW_{23}}{d\omega}\right\rangle\equiv\left\langle\frac{dW_{12}}{d\omega}\right\rangle \tag{29}$$

It is necessary to note that interchanging of orders of integrals over time and direction of the photon momentum leads to non zero value for probability of emission in the limit of no scattering and zero field ($\sigma\to0, w\to0$). It proves convenient to regulate the obtained results by subtracting this value from the corresponding terms in (24–27). Thus, we obtain finally the following results for the probability of emission

$$\left\langle\frac{dW}{d\omega}\right\rangle=I_E^{22}+2I_E^{12}+I_E^{13}, \tag{30}$$

$$\begin{aligned}
I_E^{22} &= -\frac{e^2}{\pi}\operatorname{Im}\int_0^T dt_1\int_{t_1}^T dt_2\frac{\eta}{\sinh\eta\tau}\times\\
&\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{2\varepsilon\varepsilon'}\left(\frac{\sigma}{\eta}\tanh\frac{\eta\tau}{2}+\frac{2w^2}{\eta^2}\tanh^2\frac{\eta\tau}{2}\right)\right]\exp\left(-ia\tau-\frac{w^2}{\sigma}\tau+\frac{2w^2}{\sigma\eta}\tanh\frac{\eta\tau}{2}\right)\\
&-\frac{e^2}{\pi a\gamma^2}\left(-1+\cos aT+aT\ \mathrm{Si}aT\right),
\end{aligned} \tag{31}$$

$$
\begin{aligned}
I_{\rm E}^{12} &= -\frac{e^2}{\pi}\,{\rm Im}\int_{-\infty}^{0}dt_1\int_0^T dt_2\frac{\eta}{\cosh\eta t_2\,(\tanh\eta t_2-\eta t_1)}\times \\
&\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\left(\frac{\sigma}{\eta}P_0+\frac{w^2}{\eta^2}P_0^2\right)\right]\exp\left(-ia\tau-\frac{w^2}{\sigma}t_2+\frac{w^2}{\sigma\eta}P_0\right)- \\
&\quad -\frac{e^2}{\pi a\gamma^2}\left[1-\cos aT+aT\left(\frac{\pi}{2}-{\rm Si}aT\right)\right], \qquad (32)
\end{aligned}
$$

$$
\begin{aligned}
I_{\rm E}^{13} &= -\frac{e^2}{\pi}\,{\rm Im}\int_{-\infty}^{0}dt_1\int_T^{\infty} dt_2\frac{\eta}{\cosh\eta T\,\{\eta\,(\tau-T)+[1-\eta^2 t_1\,(t_2-T)]\tanh\eta T\}}\times \\
&\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\left(\frac{\sigma}{\eta}P+\frac{w^2}{\eta^2}P^2\right)\right]\exp\left(-ia\tau-\frac{w^2}{\sigma}T+\frac{w^2}{\sigma\eta}P\right)- \\
&\quad -\frac{e^2}{\pi a\gamma^2}\left[\cos aT-aT\left(\frac{\pi}{2}-{\rm Si}aT\right)\right]. \qquad (33)
\end{aligned}
$$

The subtracted terms in (31–33) have been computed with the help of the following asymptotic behavior of the sine integral function

$$
x\left(\frac{\pi}{2}-{\rm Si}x\right)=\cos x+O\left(\frac{1}{x}\right)\quad [x\to\infty]. \qquad (34)
$$

5 Infinitely thick target. LPM limit

Our derivation of the LPM effect is valid for a target of arbitrary thickness in the presence of external field. As a first application we apply our results to the case of infinitely thick target. In this limit the term $I_{\rm E}^{13}$ in (33) vanishes; the term $I_{\rm E}^{12}$ in (32) leads to a finite value while the term $I_{\rm E}^{22}$ in (31) being proportional to the target thickness will dominate. Changing variable of integral over t_2 in this term to $\tau=t_2-t_1$ yields

$$
\begin{aligned}
I_{\rm E}^{22} &\stackrel{T\to\infty}{\longrightarrow} W_{\rm LPM}^{\rm E}=-\frac{e^2T}{\pi}\left\{\frac{\pi}{2\gamma^2}+{\rm Im}\int_0^{\infty}d\tau\frac{\eta}{\sinh\eta\tau}\times\right. \qquad (35)\\
&\left.\left[\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{2\varepsilon\varepsilon'}\left(\frac{\sigma}{\eta}\tanh\frac{\eta\tau}{2}+\frac{2w^2}{\eta^2}\tanh^2\frac{\eta\tau}{2}\right)\right]\exp\left(-ia\tau-\frac{w^2}{\sigma}\tau+\frac{2w^2}{\sigma\eta}\tanh\frac{\eta\tau}{2}\right)\right\}.
\end{aligned}
$$

The expression (35) recovers accurately the result calculated by Baryshevskii and Tikhomirov [10] for the LPM effect in the presence of external electric field.

