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Self-couplings of gauge bosons in 3-4-1 models

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ABSTRACT

Feynman rules for self-couplings of the gauge bosons in the general 3-4-1 model based on the gauge group SU(3)c⊗SU(4)$_L$⊗U(1)$_X$ will be presented in this work. The results are checked to be consistent with previous works. The results also confirm important relations between different triple and quartic gauge couplings assumed previously for constructing general one-loop contributions to physical processes searched by experiments.

Key words: cLFV, 341, ISS

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INTRODUCTION

The self-couplings of the gauge bosons always appear in the non-abelian field theories such as the standard model (SM) and models beyond the SM (BSM) and give contributions to many physical processes. In particular, this coupling kind gives the key contributions to loop-induced decay of the SM-like Higgs boson h $\to \gamma\gamma$ observed by experiments. In BSMthe new self-couplings of gauge bosons may give significant contributions to this decay as well as many other processes which are being searched for experiments such as h $\to$ Zγ[1], h $\to f\bar{f}\gamma$[2]. These cubic and quartic coupling types in 3-3-1 models were discussed in many previous works[3–5]. In this work, we will introduce them in the 3-4-1 models in the general form (G341) reported in Ref.[6]. All other particular 3-4-1 models discussed previously[7–11] can be derived from this general form. Deviations from some of these couplings with the SM predictions were searched experimentally at LEP[12]. They may be used to constrain the SU(4)$_L$ scale if the future experimental sensitivities are good enough. Experimental searches are also being paid attention at LHC[13]. They will be also interesting objects for planned experiments such as CLIC[14], LHeC, and the FCC-he[15,16]. Our results of gauge-boson couplings will be used to calculate the one-loop contributions predicted by the G341 model to various loop-induced decays relating to experiments such as h $\to \gamma\gamma$, Zγ, $f\bar{f}\gamma$.

THE MODEL AND PHYSICAL SPECTRUM

We will base on the model with the electric charge operator defined as follows[6]:

$$\widehat{Q} = T_3 + bT_8 + cT_{15} + X, \qquad (1)$$

where the coefficient in front of T$_3$ equaling to 1, is chosen to ensure that the SM group is a subgroup of the model under consideration: SU(2)$_L$ ⊗ U(1)$_Y$ ⊂ SU(4)$_L$ ⊗ U(1)$_X$. The covariant derivative of the electroweak group SU(4)$_L$ × U(1)$_X$ is:

$$D_\mu = \partial_\mu - ig\sum_{a=1}^{15} W_{a\mu} T_a - ig_X X B''_\mu T_{16}, \qquad (2)$$

where g, gX and W$_{a\mu}$, B$_\mu$'' are gauge couplings and fields of the gauge groups SU(4)$_L$ and U(1)$_X$, respectively. For any quadruplets being the fundamental representation (rep.) of the SU(4)$_L$ group, the explicit formulas of the Ta = $\lambda_a/2$ is constructed following Ref.[17]

as the expansions of the Pauli matrices, and T$_{16}$ = 1/2λ_{16} = $\frac{1}{2\sqrt{2}}$diag(1, 1, 1, 1). All of thesematrices satisfy the condition that $Tr\left[\frac{\lambda_a}{2}\frac{\lambda_b}{2}\right] = \frac{\delta_{ab}}{2}$. Consequently, all structure constants of the SU(4) group are derived as f$_{abc}$ = -2iTr ([T$_a$, T$_b$]T$_c$) with T$_a$ = λ_a/2. The non-zero values of fabc with $1 \leq a < b < c \leq 15$ are:

$f^{123} = 1$, $f^{147} = -f^{156} = f^{19(12)} = -f^{1(10)(11)} = 12$, $f^{246} = f^{257} = f^{29(11)} = f^{2(10)(12)} = 12$, $f^{345} = -f^{367} = f^{39(10)} = -f^{3(11)(12)} = 12$, $f^{458} = \frac{\sqrt{3}}{2}$, $f^{49(14)} = -f^{4(10)(13)} = 12$, $f^{59(13)} = f^{5(10)(14)} = 12$, $f^{678} = \frac{\sqrt{3}}{2}$, $f^{6(11)(14)} = -f^{6(12)(13)} = -12$, $f7^{(11)(13)} = f^{7(12)(14)} = 12$, $f^{89(10)} = f^{8(11)(12)} = \frac{1}{2\sqrt{3}}$, $f^{8(13)(14)} = -13$, $f^{9(10)(15)} = f^{(11)(12)(15)} = f^{(13)(14)(15)} = \sqrt{\frac{2}{3}}$. (3)

Other f^{abc} values are derived from the total antisymmetric property:

$f^{abc} = f^{cab} = f^{bca} = -f^{acb} = -f^{bac} = -f^{cba}$.

In this work, the left-chiral leptons are chosen as three quadruplets, namely

$$L_a = \left(\nu_a, e_a, E_a^{-q1}, {}_a, E_a^{-q2}\right)_L^T \sim (1, 4, X_L), \quad a = e, \mu, \tau, \qquad (4)$$

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where -q_1 and -q_2 are electric charges of the extra leptons E_a and $E_a^{'}$. The electric chargeoperator in (1) is written in terms of the electric charges of lepton in Eq. (4) as follows

$$b=\frac{2q_1-1}{\sqrt{3}},\ c=\frac{3q_2-q_1-1}{\sqrt{6}},\quad X_L=-\frac{q_1+q_2+1}{4}. \tag{5}$$

The right-handed leptons are arranged as $SU(4)_L$ singlets: $e_{aR}\sim(1,1,-1)$, $E_{aR}\sim(1,1,-q_1)$, and $E_{aR}^{'}\sim(1,1,-q_2)$. To cancel gauge anomalies, the three quark generationsmust be arranged as one anti-quadruplets and two quadruplets. This property is beyondthe scope of this work.

To generate the quark and lepton masses, four Higgs quadruplets are needed in general. The vacuum expectation values (vev) of the neutral Higgs components are:

$$\langle\phi_1\rangle=\left(0,0,0,\frac{V}{\sqrt{2}}\right)^T,\ \langle\phi_2\rangle=\left(0,0,0,\frac{\omega}{\sqrt{2}},0\right)^T,\quad \langle\phi_3\rangle=\left(0,\frac{v}{\sqrt{2}},0,0\right)^T,\ \langle\phi_4\rangle=\left(\frac{u}{\sqrt{2}},0,0,0\right)^T. \tag{6}$$

