Inverse dynamic analyzing of flexible link manipulators with translational and rotational joints

Bien Xuan Duong, My Anh Chu, and Khoi Bui Phan

Abstract—Inverse dynamic problem analyzing of flexible link robot with translational and rotational joints is presented in this work. The new model is developed from single flexible link manipulator with only rotational joint. The dynamic equations are built by using finite element method and Lagrange approach. The approximate force of translational joint and torque of rotational joint are found based on rigid model. The simulation results show the values of driving forces at joints of flexible robot with desire path and errors of joint variables between flexible and rigid models. Elastic displacements of end-effector are shown, respectively. There are remaining issues which need be studied further in future work because the error joints variables in algorithm to solve inverse dynamic problem of flexible with translational joint has not been mentioned yet.

Index Terms—Inverse Dynamic, flexible link manipulator, translational joint, elastic displacements.

1 INTRODUCTION

Dynamic analysis of mechanisms, especially robots, is very important. The dynamic equations of motion represent the behavior of system, so accurate modeling and equations are essential to successfully design of the control system. The analysis of robots considering the elastic characteristics of its members has been considerable attention in recent years. Flexibility in robots can affect position accuracy. Inverse dynamic of flexible robots is very essential for selecting the actuator and designing the proper control strategy. Most of the investigations on the dynamic modeling of robot manipulators with elastic arms have been confined to manipulators with only revolute joints.

In the literature, most of the investigations on the inverse dynamics of the flexible robot manipulator copies with manipulators constructed with only rotational joints [1-3]. Kwon and Book [1] present a single link robot which is described and modeled by using assumed modes method (AMM). Inverse equation is derived in a state space form from direct dynamic equations and using definitions concepts which are causal system, anti-causal system and Non-causal system. Based on these concepts, the time-domain inverse dynamic method was interpreted in the frequency-domain in detail by using the two sided Laplace transform in the frequency-domain and the convolution integral. This method is limited to linear system. Stable inversion method is studied for the same robot configuration but the nonlinear effect is taken into account [2]. An inversion-based approach to exact nonlinear output tracking control is presented. Non-causal inversion is incorporated into tracking regulators and is a powerful tool for control. Eliodoro and Miguel [3] propose a new method based on the finite difference approach to discretize the time variable for solving the inverse dynamics of the robot. This method is a non-recursive and non-iterative approach carried out in the time domain in contrast with methods previously proposed. By using either the finite element method (FEM) or AMM, some other authors consider the dynamic modeling and analysis of the flexible robots with translational joint [4-8]. Pan et al [4] presented a model R-P with FEM approach. The
result is differential algebraic equations which are solved by using Newmark method. Al-Bedoor and Khulief [5] presented a general dynamic model for R-P robot based on FEM and Lagrange approach. They defined a concept which is translational element. The stiffness of translational element is changed. The prismatic joint variable is distance from origin coordinate system to translational element. The prismatic joint variable is distance from origin coordinate system to translational element. The number of element is small because it is hard challenge to build and solve differential equations. Khadem [6] studied a three-dimensional flexible n-degree of freedom manipulator having both revolute and prismatic joint. A novel approach is presented using the perturbation method. The dynamic equations are derived using the Jourdain’s principle and the Gibbs-Appell notation. Korayem [7] also presented a systematic algorithm capable of deriving equations of motion of N-flexible link manipulators with revolute-prismatic joints by using recursive Gibbs-Appell formulation and AMM. However, the inverse dynamics modeling and analysis of the generalized flexible robot constructed with translational joint has not been much mentioned yet.

The objective of the described work in what follows was to present surveying inverse dynamics problem of flexible link robot with translational and rotational joints. The Lagrange approach in conjunction with the finite element method is employed in deriving the equations of motion. Inverse dynamics problem of model with flexible link can be approximately solved based on model with rigid links. The forward kinematic, inverse kinematic and inverse dynamics of rigid model are used to find joints values from desire path and driving force and torque which are inputs data for flexible model problems. The force and torque of joints can be found in such a way that the end point of link 2 can track the desired path even though link 2 is deformed.

2 DYNAMIC MODELING

2.1 Dynamic model

In this work, we concern the dynamic model of two link flexible robot which motions on horizontal plane with translational joint for first rigid link and rotational joint for second flexible link to formulate the inverse dynamics problem. It is shown as Fig 1.