Let us now consider in more details the limit of no external field ($w\to 0$), that corresponds to the Migdal theory. In this case the probability of photon emission becomes simpler

$$
\left\langle\frac{dW}{d\omega}\right\rangle = I^{22}+2I^{12}+I^{13}, \qquad (36)
$$

$$
I^{22} = -\frac{e^2}{\pi}\,{\rm Im}\int_0^T dt_1\int_0^{T-t_1}d\tau\frac{\eta\,{\rm e}^{-ia\tau}}{\sinh\eta\tau}\left(\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{2\varepsilon\varepsilon'}\frac{\sigma}{\eta}\tanh\frac{\eta\tau}{2}\right)-
$$

$$- \frac{e^2}{\pi a \gamma^2} \left(-1 + \cos aT + aT \;\; \mathrm{Si} aT\right), \tag{37}$$

$$I^{12} = -\frac{e^2}{\pi} \mathrm{Im} \int_0^T dt_2 \int_0^\infty dt_1 \frac{\eta \, \mathrm{e}^{-ia(t_1+t_2)}}{\cosh \eta t_2 \left(\tanh \eta t_2 + \eta t_1\right)} \left(\frac{1}{\gamma^2} + \frac{\varepsilon^2 + \varepsilon'^2}{4\varepsilon\varepsilon'} \frac{\sigma}{\eta} P_0{}'\right) -$$

$$- \frac{e^2}{\pi a \gamma^2} \left[1 - \cos aT + aT \left(\frac{\pi}{2} - \mathrm{Si} aT\right)\right], \tag{38}$$

$$I^{13} = -\frac{e^2}{\pi} \mathrm{Im} \int_0^\infty dt_1 \int_0^\infty dt_2 \left\{ \frac{\eta \, \mathrm{e}^{-iaT} \, \mathrm{e}^{-ia(t_1+t_2)}}{\cosh \eta T \left[\eta\left(t_1+t_2\right) + \left(1 + \eta^2 t_1 t_2\right) \tanh \eta T\right]} \times \right.$$

$$\left. \left(\frac{1}{\gamma^2} + \frac{\varepsilon^2 + \varepsilon'^2}{4\varepsilon\varepsilon'} \frac{\sigma}{\eta} P'\right) \right\} -$$

$$- \frac{e^2}{\pi a \gamma^2} \left[\cos aT - aT \left(\frac{\pi}{2} - \mathrm{Si} aT\right)\right]. \tag{39}$$

where

$$P_0{}' = \frac{\tanh \eta t_2 \left(2 \tanh \frac{\eta t_2}{2} + \eta t_1\right)}{\left(\tanh \eta t_2 + \eta t_1\right)},$$

$$P' = \frac{\tanh \eta T \left[2 \tanh \frac{\eta T}{2} + \eta \left(t_1 + t_2\right)\right]}{\left(1 + \eta^2 t_1 t_2\right) \tanh \eta T + \eta \left(t_1 + t_2\right)}. \tag{40}$$

As mentioned above, in the case of an infinitely thick target the leading term in (36) is

$$I^{22} \xrightarrow{T \to \infty} W_{\mathrm{LPM}} = -\frac{e^2 T}{\pi} \left[\mathrm{Im} \int_0^\infty d\tau \frac{\eta \, \mathrm{e}^{-ia\tau}}{\sinh \eta \tau} \left(\frac{1}{\gamma^2} + \frac{\varepsilon^2 + \varepsilon'^2}{2\varepsilon\varepsilon'} \frac{\sigma}{\eta} \tanh \frac{\eta \tau}{2}\right) + \frac{\pi}{2\gamma^2}\right]. \tag{41}$$

Substituting $z = \eta\tau$ in (41), and changing the path of integral in the complex plane from $z = \eta\tau$, $(0 \le \tau < \infty)$ to the path along real axis $x = \mathrm{Re} z$, $(0 \le x < \infty)$ we get the Migdal result [3]

$$I^{22} \xrightarrow{T \to \infty} W_{\mathrm{LPM}} = \left(e^2 T / 96 \pi \gamma^2 \varepsilon \varepsilon' s^2\right) \left[G(s) \omega^2 + 2\Phi(s) \left(\varepsilon^2 + \varepsilon'^2\right)\right], \tag{42}$$