They give the following mass and physical states of the non-hermitian gauge bosons:

$$m_W^2=\frac{g^2\left(v^2+u^2\right)}{4},\ m_{W13}^2=\frac{g^2\left(u^2+\omega^2\right)}{4},\quad m_{W23}^2=\frac{g^2\left(v^2+\omega^2\right)}{4},\ m_{W14}^2=\frac{g^2\left(u^2+V^2\right)}{4},\quad m_{W24}^2=\frac{g^2\left(v^2+V^2\right)}{4},\ m_{W34}^2=\frac{g^2\left(\omega^2+V^2\right)}{4}, \tag{7}$$

where $W_{ij\mu}$ is defined in the covariant part of the $SU(4)_L$ fundamental rep. as follows

$$W_\mu\equiv\sum_{i=1}^{15}\frac{\lambda}{2}W_\mu^i=\frac{1}{2}\times\begin{pmatrix}(11) & \sqrt{2}W^+ & \sqrt{2}W_{13}^{q_{13}} & \sqrt{2}W_{14}^{q_{14}}\\ \sqrt{2}W^- & (22) & \sqrt{2}W_{23}^{q_{23}} & \sqrt{2}W_{24}^{q_{24}}\\ \sqrt{2}W_{13}^{-q_{13}} & \sqrt{2}W_{23}^{-q_{23}} & (33) & \sqrt{2}W_{34}^{q_{34}}\\ \sqrt{2}W_{14}^{-q_{14}} & \sqrt{2}W_{24}^{-q_{24}} & \sqrt{2}W_{34}^{-q_{34}} & (44)\end{pmatrix}$$
$$(11)=\left(W_3+\frac{W_8}{\sqrt{3}}+\frac{W_{15}}{\sqrt{6}}\right)_\mu,\quad (22)=\left(-W_3+\frac{W_8}{\sqrt{3}}+\frac{W_{15}}{\sqrt{6}}\right)_\mu,\quad (33)=\left(-\frac{2W_8}{\sqrt{3}}+\frac{W_{15}}{\sqrt{6}}\right)_\mu,\ (44)=\left(-\frac{3W_{15}}{\sqrt{6}}\right)_\mu, \tag{8}$$

and

$$W^\pm=\frac{W_1\pm iW_2}{\sqrt{2}},\quad W_{13}^{\pm q_{13}}=\frac{W_4\pm iW_5}{\sqrt{2}},\ W_{23}^{\pm q_{23}}=\frac{W_6\pm iW_7}{\sqrt{2}},\quad W_{14}^{\pm q_{14}}=\frac{W_9\pm iW_{10}}{\sqrt{2}},W_{24}^{\pm q_{24}}=\frac{W_{11}\pm iW_{12}}{\sqrt{2}},\quad W_{34}^{\pm q_{34}}=\frac{W_{13}\pm iW_{14}}{\sqrt{2}}. \tag{9}$$

We note that the electric charges of gauge bosons given in the formula of $W\mu$ are determined from the electric charge operator $\widehat{Q}$ given in Eq. (1), where fifteen gauge bosons are included in an adjoint rep. of the SU(4)L group. Changing into the basis relating to fundamental rep. with generator being λa, the action of $\widehat{Q}$ on the gauge multiplet is: $\widehat{Q}W_\mu$ = $[Q_4W_\mu,\ W_\mu Q_4]$, where Q4 = $\widehat{Q}$ [$T_a=\lambda_a/2$] ; ∀a = $\overline{1,15}$. The electric charge qij of a gauge boson W_{ij} is $\left[\widehat{Q}W_\mu\right]_{ij}=q_{ij}W_{ij\mu}$. As a result, q_{ij} is determined in terms of the electric charges of the newleptons as follows:

$q_{12}=1$, $q_{13}=q_1$, $q_{14}=q_2$, $q_{23}=q_1-1$, $q_{24}=q_2-1$, $q_{34}=q_2-q_1$. (10)

In our calculation, we denote simply that $W_{ij}=W_{ij}^{q_{ij}}$ and $W_{ij}^*=W_{ij}^{-q_{ij}}$ with $1\leq i<j\leq 4$.

As the usual previous assumption, the spontaneous symmetry breaking follows the pattern

$SU(4)_L\otimes U(1)_X\xrightarrow{V} SU(3)_L\otimes U(1)_N\xrightarrow{\omega} SU(2)_L\otimes U(1)_Y\xrightarrow{u,v} U(1)_Q$,

which is used for constructing the matching relation of the gauge couplings and U(1) chargesof the group $SU(4)_L\times U(1)_X$ and those of the SM gauge group $SU(2)_L\times U(1)_Y$, see detailsin Ref.[6]. We just focus here necessary results used to determine the triple and quarticcouplings of all gauge bosons. The following relation should be in order: $V\gg\omega\gg u,v$.The matching with the gauge couplings of the SM gives:

$$u^2+v^2=v_{SM}^2=246^2GeV^2, \tag{11}$$

and

$$\frac{gt}{\sqrt{8+\left(b^2+c^2\right)t^2}}=g_1,\quad \frac{\widehat{Y}}{2}=bT^8+cT^{15}+XI_4, \tag{12}$$

where g_1 and $\widehat{Y}$ are the $U(1)_Y$ gauge coupling and U(1) charge in the SM. The secondformula in (12) is consistent with the identification of $\widehat{Y}$ from the definition of the electriccharge operator given in Eq. (1). Furthermore, it can be seen that $\widehat{N}\equiv cT^{15}+X$ and $b\equiv\beta\sqrt{3}$ are relations between parameters defined in the gauge groups $SU(4)_L\times U(1)_X$ and $SU(3)_L\times U(1)_N$.

From the equality $g_1/g=s_W/c_W$, where g is identified with the $SU(2)_L$ gauge couplingand $s_W^2=0.231$ from experiments, we find that

$$t=\frac{gx}{g}=\frac{2\sqrt{2}s_W}{\sqrt{1-\left(1+b^2+c^2\right)s_W^2}}, \tag{13}$$

equivalently,

$$(q_1+q_2-1)^2+2(q_1^2+q_2^2)\leq 6 \tag{14}$$

Eq. (14) results in electric charge constraints of the new exotic leptons E_a and $E_a^{'}$, namely

$$-q_1 - q_2 - q_1 q_2 + \frac{3}{2}\left(1 + q_1^2 + q_2^2\right) \leq 4 \qquad (15)$$

Eq. (15) is equivalent to $(q_1 + q_2 - 1)^2 + 2(q_1^2 + q_2^2) \leq 6$, implying that $|q_{1,2}| \leq \sqrt{3}$. The physical-states of neutral gauge bosons include one massless photon $A\mu$ and three massive bosons Z_1, Z_2, and Z_3. One of them is identified with the SM prediction, $Z_1 \equiv Z$. Denoting M_{NG}^2 isthe squared mass matrix of the neutral gauge bosons in the flavor basis $\left(W_{3\mu}, W_{8\mu}, W_{15\mu}, B_\mu^{''}\right)$, which relates to the physical basis (A_μ, $Z_{1\mu}$, $Z_{2\mu}$, $Z_{3\mu}$) through the mixing angles defined as follows[6]

$$\begin{array}{l} c_{43} \equiv \frac{ct}{\sqrt{8+c^2t^2}},\ s_{43} \equiv \frac{2\sqrt{2}}{\sqrt{8+c^2t^2}}, \\ s_{32} \equiv \frac{\sqrt{8+c^2t^2}}{\sqrt{8+(b^2+c^2)t^2}},\ c_{32} \equiv \frac{bt}{\sqrt{8+(b^2+c^2)t^2}}, \\ t_{2\alpha} = \\ \frac{4\sqrt{2}s_{43}s_{32}(-1+c_{43}s_{43}bt)\omega^2}{8s_{43}^2\omega^2 - s_{32}^2\left[(-1+c_{43}s_{43}bt)^2\omega^2+9V^2\right]}. \end{array} \qquad (16)$$

