The GAP function of a parametric mixed strong vector quasivariational inequality problem

Le Xuan Dai, Nguyen Van Hung, Phan Thanh Kieu

Abstract— **The parametric mixed strong vector quasivariational inequality problem contains many problems such as, variational inequality problems, fixed point problems, coincidence point problems, complementary problems etc. There are many authors who have been studied the gap functions for vector variational inequality problem. This problem plays an important role in many fields of applied mathematics, especially theory of optimization. In this paper, we study a parametric gap function without the help of the nonlinear scalarization function for a parametric mixed strong vector quasivariational inequality problem (in short, (SQVIP)) in Hausdorff topological vector spaces. (SQVIP) Find $\bar{x} \in K(\bar{x},\gamma)$ and $\bar{z} \in T(\bar{x},\gamma)$ such that**

$$<\bar{z}, y-\bar{x}> + f(y,\bar{x},\gamma) \in R^n_+, \forall y \in K(\bar{x},\gamma),$$

where we denote the nonnegative of R^n by

$$R^n_+ = \{t=(t_1,t_2,\ldots,t_n)^T \in R^n \mid t_i \geq 0, i=1,2,\ldots,n\}.$$

Moreover, we also discuss the lower semicontinuity, upper semicontinuity and the continuity for the parametric gap function for this problem. To the best of our knowledge, until now there have not been any paper devoted to the lower semicontinuity, continuity of the gap function without the help of the nonlinear scalarization function for a parametric mixed strong vector quasivariational inequality problem in Hausdorff topological vector spaces. Hence the results presented in this paper (Theorem 1.3 and Theorem 1.4) are new and different in comparison with some main results in the literature.

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***Index Terms*—Vector quasivariational inequality problem; parametric gap function; lower semicontinuity; upper semicontinuity, continuity.**

1 INTRODUCTION

Let X and Λ be Hausdorff topological vector spaces. Let $L(X,R^n)$ be the space of all linear continuous operators from X to R^n. $K: X\times\Lambda \to 2^X$, $T: X\times\Lambda \to 2^{L(X,R^n)}$ are set-valued mappings and let $f: X\times\Lambda \to R^n$ be continuous single-valued mappings. For $\gamma\in\Lambda$ consider the following parametric mixed strong vector quasivariational inequality problem (in short, (SQVIP)).

(**SQVIP**) Find $\bar{x} \in K(\bar{x},\gamma)$ and $\bar{z} \in T(\bar{x},\gamma)$ such that

$$<\bar{z}, y-\bar{x}> + f(y,\bar{x},\gamma) \in R^n_+, \forall y \in K(\bar{x},\gamma),$$

where we denote the nonnegative of R^n by

$$R^n_+ = \{t=(t_1,t_2,\ldots,t_n)^T \in R^n \mid t_i \geq 0, i=1,2,\ldots,n\}.$$

Here the symbol ^{T} denotes the transpose. We also denote

$$intR^n_+ = \{t=(t_1,t_2,\ldots,t_n)^T \in R^n \mid t_i > 0, i=1,2,\ldots,n\}.$$

For each $\gamma\in\Lambda$ we let $E(\gamma) := \{x\in X \mid x\in K(x,\gamma)\}$ and $\Phi: \Lambda\to 2^X$ be set-valued mapping such that $\Psi(\gamma)$ is the solution set of (SQVIP). Throughout the paper, we always assume that $\Psi(\gamma)\neq\emptyset$ for each γ in the neighborhood $\gamma_0\in\Lambda$.

The parametric mixed strong vector quasivariational inequality problem contains many problems such as, variational nequality problems, fixed point problems, coincidence point problems, complementarity problems etc,. There are many authors have been studied the gap functions for vector variational inequality problem, see ([2]-[6], [8]-[10]) and the references therein.

The structure of our paper is as follows. In Section 1 of this article, we introduce the model vector quasivariational inequality problem and recall definitions for later uses. In Section 2, we establish the lower semicontinuity, the upper semicontinuity and the continuity for the gap function of parametric mixed strong vector quasivariational inequality problem.

A. Preliminaries

Next, we recall some basic definitions and their some properties.