$$G(s) = 48 s^2 \left[\frac{\pi}{4} - \int_0^\infty \frac{\sin 2sx}{\sinh x} \mathrm{e}^{-2sx} dx\right],$$

$$\Phi(s) = 24 s^2 \left[\int_0^\infty \coth x \sin 2sx \, \mathrm{e}^{-2sx} dx - \frac{\pi}{4}\right],$$

$$s = \sqrt{\left(a / 8\sigma\gamma^2\right)}.$$

6 Thick target approximation. Corrections to the Migdal theory

If the target is not of infinite thickness but still sufficiently thick so that

$$T \gg \min\left\{ l_{\mathrm{f}},\ \frac{1}{|\eta|} \right\}, \tag{43}$$

then the term I^{12} must be taken into account while the term I^{13} is still negligible

$$I^{12} \xrightarrow{T\to\infty} W'_{\mathrm{LPM}} =$$
$$-\frac{e^2}{\pi}\left[\mathrm{Im}\int_0^\infty dt_1\int_0^\infty dt_2\ \frac{\eta\ \mathrm{e}^{-ia(t_1+t_2)}}{\cosh\eta t_2\,(\tanh\eta t_2+\eta t_1)}\left(\frac{1}{\gamma^2}+\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\frac{\sigma}{\eta}P_0'\right)+\frac{1}{a\gamma^2}\right] \tag{44}$$

Thus, in the thick target approximation, taking account for the side effect yields the corrections to the Migdal theory

$$\left\langle\frac{dW}{d\omega}\right\rangle = W_{\mathrm{LPM}} + 2W'_{\mathrm{LPM}}. \tag{45}$$

This is to explain why in the SLAC experiments [5,6] the bremsstrahlung rate is always higher than that predicted by the Migdal theory and the subtraction of two spectra for targets of different thickness is fit so good to the measurements.

7 Thin target. The Bethe-Heitler limit

In the opposite case of extremely thin targets, the Migdal theory is completely in disagreeing with the experimental data, which are closer to the Bethe-Heitler theory when the target thickness becomes smaller than $0.1\%X_0$. This can be early expected because there is too little total multiple scattering to cause suppression. As mentioned above, our derivation of the LPM effect is valid for a target of arbitrary thickness. One can find that the Bethe-Heitler limit coincides with the lowest-order term in a formal expansion of formulas (36–39) in power of T. In fact, the terms I^{12} and I^{22} is proportional to T^2 and T^3 respectively, while the term I^{13} is linear in T and will dominate. Furthermore, it is also evident from (39) that the LPM suppression disappears when

$$\mathrm{Im}\left(\frac{\eta^2T^2}{2}\right) \le aT \ \Rightarrow\ T \le T_0 = \frac{\alpha X_0}{2\pi} \approx 0.1\%X_0. \tag{46}$$

Thus, in the limit of an extremely thin target ($T \le T_0$) the probability of bremstrahlung becomes

$$I^{13} \xrightarrow{T\to 0} W_0 = \frac{e^2\sigma T}{\pi a}\left[\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\int_0^\infty dx\int_0^\infty dy\ \frac{\sin(x+y)}{(x+y)} + \int_0^\infty dx\int_0^\infty dy\ \frac{xy\cos(x+y)}{(x+y)^2}\right]. \tag{47}$$

In (47) we have made the change of variables of integrals from $t_1,\ t_2$ to $x = at_1,\ y = at_2$. The first integral in (47) can be easily computed by using the asymptotic behavior of the sine integral function (34)

$$\int_0^\infty dx \int_0^\infty dy\, \frac{\sin(x+y)}{(x+y)} = 1. \tag{48}$$

Since the photon absorption, which has not been taken into account in our treatment, can be derived by introducing a small imaginary part of the photon energy $\omega \to \omega(1 - i\epsilon)$, it is natural to use the cutoff function $\exp[-\epsilon(x+y)]$ to evaluate the second integral. The result is

$$\int_0^\infty dx \int_0^\infty dy\, \frac{xy\cos(x+y)}{(x+y)^2} = -\frac{1}{6}. \tag{49}$$