Combining the formulas in Eqs. (5), (14), and (16), the mixing angles are expressed in terms of new electric charges as follows:

$$\begin{array}{l} c_{32} = \frac{\sqrt{3-4\left(q_1^2-q_1+1\right)s_W^2}}{\sqrt{3}c_W}, \\ s_{32} = \frac{(2q_1-1)t_W}{\sqrt{3}}, \\ c_{43} = -\frac{s_W(q_1-3q_2+1)}{\sqrt{6-8\left(q_1^2-q_1+1\right)s_W^2}}, \\ s_{43} = \\ \sqrt{\frac{s_W^2\left[(3q_1-1)^2+(3q_2-1)^2-6q_1q_2+7\right]-6}{8\left(q_1^2-q_1+1\right)s_W^2-6}}. \end{array} \qquad (17)$$

Finally, the relations between physical and flavor base are:

$$\begin{array}{l} W_{3\mu} = A_\mu s_W + c_W Z_{1\mu} \\ W_{8\mu} = A_\mu c_{32} c_W - s_{32} s_W Z_{1\mu} \\ -c_\alpha s_{32} Z_{2\mu} + s_{32} s_\alpha Z_{3\mu}, \\ W_{15\mu} = A_\mu c_{43} c_W s_{32} - c_{43} s_{32} s_W Z_{1\mu} \\ +Z_{2\mu}\left(c_{32}c_{43}c_\alpha - s_{43}c_\alpha\right) \\ +Z_{3\mu}\left(-c_{32}c_{43}s_\alpha - s_{43}s_\alpha\right). \end{array} \qquad (18)$$

The above discussion is enough to derive all Feynman rules for self-couplings of gauge bosons in the G341 model.

FEYNMAN RULES FOR SELF-COUPLINGS OF GAUGE BOSONS

The self-couplings of gauge bosons are included in the covariant kinetic term of the nonabelian gauge bosons:

$$\begin{array}{l} L_g^{kin} = -\frac{1}{4}\sum_{a=1}^{15} F_{a\mu\nu}F_a^{\mu\nu}, \\ F_{a\mu\nu} = \partial_\mu W_{a\nu} - \partial_\nu W_{a\mu} \\ +g\sum_{b,c=1}^{15} f^{abc} W_{b\mu} W_{c\nu}, \\ a,\ b,\ c\ =\ 1,2,...,15. \end{array} \qquad (19)$$

We use the convention that the couplings g has the plus in $F_{a\mu\nu}$, consistent with the formulas of D_μ given in Eq. (2)[18,19]. The total antisymmetry of fabc gives a simpler form of theLagrangian parts L_{3g} and L_{4g} corresponding to the triple and quartic couplings:

$$\begin{array}{l} L_g^{kin} = -\frac{1}{4}\sum_{a=1}^{15}\left(\partial_\mu W_{a\nu} - \partial_\nu W_{a\mu}\right) \times \\ \left(\partial^\mu W_a^\nu - \partial^\nu W_a^\mu\right) + L_{3g} + L_{4g}, \\ L_{3g} = -g f^{abc}\left(\partial_\mu W_\nu^a\right) W^{b\mu} W^{c\nu}, \\ L_{4g} = -\frac{g^2}{4} f^{abc} f^{ab'c'} W_\mu^b W_\nu^c W^{b'\mu} W^{c'\nu} \\ = -\frac{g^2}{4} f^{abc} f^{ab'c'}\left(W^b.W^{b'}\right)\left(W^c.W^{c'}\right). \end{array} \qquad (20)$$

For simplicity, we omit the sum over all repeated indices a, b, c, a′, b′, c′ = 1, 2, ..., 15. In our calculation, other conventions will be noted if needed.

Lagrangian L_{3g} in Eq. (20) is written in the following form:

$$\begin{array}{l} L_{3g} = g\sum_{a=3,8,15}\sum_{b<c}[f^{abc}\left(\partial_\mu W_\nu^a\right) \\ \left(W^{b\mu}W^{c\nu} - W^{c\mu}W^{b\nu}\right) + \\ (permutation\,[a,\ b,\ c])] \\ +g\sum_{a<b<c\neq 3,8,15}[f^{abc}\left(\partial_\mu W_\nu^a\right) \\ \left(W^{b\mu}W^{c\nu} - W^{c\mu}W^{b\nu}\right) + \\ (permutation\,[a,\ b,\ c])], \end{array} \qquad (21)$$

where permutation [a, b, c] in the second sum stands for the two remaining terms generatedby the following permutations [a, b, c] → [b, c, a] → [c, a, b]. Here, b, c ≠ 3, 8, 15 in both sumsabove, because f^{abc} does not contain simultaneously two indices of neutral gauge bosons. This implies that there are no triple couplings consisting of two real neutral gauge bosons, which is consistent with the electric charge conversation and leads to the coupling type $V^{0\mu}V_1^{q\nu}V_2^{-q\lambda}$. The first term in Eq. (21) consists of all couplings with one neutral real gaugeboson. In contrast, all couplings in the second term have three charged gauge bosons. The conservation of all electric charges given in Eq. (10) will constrain allowed couplings, namely

$$W^- W_{13} W_{23}^* \rightarrow h.c. = W^+ W_{13}^* W_{23}, ... \qquad (22)$$

Now, we start from the first line in Eq. (21), which is rewritten precisely as follows:

$$\begin{aligned}
L_{3g}^{G^0G^{\pm}} &= [\left(\partial_\mu W_\nu^3\right)\left(W^{1\mu}W^{2\nu} - W^{2\mu}W^{1\nu}\right)\\
&+ (permutation)]\\
&+[\left(\partial_\mu W_{13\nu}^0\right)\left(W^{4\mu}W^{5\nu} - W^{5\mu}W^{4\nu}\right)\\
&+ (permutation)]\\
&-[\left(\partial_\mu W_{23\nu}^0\right)\left(W^{6\mu}W^{7\nu} - W^{7\mu}W^{6\nu}\right)\\
&+ (permutation)]\\
&+[\left(\partial_\mu W_{24\nu}^0\right)\left(W^{9\mu}W^{10\nu} - W^{10\mu}W^{9\nu}\right)\\
&+ (permutation)]\\
&-[\left(\partial_\mu W_{24\nu}^0\right)\left(W^{11\mu}W^{12\nu} - W^{12\mu}W^{11\nu}\right)\\
&+ (permutation)]\\
&+[\left(\partial_\mu W_{34\nu}^0\right)\left(W^{13\mu}W^{14\nu} - W^{14\mu}W^{13\nu}\right)\\
&+ (permutation)].
\end{aligned} \tag{23}$$