![Figure 1. Flexible links robot with translational and rotational joints](image)

The coordinate system \( XOY \) is the fixed frame. Coordinate system \( X_1O_1Y_1 \) is attached to end point of link 1. Coordinate system \( X_2O_2Y_2 \) is attached to first point of link 2. The translational joint variable \( d(t) \) is driven by \( F_t \) force. The rotational joint variable \( \tau(t) \) is driven by \( \tau(t) \) torque. Both joints are assumed rigid. Link 1 with length \( L_1 \) is assumed rigid and link 2 with length \( L_2 \) is assumed flexibility. Link 2 is divided \( n \) elements. The elements are assumed interconnected at certain points, known as nodes. Each element has two nodes. Each node of element \( j \) has 2 elastic displacement variables which are the flexural \((u_{2j-1}, u_{2j+1})\) and the slope displacements \((u_{2j}, u_{2j+2})\). Symbol \( m_i \) is the mass of payload on the end-effector point. The coordinate \( r_{01} \) of end point of link 1 on \( XOY \) is computed as

\[
r_{01} = [L_1 \; d(t)]^T
\]

The coordinate \( r_{2j} \) of element \( j \) on \( X_2O_2Y_2 \) can be given as

\[
r_{2j} = [(j-1)l_e + x_j \; w_j(x_j, t)]^T : (x_j = 0, L_e)
\]

Where, length of each element is \( l_e = \frac{L_2}{n} \) and \( w_j(x_j,t) \) is the total elastic displacement of element \( j \) which is defined by [10]

\[
w_j(x_j, t) = N_j(x_j)Q_j(t)
\]
\[ \mathbf{N}_j(x_j) = \begin{bmatrix} f_1(x_j) & f_2(x_j) & f_3(x_j) & f_4(x_j) \end{bmatrix} \]  

Mode shape function \( f_i(x_j); (i = 1...4) \) can be presented in [10]. The elastic displacement \( \mathbf{Q}_j(t) \) of element \( j \) is given as

\[ \mathbf{Q}_j(t) = \begin{bmatrix} u_{2j-1} & u_{2j} & u_{2j+1} & u_{2j+2} \end{bmatrix}^T \]  

The coordinate \( \mathbf{r}_{2j} \) of element \( j \) on \( X,O,Y \) can be written as

\[ \mathbf{r}_{2j} = \mathbf{T}_2 \mathbf{r}_{2j} \]  

Where, \( \mathbf{T}_2 = \begin{bmatrix} \cos q(t) & -\sin q(t) \\ \sin q(t) & \cos q(t) \end{bmatrix} \) is the transformation matrix from \( X,O,Y \) to \( X,O,Y \). The coordinate \( \mathbf{r}_{0j} \) of element \( j \) on \( XOY \) can be computed as

\[ \mathbf{r}_{0j} = \mathbf{r} + \mathbf{r}_{2j} \]  

The elastic displacement \( \mathbf{Q}_n(t) \) of element \( n \) is given as

\[ \mathbf{Q}_n(t) = \begin{bmatrix} u_{2n+1} & u_{2n} & u_{2n+1} & u_{2n+2} \end{bmatrix}^T \]  

The coordinate \( \mathbf{r}_{0e} \) of end point of flexible link 2 on \( XOY \) can be computed as

\[ \mathbf{r}_{0e} = \begin{bmatrix} L_1 + L_2 \cos q(t) - u_{2n+1} \sin q(t) \\ d(t) + L_2 \sin q(t) + u_{2n+1} \cos q(t) \end{bmatrix} \]  

If assumed that robot with all of links are rigid, the coordinate \( \mathbf{r}_{0e_{-rigid}} \) on \( XOY \) can be written as

\[ \mathbf{r}_{0e_{-rigid}} = \begin{bmatrix} L_1 + L_2 \cos q(t) \\ d(t) + L_2 \sin q(t) \end{bmatrix} \]  

The kinematic energy of link 1 can be computed as

\[ T_1 = \frac{1}{2} m_1 \dot{\mathbf{r}}_{01}^2 \]  

Where \( m_1 \) is the mass of link 1. The kinetic energy of element \( j \) is determined as

\[ T_{2j} = \frac{1}{2} \int_0^t m_2 \left[ \frac{\partial \mathbf{r}_{2j}}{\partial t} \right]^2 \, dt = \frac{1}{2} \mathbf{Q}^T_j(t) \mathbf{M}_j \mathbf{Q}_j(t) \]  