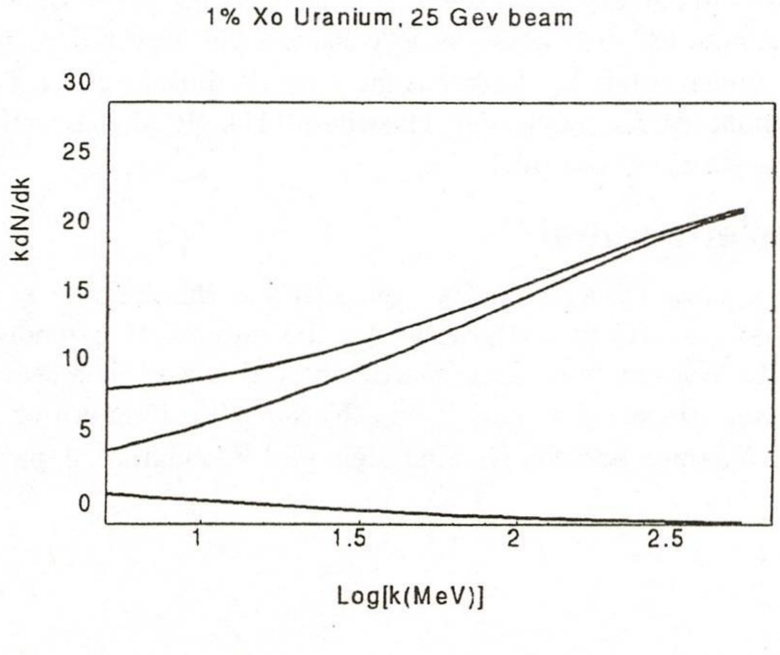
This yields finally

$$\left\langle \frac{dW}{d\omega} \right\rangle \xrightarrow{T\to 0} W_0 = \frac{e^2\gamma^2\sigma T}{6\pi\omega}\left[\frac{2(\varepsilon^2 + \varepsilon'^2)}{\varepsilon^2} + \frac{\omega^2}{\varepsilon^2}\right] \equiv \frac{dW_{\rm BH}}{d\omega}. \tag{50}$$

Thus, our treatment includes as limiting case Bethe-Heitler bremsstrahlung relevant for very thin target, that is confirmed by the SLAC experiments [5,6].

8 Numerical Results

In this section we give some of our preliminary numerical results. The theoretical formulas (36-40) enable us to calculate the expected bremsstrahlung rate for a case of intermediate thickness. As pictured in Fig.1 for not very thick targets taking into account for the boundary effect yields extra contributions to the total bremsstrahlung rate. In the case of extremely thin targets, the boundary effect cancels out the LPM suppression and we are back to the Bethe–Heitler regime for the case of isolated atom (50).



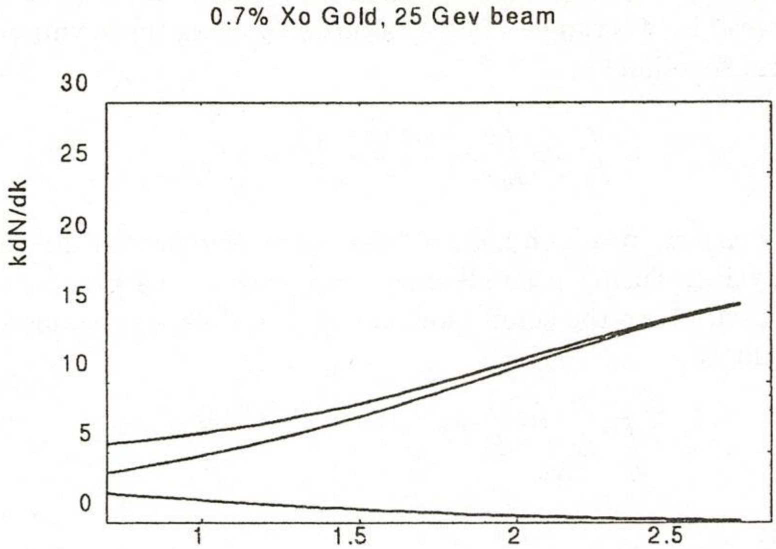


Figure 1. The bremsstrahlung spectra kdN/dk in relative unit versus Log[k(MeV)] for 25 GeV electrons incident on (a) 1%, (b) 0.7% radiation length uranium and gold targets. The Landau–Pomeranchuk–Migdal theory is denoted by middle curves, and our theory, the contributions of the boundary effect are denoted by lower curves and the total bremsstrahlung rate is denoted by upper curves.

9 Conclusion

In this report we have given outline of the application of Feynman path integral method in performing the statistical averages that treats the LPM suppression effect in the presence of an uniform electric field. Our derivation is valid for a target of arbitrary thickness. It includes as limiting cases Bethe-Heitler bremsstrahlung relevant for very thin target and the LPM theory for an infinite medium. In the thick target approximation we have made corrections to the well-known Migdal formulas. Our theory is completely in agreeing with the SLAC experiments on the LPM effect [5,6]. The theoretical results (36-40) enable us to calculate the expected bremsstrahlung rate for a case of intermediate thickness. A more detailed description of our present work is being completed for publication elsewhere [17]. It includes our numerical calculation for targets of various thickness.

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