Definition 1.1 *(See [1], [7]) Let X and Z be Hausdorff topological vector spaces and $F : X \to 2^Z$ be a multifunction.*

i) F is said to be *lower semicontinuous (lsc)* at x_0 if $F(x_0) \cap U \neq \varnothing$ for some open set $U \subseteq Z$ implies the existence of a neighborhood N of x_0 such that, for all $x \in N, F(x) \cap U \neq \varnothing$. An equivalent formulation is that: F is lsc at x_0 if $\forall x_\alpha \to x_0$, $\forall z_0 \in F(x_0)$, $\exists z_\alpha \in F(x_\alpha)$, $z_\alpha \to z_0$. F is said to be lower semicontinuous in X if it is lower semicontinuous at each $x_0 \in X$.

ii) F is said to be *upper semicontinuous (usc)* at x_0 if for each open set $U \supseteq F(x_0)$, there is a neighborhood N of x_0 such that $U \supseteq F(N)$. F is said to be upper semicontinuous in X if it is upper semicontinuous at each $x_0 \in X$.

iii) F is said to be *continuous* at x_0 if it is both lsc and usc at x_0. F is said to be continuous at x_0 if it is continuous at each $x_0 \in X$.

iv) F is said to be *closed* at $x_0 \in X$ if and only if $\forall x_n \to x_0$, $\forall y_n \to y_0$ such that $y_n \in F(x_n)$, we have $y_0 \in F(x_0)$. F is said to be closed in X if it is closed at each $x_0 \in X$.

Lemma 1.1 *(See [1], [7]) If F has compact values, then F is usc at x_0 if and only if, for each net $\{x_\alpha\} \subseteq X$ which converges to x_0 and for each net $\{y_\alpha\} \subseteq F(x_\alpha)$, there are $y \in F(x)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \to y$.*

B. Main Results

In this section, we introduce the parametric gap functions for parametric mixed strong vector quasivariational inequality problem, then we study some properties of this gap function.

Definition 1.2 *A function $h : X \times \Lambda \to R$ is said to be a parametric gap function of (SQVIP) if it satisfies the following properties [i)]*

i) $h(x,\gamma) \geq 0$ for all $x \in E(\gamma)$.

i) $h(x_0,\gamma_0) = 0$ if and only if $x_0 \in \Psi(\gamma_0)$.

Now we suppose that $K(x,\gamma)$ and $T(x,\gamma)$ are compact sets for any $(x,\gamma) \in X \times \Lambda$. We define function $h : X \times \Lambda \to R$ as follows

$$h(x,\gamma) = \min_{z \in T(x,\gamma)} \max_{y \in K(x,\gamma)} (< z, x-y > - f(y,x,\gamma))_i \quad (1)$$

where $(< z, x-y > - f(y,x,\gamma))_i$ is the i th component of $< z, x-y > - f(y,x,\gamma)$, $i = 1,2,\ldots,n$.

Since $K(x,\gamma)$ and $T(x,\gamma)$ are compact sets, $h(x,\gamma)$ is well-defined.

In the following, we will always assume that $f(x,x,\gamma) = 0$ for all $x \in E(\gamma)$.

Theorem 1.2 *The function $h(x,\gamma)$ defined by (1) is a parametric gap function for the (SQVIP).*

Proof. We define a function $h_1 : X \times L(X,R^n) \to R^n$ as follows

$$h_1(x,z) = \max_{y \in K(x,\gamma)} \max_{1 \leq i \leq n} (< z, x-y > - f(y,x,\gamma))_i,$$

where $x \in E(\gamma), z \in T(x,\gamma)$.

i) It is easy to see that $h_1(x,z) \geq 0$. Suppose to the contrary that there exists $x_0 \in E(\gamma)$ and $z_0 \in T(x_0,\gamma)$ such that $h_1(x_0,z_0) < 0$, then

$$0 > h_1(x_0,z_0) = \max_{y \in K(x_0,\gamma)} \max_{1 \leq i \leq n} (< z_0, x-y > - f(y,x_0,\gamma))_i$$

$$\geq \max_{1 \leq i \leq n} (< z_0, x-y > - f(y,x_0,\gamma))_i, \forall y \in K(x_0,\gamma),$$

which is impossible when $y = x_0$. Hence,

$$h_1(x,z) = \max_{y \in K(x,\gamma)} \max_{1 \leq i \leq n} (< z, x-y > - f(y,x,\gamma))_i \geq 0,$$

where $x \in E(\gamma), z \in T(x,\gamma)$. Thus, since $z \in T(x,\gamma)$ is arbitrary, we have

$$h(x,\gamma) = \min_{z \in T(x,\gamma)} \max_{y \in K(x,\gamma)} (< z, x-y > - f(y,x,\gamma))_i \geq 0.$$