where $W_{ij,\nu}^0$ is the linear combinations of W_ν^3, W_ν^8, and W_ν^{15}, namely

$$\begin{aligned}
W_{12\mu}^0 &= W_\mu^3 = A_\mu s_W + c_W Z_{1\mu},\\
W_{13\mu}^0 &= \tfrac{1}{2}\left(W_\mu^3 + \sqrt{3}W_\mu^8\right) = A_\mu q_{13} s_W\\
&- \tfrac{c_\alpha k_1 Z_{2\mu}}{2c_W} + \tfrac{s_\alpha k_1 Z_{3\mu}}{2c_W} + \tfrac{Z_{1\mu}\left(1-2q_{13}s_W^2\right)}{2c_W},\\
W_{34\mu}^0 &= \tfrac{1}{\sqrt{3}}\left(-W_\mu^8 + \sqrt{2}W_\mu^{15}\right)\\
&= A_\mu q_{34} s_W - \tfrac{s_W^2 Z_{1\mu} q_{34}}{c_W}\\
&+ \tfrac{c_\alpha Z_{2\mu}\left(-2c_W k_2 t_\alpha + k_1^2 + 2k_3 s_W^2 - 1\right)}{2c_W k_1}\\
&- \tfrac{c_\alpha Z_{3\mu}\left(2c_W k_2 + t_\alpha\left(k_1^2 + 2k_3 s_W^2 - 1\right)\right)}{2c_W k_1},\\
W_{14\mu}^0 &= \tfrac{1}{2\sqrt{3}}\left(\sqrt{3}W_\mu^3 + W_\mu^8 + 2\sqrt{2}W_\mu^{15}\right)\\
&= W_{13\mu}^0 + W_{34\mu}^0\\
&= A_\mu q_2 s_W + \tfrac{z_{1\mu}\left(1-2q_{14}s_W^2\right)}{2c_W}\\
&- \tfrac{c_\alpha Z_{2\mu}\left(2c_W k_2 t_\alpha - 2k_3 s_W^2 + 1\right)}{2c_W k_1}\\
&- \tfrac{c_\alpha Z_{3\mu}\left(2c_W k_2 + t_\alpha\left(2k_3 s_W^2 - 1\right)\right)}{2c_W k_1},\\
W_{24\mu}^0 &= \frac{1}{2\sqrt{3}}\left(\sqrt{3}W_\mu^3 - W_\mu^8 - 2\sqrt{2}W_\mu^{15}\right)\\
&= W_{23\mu}^0 - W_{34\mu}^0\\
&= A_\mu q_{24} s_W - \frac{z_{1\mu}\left(1+2q_{24}s_W^2\right)}{2c_W}\\
&- \frac{c_\alpha Z_{2\mu}\left(2c_W k_2 t_\alpha - 2k_3 s_W^2 + 1\right)}{2c_W k_1}\\
&- \frac{c_\alpha Z_{3\mu}\left(2c_W k_2 + t_\alpha\left(2k_3 s_W^2 - 1\right)\right)}{2c_W k_1},
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
k_1 &\equiv \sqrt{3 - 4\left(q_1^2 - q_1 + 1\right)s_W^2},\\
k_2 &= 2 - s_W^2(3q_1^2 - 2q_1\left(q_2 + 1\right)\\
&+ q_2\left(3q_2 - 2\right) + 3),\\
k_3 &\equiv 2q_1 q_2 + 1 - q_1 - q_2.
\end{aligned} \tag{25}$$

Every gauge boson pair W_μ^b and W_μ^{b+1} with b ∈ {1, 4, 6, 9, 11, 13} given in Eq. (9) relatesto the two physical states $G_\mu^{\pm Q} \sim$ {$W^{\pm}$, W_{13}, W_{23}, W_{14}, W_{24}, W_{34}} by the same followingformulas:

$$\begin{aligned}
W_\mu^b &= \frac{G_\mu^Q + G_\mu^{-Q}}{\sqrt{2}},\\
W_\mu^{b+1} &= \frac{i\left(G_\mu^{+Q} - G_\mu^{-Q}\right)}{\sqrt{2}}.
\end{aligned} \tag{26}$$

Then Eq. (23) is written in the following form:

$$\begin{aligned}
L_{3g}^{G^0G^{\pm}} &= \textstyle\sum_{i\le j\le 4}[\left(\partial_\mu W_{ij,\nu}^0\right) \times\\
&\left(W^{b\mu}W^{(b+1)\nu} - W^{(b+1)\mu}W^{b\nu}\right)\\
&+ (permutation)],
\end{aligned} \tag{27}$$

Then, each line in Eq. (23) is written in term of the physical states as follows:

$$\begin{aligned}
&\left(\partial_\mu G_\nu^0\right)\left[W^{b\mu}W^{(b+1)\nu} - W^{(b+1)\mu}W^{b\nu}\right]\\
&= i\left(\partial_\mu G_\nu^0\right)\left(-G^{+\mu}G^{-\nu} + -G^{-\mu}G^{+\nu}\right),\\
&= -\left(p_1 G^+\right)\left(V^0 G^-\right) + \left(p_1 G^-\right)\left(V^0 G^+\right),\\
&G^{0\nu}\left[\left(\partial_\mu W_\nu^b\right)W^{(b+1)\mu} - \left(\partial_\mu W_\nu^{b+1}\right)W^{b\mu}\right]\\
&= iG^{0\nu}\left[-\left(\partial_\mu G_\nu^+\right)G^{-\mu} + \left(\partial_\mu G_\nu^-\right)G^{+\mu}\right]\\
&= -\left(p_2 G^-\right)\left(V^0 G^+\right) + \left(p_3 G^+\right)\left(V^0 G^-\right),\\
&-G^{0\mu}\left[\left(\partial_\mu W_\nu^b\right)W^{(b+1)\nu} - \left(\partial_\mu W_\nu^{b+1}\right)W^{b\nu}\right]\\
&= -iG^{0\nu}\left[-\left(\partial_\mu G_\nu^+\right)G^{-\nu} + \left(\partial_\mu G_\nu^-\right)G^{+\nu}\right]\\
&= -\left(p_2 V^0\right)\left(G^- G^+\right) - \left(p_3 V^0\right)\left(G^+ G^-\right),
\end{aligned} \tag{28}$$

where we have replace the derivatives with the momenta incoming vertex containing therespective field: $\partial_\mu G_\nu^0 = -ip_{1\mu}G_\nu^0$, $\partial_\mu G_\nu^+ = -ip_{2\mu}G_\nu^+$ and $\partial_\mu G_\nu^- = -ip_{3\mu}G_\nu^-$. In addition,$G^0$ = A, Z_1, Z_2, Z_3 belong to the linear combinations of neutral gauge bosons. Then, theLagrangian (23) is written in a standard form as follows:

$$\begin{aligned}
L_{3g}^{G^0G^{\pm}} &= \textstyle\sum_{G^0,G^{\pm}}(-1) \times g_{G^0,ij} \times \tau_{\mu\nu\alpha}\left(p_1, p_2, p_3\right)\\
&\left[G^{0\mu}\left(p_1\right)W_{ij}^{\nu}\left(p_2\right)W_{ij}^{*\alpha}\left(p_3\right)\right],
\end{aligned} \tag{29}$$

where the factor $-ig_{G^0,ij}$ is the vertex factors of the respective Feynman rule and

$$\begin{aligned}
\tau_{\mu\nu\alpha}\left(p_1, p_2, p_3\right) &= g_{\mu\nu}(p_1 - p_2)_\alpha\\
&+ g_{\nu\alpha}(p_2 - p_3)_\mu + g_{\mu\alpha}(p_3 - p_1)_\nu
\end{aligned} \tag{30}$$