Where \( m_2 \) is mass per meter of link 2. The generalized elastic displacement \( \mathbf{Q}_{je}(t) \) of element \( j \) is given as

\[ \mathbf{Q}_{je}(t) = \begin{bmatrix} d(t) & q(t) & u_{2j-1} & u_{2j} & u_{2j+1} & u_{2j+2} \end{bmatrix}^T \]  

Each element of inertial mass matrix \( \mathbf{M}_j \) can be computed as

\[ \mathbf{M}_j(s,e) = \int_0^t m_1 \left[ \frac{\partial \mathbf{r}_{0j}}{\partial t} \right]^2 \, dt = \frac{1}{2} \mathbf{Q}^T_{je}(t) \mathbf{M}_j \mathbf{Q}_{je}(t) \]  

Where \( \mathbf{Q}_{je} \) and \( \mathbf{Q}_{je} \) are the \( s^{th}, e^{th} \) element of \( \mathbf{Q}_{je} \) vector. It can be shown that \( \mathbf{M}_j \) is of the form

\[ \mathbf{M}_j = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} & m_{46} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & m_{56} \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix} \]  

And,

\[ m_{11} = m_2 L_1^2; m_{12} = -\frac{1}{12} m_1 L_1 \left( (6u_{2j-1} + 6u_{2j+1}) + \frac{L_1^2}{L_1} \right) \sin q(t) + 6L_1 (1-2 \cos q(t)) \cos q(t) \]
\[ m_{13} = m_2 = \frac{1}{2} m_1 L_2 \cos q(t); m_{14} = m_{16} = \frac{1}{2} m_1 L_2 \cos q(t); m_{15} = m_{33} = \frac{1}{2} m_1 L_2 \cos q(t); m_{15} = m_{33} \]
\[ m_{22} = \frac{1}{20} m_1 L_1^2 (10j - 7); m_{23} = \frac{1}{60} m_1 L_1^2 (5j - 3); m_{25} = \frac{1}{20} m_1 L_1^2 (10j - 3); \]
\[ m_{24} = \frac{1}{210} m_1 L_2 \left\{ \begin{array}{l} 210 L_1^2 (j + 1) + 240 L_1 L_2 (3j - 2) (u_{2j-1} - 2u_{2j+1} + 2u_{2j+2}) \\
+ 240 L_2 (9j + 3) (u_{2j-1} - 3u_{2j+1} + 3u_{2j+2}) \\
+ 720 L_1^2 (9j + 3) (u_{2j-1} - 3u_{2j+1} + 3u_{2j+2}) \end{array} \right\} \]
\[ m_{26} = \frac{1}{60} m_1 L_2 (5j - 2); m_{31} = m_{32} = m_{33} = m_{34} = m_{35} = m_{36} = m_{41} = m_{42} = m_{43} = m_{44} = m_{45} = m_{46} = m_{51} = m_{52} = m_{53} = m_{54} = m_{55} = m_{56} = m_{61} = m_{62} = m_{63} = m_{64} = m_{65} = m_{66} \]

The total elastic kinetic energy of link 2 is yielded as

\[ T_{el} = \sum_{j=1}^n T_{2j} = \frac{1}{2} \mathbf{Q}^T(t) \mathbf{M}_d \mathbf{Q}(t) \]  

The inertial mass matrix \( \mathbf{M}_d \) is constituted from matrices of elements follow FEM theory, respectively. Vector \( \mathbf{Q}(t) \) represents the generalized coordinate of system and is given as

\[ \mathbf{Q}(t) = \begin{bmatrix} d(t) & q(t) & u_{2j-1} & u_{2j} & u_{2j+1} & u_{2j+2} \end{bmatrix}^T \]  

The kinetic energy of payload is given as
\[ T_r = \frac{1}{2} m_r \dot{x}_{ru}^2 \]  

(19)

The kinetic energy of system is determined as

\[ T = T_1 + T_0 + T_r = \frac{1}{2} \dot{Q}^T(t)M \dot{Q}(t) \]  

(20)

Matrix \( M \) is mass matrix of system. The gravity effects can be ignored as the robot movement is confined to the horizontal plane. Defining \( E \) and \( I \) are Young’s modulus and inertial moment of link 2, the elastic potential energy of element \( j \) is shown as \( P_j \) with the stiffness matrix \( K_j \) and presented as [10]

\[ P_j = \frac{1}{2} \int_0^l EI \left[ \frac{\partial^2 w(x_j,t)}{\partial x_j^2} \right]^2 dx_j = \frac{1}{2} Q_j^T(t)K_jQ_j(t) \]  