ii) By definition, $h(x_0,\gamma_0) = 0$ if and only if there exists $z_0 \in T(x_0,\gamma_0)$ such that $h_1(x_0,z_0) = 0$, i.e.,

$$\max_{y \in K(x_0,\gamma_0)} \max_{1 \leq i \leq n} (< z_0, x_0-y > - f(y,x_0,\gamma_0))_i = 0,$$

for $x_0 \in E(\gamma_0)$ if and only if, for any $y \in K(x_0,\gamma_0)$,

$$\max_{1 \leq i \leq n} (< z_0, x_0-y > - f(y,x_0,\gamma_0))_i \leq 0,$$

namely, there is an index $1 \leq i_0 \leq n$, such that $(< z_0, x_0-y > - f(y,x_0,\gamma_0))_{i_0} \geq 0$, which is equivalent to

$$< z_0, x_0-y > - f(y,x_0,\gamma_0) \in R^n_+, \forall y \in K(x_0,\gamma_0),$$

that is, $x_0 \in \Psi(\gamma_0)$.

Remark 1.1 *As far as we know, there have not been any works on parametric gap functions for mixed strong vector quasiequilibrium problems, and hence our the parametric gap functions is new and cannot compare with the existing ones in the literature.*

Example 1.1 *Let* $X = R, n = 2, \Lambda = [0,1]$, $K(x,\gamma) = [0,1]$, $T(x,\gamma) = \left[\frac{1}{2}, 3\gamma^2 x^2 + x^4\right]$ *and* $f(y,x,\gamma) = 0$. *Now we consider the problem (QVIP), finding* $x \in K(x,\gamma)$ *and* $z \in T(x,\gamma)$ *such that*

$$< z, y - x > + f(y,x,\gamma) = \left(\frac{1}{2}(y-x), (3\gamma^2 x^2 + x^4)(y-x)\right)$$

$\in R_+^2$. It follows from a direct computation $\Psi(\gamma) = \{0\}$ for all $\gamma \in [0,1]$. Now we show that $h(.,.)$ is a parametric gap function of (SQVIP). Indeed, taking $e = (1,1) \in intR_+^n$, we have

$$h(x,\gamma) = \min_{z \in T(x,\gamma)} \max_{y \in K(x,\gamma)} \max_{1 \le i \le n} (< z, x - y > - f(y,x,\gamma))_i$$

$$= \max_{y \in K(x,\gamma)} ((3\gamma^2 x^2 + x^4)(x - y)) = \begin{cases} 0, & if\ x = 0 \\ \gamma^2 x^3 + x^5, & if\ x \in (0,1] \end{cases}$$

Hence, $h(.,.)$ is a parametric gap function of (SQVIP).

The following Theorem 1.3 gives sufficient condition for the parametric gap function $h(.,.)$ is continuous in $X \times \Lambda$.

Theorem 1.3 *Consider (SQVIP). If the following conditions hold:*

i) $K(.,.)$ is continuous with compact values in $X \times \Lambda$;

ii) $T(.,.)$ is upper semicontinuous with compact values in $X \times \Lambda$.

Then $h(.,.)$ is lower semicontinuous in $X \times \Lambda$.

Proof. First, we prove that $h(.,.)$ is lower semicontinuous in $X \times \Lambda$. Indeed, we let $a \in R$ and suppose that $\{(x_\alpha, \gamma_\alpha)\} \subseteq X \times \Lambda$ satisfying $h(x_\alpha, \gamma_\alpha) \le a, \forall \alpha$ and $(x_\alpha, \gamma_\alpha) \to (x_0, \gamma_0)$ as $\alpha \to \infty$. It follows that $h(x_\alpha, \gamma_\alpha) =$

$$= \min_{z \in T(x_\alpha, \gamma_\alpha)} \max_{y \in K(x_\alpha, \gamma_\alpha)} \max_{1 \le i \le n} (< z, x_\alpha - y > - f(y, x_\alpha, \gamma_\alpha))_i \le a.$$

We define the function $h_0 : X \times L(X, R^n) \times \Lambda \to R$ by

$$h_0(x,z,\gamma) = \max_{y \in K(x,\gamma)} \max_{1 \le i \le n} (< z, x - y > - f(y,x,\gamma))_i \in E(\gamma).$$