The explicit expressions of $g_{G^0,ij}$ is defined in Eq. (29) are listed in the table I. They are derived from the linear combinations of neutral gauge bosons given in Eq. (23), namely,they are exactly the factors in front of the physical states of W_{ij}^0 defined in Eq. (24):

$$\begin{aligned}
W_{ij\mu}^0 &= \textstyle\sum_{G^0} g_{G^0,ij}G_\mu^0 = g_{A,ij}Z_{1\mu}\\
&+ g_{Z_{2,ij}}Z_{2\mu} + g_{Z_{3,ij}}Z_{3\mu}.
\end{aligned} \tag{31}$$

We can see that the couplings of gauge boson W with photon and Z_1 are consistent with the results from

Table 1: Feynman rules for triple gauge couplings consisting of a neutral gauge boson in thegeneral 3-4-1 model i.e., the coefficients $g_{G^0,ij}$ of Eq. (29) Here $G^0_\mu = A_\mu,\ Z_{k\mu}$ with k = 1, 2, 3.

$G^{0\mu}W^{\nu}_{ij}W^{*\alpha}_{ij}$	$-ig_{G^0,ij}$	$G^{0\mu}W^{\nu}_{ij}W^{*\alpha}_{ij}$	$-ig_{G^0,ij}$
AW^+W^-	-ie	$AW_{ij}W^*_{ij}$	$-ie\times q_{ij}$
$Z_1W^+W^-$	$-igc_W$	$Z_1W_{13}W^*_{13}$	$-\frac{ig\left(1-2q_1s_W^2\right)}{2c_W}$
$Z_1W_{23}W^*_{23}$	$\frac{ig\left(1+2q_{23}s_W^2\right)}{2c_W}$	$Z_1W_{14}W^*_{14}$	$-\frac{ig\left(1-2q_{14}s_W^2\right)}{2c_W}$
$Z_1W_{24}W^*_{24}$	$\frac{ig\left(1+2q_{24}s_W^2\right)}{2c_W}$	$Z_1W_{34}W^*_{34}$	$-\frac{igq_{34}s_W^2}{c_W}$
$Z_2W_{13}W^*_{13}$	$\frac{igc_\alpha k_1}{2c_W}$	$Z_2W_{23}W^*_{23}$	$-\frac{igc_\alpha k_1}{2c_W}$
$Z_2W_{14}W^*_{14}$	$\frac{igc_\alpha\left(2c_Wk_2t_\alpha-2k_3s_W^2+1\right)}{2c_Wk_1}$	$Z_2W_{24}W^*_{24}$	$\frac{igc_\alpha\left(2c_Wk_2t_\alpha-2k_3s_W^2+1\right)}{2c_Wk_1}$
$Z_2W_{34}W^*_{34}$	$-\frac{igc_\alpha\left(-2c_Wk_2t_\alpha+k_1^2+2k_3s_W^2-1\right)}{2c_Wk_1}$	$Z_3W_{13}W^*_{13}$	$-\frac{gc_\alpha k_1t_\alpha}{2c_W}$
$Z_3W_{23}W^*_{23}$	$-\frac{gc_\alpha k_1t_\alpha}{2c_W}$	$Z_3W_{14}W^*_{14}$	$\frac{igc_\alpha\left(2c_Wk_2+t_\alpha\left(2k_3s_W^2-1\right)\right)}{2c_Wk_1}$
$Z_3W_{24}W^*_{24}$	$\frac{igc_\alpha\left(2c_Wk_2+t_\alpha\left(2k_3s_W^2-1\right)\right)}{2c_Wk_1}$	$Z_3W_{34}W^*_{34}$	$\frac{igc_\alpha\left(2c_Wk_2+t_\alpha\left(k_1^2+2k_3s_W^2-1\right)\right)}{2c_Wk_1}$

SM. In addition, the photon always couples with two identical charged gauge bosons Wij, confirming the consequence of Ward Identity shown in Ref.[20]. This form of photon couplings is also consistent with that assumed in Ref.[21] necessary for calculating one-loop contributions of gauge bosons to lepton flavor violating (LFV) decays as well as the anomalous magnetic moments (AMM) of charged leptons.

Now we pay attention to the triple couplings of three non-hermitian gauge bosons given in the second line of Eq. (21). Because the relations between the two flavor and physical base of these gauge bosons are generalized simply by Eq. (26), the vertex factors corresponding to the Feynman rules are $(\pm)\,\frac{g}{2}\times\tau_{\mu\nu\alpha}\left(p_1,p_2,p_3\right)$.

In general, Lagrangian for all triple couplings is always written in the following form:

$$L_{3g}=\sum_{G_1,G_2,G_3}(-1)\times g_{123}\tau_{\mu\nu\alpha}\left(p_1,p_2,p_3\right)\left[G_1^\mu\left(p_1\right)G_2^\nu\left(p_2\right)G_3^\alpha\left(p_3\right)\right], \quad (32)$$

which the Feynman rule for a coupling $G_1^\mu\left(p_1\right)G_2^\nu\left(p_2\right)G_3^\alpha\left(p_3\right)$ is the vertex factor ig_{123}, where $\mathrm{g}_{123}\equiv\mathrm{g}_{0\pm}$ for the vertex consisting of a neutral gauge boson, as given in Table I. For allnon-zero couplings between three non-hermitian gauge bosons, the Feynman rules are listed in the table II.