(21)

Where,

\[
K_j = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{12EI}{l^2} & 6EI & -\frac{12EI}{l^2} & 6EI & 0 \\
0 & 0 & \frac{6EI}{l^2} & \frac{4EI}{l} & -\frac{6EI}{l^2} & \frac{2EI}{l} & 0 \\
0 & 0 & -\frac{12EI}{l^2} & 6EI & -\frac{12EI}{l^2} & 6EI & 0 \\
0 & 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & -\frac{6EI}{l^2} & \frac{4EI}{l} & 0
\end{bmatrix}
\]  

(22)

The total elastic potential energy of system is yielded as

\[ P = \sum_{j=1}^{n} P_j = \frac{1}{2} Q^T(t)KQ(t) \]  

(23)

The stiffness matrix \( K \) is constituted from matrices of elements follow FEM theory similar \( M \) matrix, respectively.

2.2 Dynamic equations of motion

Fundamentally, the method relies on the Lagrange equations with Lagrange function \( L = T - P \) are given by

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{Q}(t)} \right) = \frac{\partial L}{\partial Q(t)} = F(t) \]  

(24)

Vector \( F(t) \) is the external generalized forces acting on specific generalized coordinate \( Q(t) \) and is determined as

\[ F(t) = \left[ F_r(t) \quad \tau(t) \quad 0 \quad \ldots \quad 0 \right]^T \]  

(25)

Size of matrices \( M, K \) is \( (2n+4) \times (2n+4) \) and size of \( F(t) \) and \( Q(t) \) is \( (2n+4) \times 1 \). The rotational joint of link 2 is constrained so that the elastic displacements of first node of element 1 on link 2 can be zero. Thus variables \( u_1, u_2 \) are zero. By enforcing these boundary conditions and FEM theory, the generalized coordinate \( Q(t) \) becomes

\[ Q(t) = \left[ d(t) \quad q(t) \quad u_3 \quad \ldots \quad u_{2n+1} \quad u_{2n+2} \right]^T \]  

(26)

So now, size of matrices \( M, K \) is \( (2n+2) \times (2n+2) \) and size of \( F(t) \) and \( Q(t) \) is \( (2n+2) \times 1 \). When kinetic and potential energy are known, it is possible to express Lagrange equations as shown

\[ M(Q)\ddot{Q} + C(Q,\dot{Q})\dot{Q} + D\dot{Q} + KQ = F(t) \]  

(27)

Where structural damping \( D \) and coriolis force \( C \) matrices are calculated as

\[ C(Q,\dot{Q}) = M(Q)\dot{Q} - \frac{1}{2} \frac{\partial}{\partial Q} \frac{\partial}{\partial \dot{Q}} (Q^T M(Q) \dot{Q}) \]  

(28)

\[ D = \alpha M + \beta K \]  

(29)

Where \( \alpha \) and \( \beta \) are the damping ratios of the system which are determined by experience.

3 Inverse dynamic analyzing

Solving inverse dynamics problem can be computed a feed-forward control to follow a trajectory more accurately. Inverse dynamics of flexible robot is the process of determining load profiles to produce given displacement profiles as function of time. Forward dynamics of flexible robot is the process of finding displacements given the loads. This is much simpler than inverse dynamics process because elastic displacements do not to know before if there are not external forces which effect on system. Unlike the rigid link, the inverse dynamics of flexible robot is more complex because of links deformations. We need to determine the force and torque of actuators in such a way that the end point of link 2 can still track the desire path even though link 2 is deformed. Inverse dynamics problem of model with flexible link can be approximately solved based on model with rigid links. Steps to solve are shown as Fig 2. The detail of blocks in Fig 2 is presented in Fig 3, Fig 4, Fig 5 and Fig 6.
Figure 2. General diagram of inverse dynamic flexible robot algorithm

Figure 3. Diagram of inverse kinematic rigid model block

Figure 4. Diagram of inverse dynamic rigid model block

Figure 5. Diagram of forward dynamic flexible robot block

Figure 6. Diagram of inverse dynamic flexible robot block
Firstly, assuming that two link is rigid. The translational and rotational joints of rigid model are computed from desire path by solving inverse kinematic rigid problem [9] which is shown in Fig. 3. Then driving force and torque at joints of rigid model are computed by solving inverse dynamic rigid [9] (Fig. 4). Results are input data for forward dynamic flexible model follow equation (27) and are shown in Fig. 5. Finally, the approximates force and torque of joints are found by solving inverse dynamic flexible problem with inputs data which are joints values of rigid model and elastic displacements. It is presented by block in Fig. 6.