Since g and f are continuous, we have $(< z, x - y > - f(y,x,\gamma))_i$ is continuous, and since $K(.,.)$ is continuous with compact values in $X \times \Lambda$. Thus, by Proposition 19 in Section 3 of Chapter 1 [1] we can deduce that $h_0(x,z,\gamma)$ is continuous. By the compactness of $T(x_\alpha, \gamma_\gamma)$, there exists $z_\alpha \in T(x_\alpha, \gamma_\gamma)$ such that $h(x_\alpha, \gamma_\alpha) =$

$$= \min_{z \in T(x_\alpha, \gamma_\alpha)} \max_{y \in K(x_\alpha, \gamma_\alpha)} \max_{1 \le i \le n} (< z, x_\alpha - y > - f(y, x_\alpha, \gamma_\alpha))_i$$

$$= h_0(x_\alpha, z_\alpha, \gamma_\alpha) =$$

$$= \max_{y \in K(x_\alpha, \gamma_\alpha)} \max_{1 \le i \le n} (< z, x_\alpha - y > - f(y, x_\alpha, \gamma_\alpha))_i \le a.$$

Since $K(.,.)$ is lower semicontinuous in $X \times \Lambda$, for any $y_0 \in K(x_0, \gamma_0)$, there exists $y_\alpha \in K(x_\alpha, \lambda_\alpha)$ such that $y_\alpha \to y_0$. For $y_\alpha \in K(x_\alpha, \lambda_\alpha)$, we have

$$\max_{1 \le i \le n} (< z, x_\alpha - y_\alpha > - f(y_\alpha, x_\alpha, \gamma_\alpha))_i \le a. \quad (2)$$

Since $T(.,.)$ is upper semicontinuous with compact values in $X \times \Lambda$, there exists $z_0 \in T(x_0, \gamma_0)$ such that $z_\alpha \to z_0$ (taking a subnet $\{z_\beta\}$ of $\{z_\alpha\}$ if necessary) as $\alpha \to \infty$. Since $\max_{1 \le i \le n} (< z, x - y > - f(y,x,\gamma))_i$ is continuous. Taking the limit in (2), we have

$$\max_{1 \le i \le n} (< z_0, x_0 - y_0 > - f(y_0, x_0, \gamma_0))_i \le a. \quad (3)$$

Since $y \in K(x_0, \gamma_0)$ is arbitrary, it follows from (3) that

$$h_0(x_0, z_0, \gamma_0) =$$

$$= \max_{y \in K(x_0, \gamma_0)} \max_{1 \le i \le n} (< z_0, x_0 - y > - f(y, x_0, \gamma_0))_i \le a.$$

and so, for any $z \in T(x_0, \gamma_0)$, we have $h(x_0, \gamma_0) =$

$$= \min_{z \in T(x_0, \gamma_0)} \max_{y \in K(x_0, \gamma_0)} \max_{1 \le i \le n} (< z, x_0 - y > - f(y, x_0, \gamma_0))_i \le a.$$

This proves that, for $a \in R$, the level set $\{(x,\gamma) \in X \times \Lambda \mid h(x,\gamma) \le a\}$ is closed. Hence, $h(.,.)$ is lower semicontinuous in $X \times \Lambda$.

Theorem 1.4 *Consider (SQVIP). If the following conditions hold: [i)]*

1. $K(.,.)$ is continuous with compact values in $X \times \Lambda$;
2. $T(.,.)$ is continuous with compact values in $X \times \Lambda$.

Then $h(.,.)$ is continuous in $X \times \Lambda$.

Proof. Now, we need to prove that $h(.,.)$ is upper semicontinuous in $X \times \Lambda$. Indeed, let $a \in R$ and suppose that $\{(x_\alpha, \gamma_\alpha)\} \subseteq X \times \Lambda$ satisfying $h(x_\alpha, \gamma_\alpha) \ge a$, for all α and $(x_\alpha, \gamma_\alpha) \to (x_0, \gamma_0)$ as $\alpha \to \infty$, then $h(x_\alpha, \gamma_\alpha) =$

$$= \min_{z \in T(x_\alpha, \gamma_\alpha)} \max_{y \in K(x_\alpha, \gamma_\alpha)} \max_{1 \le i \le n} (< z, x_\alpha - y > - f(y, x_\alpha, \gamma_\alpha))_i \ge a$$

and so

$$\max_{y\in K(x_\alpha,\gamma_\alpha)}\max_{1\le i\le n}(<z,x_\alpha-y>-f(y,x_\alpha,\gamma_\alpha))_i\ge a,\forall z\in T(x_\alpha,\gamma_\alpha) \quad (4)$$