The quartic couplings is included in the following Lagrangian

$$L_{4g}=-\frac{g^2}{4}f^{abc}f^{ab'c'}\left(W^bW^{b'}\right)\left(W^cW^{c'}\right) \quad (33)$$

In the physical basis of all $G_1^\mu G_2^\nu G_3^\alpha G_4^\beta$, this Lagrangian is written in the following form:

$$L_{4g}\sim G_1^\mu G_2^\nu G_3^\alpha G_4^\beta\times\Gamma_{\mu\nu,\alpha\beta}\to ig_{G_{1234}}\times$$
$$\Gamma'_{\mu\nu,\alpha\beta}=i\frac{\partial^4L_{4g}}{\left(\partial G_1^\mu\right)\left(\partial G_2^\nu\right)\left(\partial G_3^\alpha\right)\left(\partial G_4^\beta\right)}, \quad (34)$$

where the last expression in Eq. (34) is the Feynman rule for calculating vertex factors of quartic couplings. In general, $\Gamma_{\mu\nu,\alpha\beta}\neq\Gamma'_{\mu\nu,\alpha\beta}$, depending on the relations between four physical states, see for example for 3-3-1 models introduced in ref.[4]. Here the tensor structures are:

$$\Gamma'_{\mu\nu,\alpha\beta}\to S_{\mu\nu,\alpha\beta}\equiv 2g_{\mu\alpha}g_{\nu\beta}-g_{\mu\nu}g_{\alpha\beta}-g_{\mu\beta}g_{\nu\alpha} \quad (35)$$

Applying the same step of calculations shown precisely in ref.[22], the Feynman rules ofthese couplings are divided into different classes as follows.

Firstly, all couplings consisting of only charged bosons $W_{ij}^\mu W_{ij}^\nu W_{ij}^{*\alpha}W_{ij}^{*\beta}$, . . . are writtenin the following

Table 2: Feynman rules for triple non-hermitian gauge couplings in the G341 model i.e., the coefficients g_{123} of Eq. (32).

$G_1^\mu G_2^\nu G_3^\alpha$	-ig_{123}	$G_1^\mu G_2^\nu G_3^\alpha$	-ig_{123}
$W^+W_{13}^*W_{23}$	$\frac{ig}{\sqrt{2}}$	$W^-W_{13}W_{23}^*$	$-\frac{ig}{\sqrt{2}}$
$W^+W_{14}^*W_{24}$	$\frac{ig}{\sqrt{2}}$	$W^-W_{14}W_{24}^*$	$-\frac{ig}{\sqrt{2}}$
$W_{13}W_{14}^*W_{34}$	$\frac{ig}{\sqrt{2}}$	$W_{13}^*W_{14}W_{34}^*$	$-\frac{ig}{\sqrt{2}}$
$W_{23}W_{24}^*W_{34}$	$\frac{ig}{\sqrt{2}}$	$W_{23}^*W_{24}W_{34}^*$	$-\frac{ig}{\sqrt{2}}$

forms:

$$\begin{aligned}
\frac{L_{4g}^c}{g^2} &= \sum_{i<j=1}^{4}[(W_{ij}.W_{ij}^*)^2-(W_{ij}W_{ij})(W_{ij}^*W_{ij}^*)]\\
&+\Sigma_{(i,j,kl)\in X}[(W_{ij}W_{kl})\left(W_{ij}^*W_{kl}^*\right)-\\
&\frac{1}{2}((W_{ij}.W_{ij}^*)(W_{kl}.W_{kl}^*)+(W_{ij}.W_{kl}^*)(W_{kl}.W_{ij}^*))]\\
&+\Sigma_{(i,j,kl)\in Y}[(W_{ij}W_{kl}^*)\left(W_{ij}^*W_{kl}\right)-\\
&\frac{1}{2}((W_{ij}.W_{ij}^*)(W_{kl}.W_{kl}^*)+(W_{ij}.W_{kl})(W_{kl}^*.W_{ij}^*))]\\
&+[-\frac{1}{2}W^+W_{24}W_{13}^*W_{34}^*-\frac{1}{2}W_{24}W_{34}^*W^+W_{13}^*\\
&+W_{13}^*W_{24}W^+W_{34}^*+h.c.]\\
&+[-\frac{1}{2}W^+W_{23}W_{14}^*W_{34}^*+W_{23}W_{14}^*W^+W_{34}^*\\
&-\frac{1}{2}W_{23}^*W_{34}W^+W_{14}^*+h.c.]\\
&+[-\frac{1}{2}W_{13}W_{23}^*W_{14}^*W_{24}-\frac{1}{2}W_{13}W_{14}^*W_{23}^*W_{24}\\
&+W_{13}W_{24}W_{23}^*W_{14}^*+h.c.]
\end{aligned} \quad (36)$$

where $W^+ \equiv W_{12}$ is the SM charged gauge boson and

$$\begin{aligned}
X &= \{(12,\ 13),\ (12,\ 14),\ (13,\ 14),\ (13,\ 23),\\
&(13,\ 14),\ (23,\ 24),\ (14,\ 24),\ (14,\ 34),\ (24,\ 34)\}\\
Y &= \{(12,\ 23),\ (12,\ 24),\ (13,\ 34),\ (23,\ 34)\}.
\end{aligned} \quad (37)$$

The respective Feynman rules for couplings given in Eq. (36) are listed in table III. It can be seen that the Lorentz structures have the same form as the SM coupling $W^+W^+W^-W^-$. Regarding the quartic couplings consisting of two neutral gauge bosons. Keeping all neutral states as linear combinations W_{ij}^0 of $W_{3,8,15}$ given in Eq. (24), we have derived that:

$$L_{G_{1,2}^0G_{3,4}} = \Sigma_{i<j=1}^4 g^2[\left(W_{ij}^0W_{ij}\right)\left(W_{ij}^0W_{ij}^*\right)-\left(W_{ij}W_{ij}^*\right)\left(W_{ij}^0W_{ij}^0\right)], \quad (38)$$

which results in the following forms in terms of the physical states:

$$L_{G^0W_{ij}=g_{G^0G^0,ij}}[(G^0G^0)\left(W_{ij}W_{ij}^*\right)-(G^0W_{ij})\left(G^0W_{ij}^*\right)] \rightarrow G^{0\mu}G^{0\nu}W_{ij}^{\alpha}W_{ij}^{*\beta},\ ig_{G^0G^0,ij}\times S_{\mu\nu,\alpha\beta} \quad (39)$$

and

$$\begin{aligned}
&L_{G_1^0G_2^0W_{ij}=g_{G_1^0G_2^0,ij}}[2\left(G_1^0G_2^0\right)\left(W_{ij}W_{ij}^*\right)\\
&-\left(G_1^0W_{ij}\right)\left(G_2^0W_{ij}^*\right)-\left(G_2^0W_{ij}\right)\left(G_1^0W_{ij}^*\right)]\\
&\rightarrow G_1^{0\mu}G_2^{0\nu}W_{ij}^{\alpha}W_{ij}^{*\beta},\ ig_{G_1^0G_2^0,ij}\times S_{\mu\nu,\alpha\beta}
\end{aligned} \quad (40)$$

where $G^0, G_1^0 \neq G_2^0 = A$ with k = 1, 2, 3. The second line in Eq. (39) or (40) shows the Feynman rules derived based on Eq. (34). Using the relations of W_{ij}^0 given in Eq. (24), which is written generally in Eq. (31), we can derive all formulas of $g_{G_1^0G_2^0,ij}$, namely

$$\begin{aligned}
&g_{G_1^0G_2^0,ij} \sim W_{ij}^0W_{ij}^0 \rightarrow g_{G_1^0G_2^0,ij} =\\
&-g_{G_1^0,ij}^2--g_{G_2^0,ij}^2-g_{G_1^0,ij}g_{G_2^0,ij}.
\end{aligned} \quad (41)$$