4 NUMERICAL SIMULATIONS

Simulation specifications of flexible model are given by Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of link 1 (m)</td>
<td>L₁</td>
<td>0.05</td>
</tr>
<tr>
<td>Mass of link 1 and base (kg)</td>
<td>m₁</td>
<td>1.4</td>
</tr>
<tr>
<td>Length of link 2 (m)</td>
<td>L₂</td>
<td>0.3</td>
</tr>
<tr>
<td>Width (m)</td>
<td>b</td>
<td>0.02</td>
</tr>
<tr>
<td>Thickness (m)</td>
<td>h</td>
<td>0.001</td>
</tr>
<tr>
<td>Number of element</td>
<td>n</td>
<td>5</td>
</tr>
<tr>
<td>Cross section area (m²)</td>
<td>A</td>
<td>b.h</td>
</tr>
<tr>
<td>Mass density (kg/m³)</td>
<td>ρ</td>
<td>7850</td>
</tr>
<tr>
<td>Mass per meter (kg/m)</td>
<td>m</td>
<td>ρ.A</td>
</tr>
<tr>
<td>Young’s modulus (N/m²)</td>
<td>E</td>
<td>2.10¹⁰</td>
</tr>
<tr>
<td>Inertial moment of cross section (m⁴)</td>
<td>I</td>
<td>b.h³/12</td>
</tr>
<tr>
<td>Damping ratios</td>
<td>α, β</td>
<td>0.005, 0.007</td>
</tr>
<tr>
<td>Mass of payload (g)</td>
<td>mₚ</td>
<td>10</td>
</tr>
<tr>
<td>Desire path on workspace in OX axis (m)</td>
<td>xE</td>
<td>0.25-0.1.sin(t-)π/2</td>
</tr>
<tr>
<td>Desire path on workspace in OX axis (y)</td>
<td>yE</td>
<td>0.1.sin(t)</td>
</tr>
<tr>
<td>Time simulation (s)</td>
<td>T</td>
<td>2</td>
</tr>
</tbody>
</table>

Simulation results for inverse dynamic of flexible robot with translational and rotational joints are shown from Fig 7 to Fig 16. It is noteworthy to mention that we need to find the initial values of joints variable at t=0 when inverse kinematic of rigid model is solved.

Fig. 7 and fig. 8 show the values of joint variables between rigid and flexible model. Translational and rotational joints values are small because of short time simulation. Fig. 9 and fig. 10 describe deviation of these values. Maximum deviation value of translational joint is 25 mm and rotational joint variable is 0.17 rad. These deviations appear from effect of elastic displacements and error of numerical method which is used to solving problems.
Fig. 10 to fig 14 present values of driving forces at joints and these deviations between rigid and flexible model. The values of driving force at translational joint are not too difference because first link of both models is assumed rigid. Maximum force is 0.6 N. Driving torque values at rotational joint are more difference because of effect of elastic displacements of flexible link.
Fig. 15 shows flexural displacement value at end-effect point. Maximum value is 0.7 mm. Fig. 16 shown slope displacement at end-effect point. Maximum value is 0.035 rad. Both values are small because of short time simulation and small values of joint variables.

In general, simulation results show that elastic displacements of flexible link effect on dynamic behaviors of system. Different between rigid model and flexible model are clearly visible.

5 CONCLUSION

Nonlinear dynamic modeling and equations of motion of flexible manipulators with translational and rotational joints are built by using finite element method and Lagrange approach. Model is developed based on single link manipulator with only rotational joint. Inverse dynamic problem of flexible link manipulator is surveyed with an algorithm which is based on rigid model. Approximate driving force and torque at joints of flexible link manipulator are found with desire path. Derivation values of these also are shown. Elastic displacements at end-effector point are presented. However, there are remaining issues which need be studied further in future work because the error joints variables in algorithm to solve inverse dynamic problem of flexible with translational joint has not been mentioned yet.

REFERENCES


Phân tích động lực học rò bọt có khâu đàn hồi với các khớp tính tiến và khớp quay

Dương Xuân Biên, Chu Anh Mỹ, Phan Bùi Khôi


Tiêu khoá - Động lực học ngược, khâu đàn hồi, khớp tính tiến, chuyển vị đàn hồi