As a result, all Feynman rules for $g_{G_1^0G_2^0,ij}$ of two neutral gauge boson are listed in Table IV, where the formulas of $g_{G^0,ij} \equiv g_{G_1^0,ij}, g_{G_2^0,ij}$ are shown explicitly in table I. We note here some interesting results. Firstly, the vertex factors always contain two identical charged gauge bosons. The couplings consist of photon and only SM gauge bosons Z1, $W^{\pm}$ are the same as the SM forms. The relations between vertex factors $AZ_kW_{ij}W_{ij}^*$, $AW_{ij}W_{ij}^*$, and $Z_kW_{ij}W_{ij}^*$ have same as those assumed in Ref.[23]. Therefore, the general one-loop contributions to the decays $h \rightarrow Z\gamma, f\bar{f}\gamma$ given in Ref.[23,24] are applicable in the G341 model.

Table 3: Feynman rules for quartic charged gauge couplings with G = W, W_{ij} with i < j = $\overline{1,4}$. Here gG1234 and $\Gamma'_{\mu\nu,\,\alpha\beta}$ are respectively the scalar factors and Lorentz structure derived from Eq. (34) for specific couplings appearing in Eq. (36).

$G_1^{\mu} G_2^{\nu} G_3^{\alpha} G_4^{\beta}$	$ig_{G_{1234}}\Gamma'_{\mu\nu,\,\alpha\beta}$
$W_{ij}W_{ij}W^*_{ij}W^*_{ij}$	$ig^2 S_{\mu\nu,\,\alpha\beta}$
$W_{ij}W^*_{ij}W_{kl}W^*_{kl},\ (ij,kl)\in X$	$\frac{ig^2}{2}S_{\mu\alpha,\,\nu\beta}$
$W_{ij}W^*_{ij}W_{kl}W^*_{kl},\ (ij,kl)\in Y$	$\frac{ig^2}{2}S_{\mu\beta,\,\nu\alpha}$
$W^+W^*_{13}W_{24}W^*_{34},\ h.c.$	$\frac{ig^2}{2}S_{\mu\beta,\,\nu\alpha}$
$W^+W^*_{23}W_{14}W^*_{34},\ h.c.$	$\frac{ig^2}{2}S_{\mu\beta,\,\nu\alpha}$
$W_{13}W^*_{23}W^*_{14}W_{24},\ h.c.$	$ig^2 S_{\mu\nu,\,\alpha\beta}$

Table 4: Feynman rules for quartic couplings with two neutral gauge bosons in the G341 modeli.e., the coefficients $g_{G_1^0G_2^0,ij}$ defined in Eq. (40), where $W_{12\mu}\equiv W^+_\mu;\ 1\le i<j\le 4;$ and k = 1, 2, 3.The case of $G_1^0=G_2^0$ is allowed here.

$G_1^{0\mu}G_2^{0\nu}W^{\alpha}_{ij}G^{*\beta}_{ij}$	$ig_{G_1^0G_2^0,ij}$
$AAW_{ij}W^*_{ij}$	$-ie^2q^2_{ij}$
$Z_kZ_kW_{ij}W^*_{ij}$	$-i\left(gz_{k,ij}\right)^2$
$AZ_kW_{ij}W^*_{ij}$	$-ieq_{ij}gz_{k,ij}$
$Z_kZ_lW_{ij}W^*_{ij}\,(k\neq l)$	$-i\left(gz_{k,ij}gz_{l,ij}\right)$

The quartic couplings containing one neutral real bosons are:

$$\begin{aligned}
&L_{G_{0234}}/g^2=\frac{1}{\sqrt{2}}\{-\sqrt{3}\left(W^8W^+\right)\left(W_{23}W^*_{13}\right)\\
&+\left[\left(2W^0_{13}+W^0_{23}\right)W_{23}\right]\left(W^+W^*_{13}\right)\\
&-\left[\left(W^0_{13}+2W^0_{23}\right)W^*_{13}\right]\left(W^+W_{23}\right)+h.c.\}\\
&+\frac{1}{\sqrt{2}}\{\left[\left(-W^0_{13}+W^0_{23}-2W^0_{34}\right)W^+\right]\left(W_{24}W^*_{14}\right)\\
&+\left[\left(2W^0_{13}+W^0_{23}+W^0_{34}\right)W_{24}\right]\left(W^+W^*_{14}\right)+\\
&\left(-W^0_{13}-2W^0_{23}+W^0_{34}\right)W^*_{14}\left(W^+W_{24}\right)+h.c.\}\\
&+\frac{1}{\sqrt{2}}\{-W_{13}W_{34}W^*_{14}\left(W^0_{13}+2W^0_{34}\right)\\
&+W_{34}W_{13}W^*_{14}\left(2W^*_{13}+W^0_{34}\right)\\
&+W^*_{14}W_{13}W_{34}\left(W^0_{34}-W^0_{13}\right)+h.c.\}\\
&+\frac{1}{\sqrt{2}}\{\left[\left(W^0_{23}-2W^0_{34}\right)W_{23}\right]\left(W^*_{24}W_{34}\right)\\
&+\left[\left(W^0_{23}-W^0_{13}\right)W_{34}\right]\left(W_{23}W^*_{24}\right)+\\
&\left[\left(2W^0_{13}+W^0_{23}\right)W^*_{24}\right]\left(W_{23}W_{34}\right)+h.c.\},
\end{aligned}\tag{42}$$

where W^0_{ij} is given in Eq. (24). Changing into the physical states of neutral gauge bosons G^0, it can be written in the following forms:

$$\begin{aligned}
&L_{G_{0234}}\sim\left(a_3+a_4\right)\left(G^0G_2\right)\left(G_3G_4\right)\\
&+a_3\left(G^0G_3\right)\left(G_2G_4\right)+a_4\left(G^0G_4\right)\left(G_2G_3\right)\\
&\to i\Gamma_{\mu\nu,\alpha\beta}G^{0\mu}G^{\nu}_2G^{\alpha}_3G^{\beta}_4,\\
&\Gamma_{\mu\nu,\alpha\beta}\equiv-\left(a_3+a_4\right)g_{\mu\nu}g_{\alpha\beta}+\\
&a_3g_{\mu\alpha}g_{\nu\beta}+a_4g_{\mu\alpha}g_{\nu\beta},
\end{aligned}\tag{43}$$

where the two factors (a_3, a_4) are derived from the particular case of (G^0, G_2, G_3, G_4). For example the first line of Eq. (42) gives (G^0, G_2, G_3, G_4) = (G^0, W^+, W^*_{13}, W_{23}) then $a_3G^0W^*_{13}\in\left[-\left(W^0_{13}+2W^0_{23}\right)W^*_{13}\right]$ and $a_4G^0W^*_{23}\in\left[\left(2W^0_{13}+W^0_{23}\right)W^*_{13}\right]$. The vertex factor corresponding to the Feynman rule is $I\Gamma_{\mu\nu,\alpha\beta}$. Values of ($a_3$, a_4) are shown in two tables V and VI corresponding to the two classes of vertices containing SM-like and exotic neutral gauge bosons. The analytic formulas of A, B, and C are:

$$\begin{aligned}
A&=\frac{\sqrt{2+\left[2\left(q_1q_2+q_1+q_2\right)-3\left(q_1^2+q_2^2+1\right)\right]s_W^2}}{\sqrt{6-8\left(q_1^2-q_1+1\right)s_W^2}}\\
B&=\frac{5-2s_W^2\left(4q_1^2-2q_1q_2-3q_1+q_2+3\right)}{c_W\sqrt{6-8\left(q_1^2-q_1+1\right)s_W^2}},\\
C&=\frac{s_W^2\left(2q_1^2+2q_1q_2-3q_1-q_2+3\right)-2}{c_W\sqrt{6-8\left(q_1^2-q_1+1\right)s_W^2}}.
\end{aligned}\tag{44}$$

We can see that the couplings relating with photons are always proportional to the electric charges.

CONCLUSIONS

Feynman rules for all triple and quartic self-couplings of gauge bosons in the G341 model were presented

Table 5: Formulas of a3 and a4 defined in Eq. (43) relating to the Feynman rules for quarticgauge couplings consisting of one neutral real SM-like gauge boson in the G341 model.

$G^{0\mu} G_2^{\nu} G_3^{\alpha} G_4^{\beta}$	$\sqrt{2}a_3$	$\sqrt{2}a_4$
$AW^{+}W_{13}^{*}W_{23},\ h.c.$	$(q_1-2)\,e$	$(q_1+1)\,e$
$Z_1W^{+}W_{13}^{*}W_{23},\ h.c.$	$-\frac{g\left[3+2(q_1-2)s_W^2\right]}{2c_W}$	$\frac{g\left[3-2(q_1+1)s_W^2\right]}{2c_W}$
$AW^{+}W_{14}^{*}W_{24},\ h.c.$	$\sqrt{2}\,(q_2-2)\,e$	$\sqrt{2}\,(q_2+1)\,e$
$Z_1W^{+}W_{14}^{*}W_{24},\ h.c.$	$-\frac{g\left[3+2(q_2-2)s_W^2\right]}{\sqrt{2}c_W}$	$\frac{g\left[3-2(q_2+1)s_W^2\right]}{\sqrt{2}c_W}$
$AW_{13}W_{14}^{*}W_{34},\ h.c.$	$\sqrt{2}e\,(q_2-2q_1)$	$\sqrt{2}e\,(q_1+q_2)$
$Z_1W_{13}W_{14}^{*}W_{34},\ h.c.$	$\frac{g\left[-1+2(2q_1-q_2)s_W^2\right]}{\sqrt{2}c_W}$	$\frac{g\left[1-(q_1+q_2)s_W^2\right]}{c_W}$
$AW_{23}W_{24}^{*}W_{34},\ h.c.$	$\sqrt{2}e\,(-2q_1+q_2+1)$	$\sqrt{2}e\,(q_1+q_2-2)$
$Z_1W_{23}W_{24}^{*}W_{34},\ h.c.$	$\frac{s_W^2\,[4q_1-2(q_2+1)+1]}{\sqrt{2}c_W}$	$\frac{\sqrt{2}\left[s_W^2\,(q_1+q_2-2)+1\right]}{c_W}$

Table 6: Formulas of a_3 and a_4 defined in Eq. (43) relating to the Feynman rules for quarticgauge couplings consisting of one neutral real exotic gauge boson in the G341 model.

$G^{0\mu} G_2^{\nu} G_3^{\alpha} G_4^{\beta},\ h.c.$	$\sqrt{2}a_3$	$\sqrt{2}a_4$
$Z_2W^{+}W_{13}^{*}W_{23},\ h.c.$	$-\frac{gc_\alpha\sqrt{3-4\left(q_1^2-q_1+1\right)s_W^2}}{2c_W}$	$-\frac{gc_\alpha\sqrt{3-4\left(q_1^2-q_1+1\right)s_W^2}}{2c_W}$
$Z_3W^{+}W_{13}^{*}W_{23},\ h.c.$	$-\frac{gs_\alpha\sqrt{3-4\left(q_1^2-q_1+1\right)s_W^2}}{2c_W}$	$-\frac{gs_\alpha\sqrt{3-4\left(q_1^2-q_1+1\right)s_W^2}}{2c_W}$
$Z_2W^{+}W_{14}^{*}W_{24},\ h.c.$	$-\frac{gc_\alpha\left[2s_W^2\,(2q_1q_2-q_1-q_2+1)-1\right]}{c_W\sqrt{6-8\left(q_1^2-q_1+1\right)s_W^2}}-\frac{2es_\alpha}{c_W}A$	$\sqrt{2}a_3$
$Z_3W^{+}W_{14}^{*}W_{24},\ h.c.$	$-\frac{gs_\alpha\left[2s_W^2\,(2q_1q_2-q_1-q_2+1)-1\right]}{c_W\sqrt{6-8\left(q_1^2-q_1+1\right)s_W^2}}-\frac{2es_\alpha}{c_W}A$	$\sqrt{2}a_3$
$Z_2W_{13}W_{14}^{*}W_{24},\ h.c.$	$gc_\alpha B-2gs_\alpha A$	$2gc_\alpha C-2gs_\alpha A$
$Z_3W_{13}W_{14}^{*}W_{24},\ h.c.$	$-gs_\alpha B-2gc_\alpha A$	$-2gs_\alpha C-2gc_\alpha A$
$Z_2W_{23}W_{24}^{*}W_{34},\ h.c.$	$gc_\alpha B-2gs_\alpha A$	$2gc_\alpha C-2gs_\alpha A$
$Z_3W_{23}W_{24}^{*}W_{34},\ h.c.$	$-gs_\alpha B-2gc_\alpha A$	$-2gs_\alpha C-2gc_\alpha A$

precisely in this work. They will be necessary for calculating many importantprocesses paid attention by experiments such as h $\to \gamma\gamma$, Zγ, f$\bar{f}\gamma$; cLFV decays of SM-likeHiggs boson, Z boson, and charged leptons. The results showed that all of the couplings of the photon with other gauge bosons predicted by the G341 model satisfy all relations assumed in previous works [21,23,24] needed for implementing these general one-loop formulas in the G341 model framework. The detailed investigation of these contributionswill be discussed elsewhere.

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AUTHOR'S CONTRIBUTIONS

L.T. Hue and N.H.Thao contributed to the design and implementation of the research, to the analysis of the xxx results and to the writing of the manuscript.

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COMPETING INTERESTS

The authors declare that they have no competing interests.

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