Since $T(.,.)$ is lower semicontinuous with compact values in $X\times\Lambda,$ for any $z_0\in T(x_0,\gamma_0),$ there exists $z_\alpha\in T(x_\alpha,\gamma_\alpha)$ such that $z_\alpha\to z_0$ as $\alpha\to\infty.$

Since $z_\alpha\in T(x_\alpha,\gamma_\alpha),$ it follows (4) that

$$\max_{y\in K(x_\alpha,\gamma_\alpha)}\max_{1\le i\le n}(<z_\alpha,x_\alpha-y>-f(y,x_\alpha,\gamma_\alpha))_i\ge a, \quad (5)$$

Since f and g are continuous, so $\max_{1\le i\le n}(<z_\alpha,x-y>-f(y,x,\gamma))_i$ is continuous. By the compactness of $K(.,.)$ there exists $y_\alpha\in K(x_\alpha,\gamma_\gamma)$ such that

$$\max_{1\le i\le n}(<z_\alpha,x_\alpha-y_\alpha>-f(y_\alpha,x_\alpha,\gamma_\alpha))_i\ge a. \quad (6)$$

Since $K(.,.)$ is upper semicontinuous with compact values, there exists $y_0\in K(x_0,\gamma_0)$ such that $y_\alpha\to y_0$ (taking a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ if necessary) as $\alpha\to\infty.$ Since $\max_{1\le i\le n}(<z,x-y_\alpha>-f(y,x,\gamma))_i$ is continuous. Taking limit in (6), we have

$$\max_{1\le i\le n}(<z_0,x_0-y_0>-f(y_0,x_0,\gamma_0))_i\ge a. \quad (7)$$

For any $y\in K(x_0,\gamma_0),$ we have

$$\max_{y\in K(x_0,\gamma_0)}\max_{1\le i\le n}(<z_0,x_0-y>-f(y,x_0,\gamma_0))_i\ge a. \quad (8)$$

Since $z\in T(x_0,\gamma_0)$ is arbitrary, it follows from (8) that $h(x_0,\gamma_0)=$ f

$$=\min_{z\in T(x_0,\gamma_0)}\max_{y\in K(x_0,\gamma_0)}\max_{1\le i\le n}(<z,x_0-y>-f(y,x_0,\gamma_0))_i\ge a$$

This proves that, for $a\in R,$ the level set $\{(x,\gamma)\in X\times\Lambda\,|\,h(x,\gamma)\ge a\}$ is closed. Hence, $h(.,.)$ is upper semincontinuous in $X\times\Lambda.$

2 Conclusion

To the best of our knowledge, until now there have not been any paper devoted to the lower semicontinuity, continuity of the gap function without the help of the nonlinear scalarization function for a parametric mixed strong vector quasivariational inequality problem in Hausdorff topological vector spaces. Hence our results, Theorem 1.3 and Theorem 1.4 are new.

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Hàm GAP cho bài toán bất đẳng thức tựa biến phân véc tơ tham số hỗn hợp mạnh

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***Tóm tắt* - Bài toán bất đẳng thức tựa biến phân véc tơ tham số hỗn hợp mạnh bao gồm nhiều vấn đề như bài toán bất đẳng thức biến phân, bài toán điểm bất động, bài toán điểm trùng lặp, bài toán bù nhau, v.v. Có nhiều tác giả đang nghiên cứu tìm hàm gap cho bài toán bất đẳng thức biến phân véc tơ. Bài toán này đóng vai trò quan trọng trong nhiều lĩnh vực toán ứng dụng, đặc biệt là lý thuyết tối ưu. Trong bài báo này, chúng tôi nghiên cứu hàm gap tham số với sự hỗ trợ của hàm phi tuyến vô hướng cho bài toán bất đẳng thức tựa biến phân véc tơ tham số hỗn hợp mạnh (viết tắt (SQVIP)) trong không gian tô pô véc tơ Hausdorff. (SQVIP) Tìm**

$\bar{x} \in K(\bar{x},\gamma)$ **và** $\bar{z} \in T(\bar{x},\gamma)$ **sao cho**

$$<\bar{z}, y-\bar{x}> + f(y,\bar{x},\gamma) \in R_+^n, \forall y \in K(\bar{x},\gamma),$$

với

$$R_+^n = \{t = (t_1, t_2, \ldots, t_n)^T \in R^n \mid t_i \geq 0, i = 1,2,\ldots,n\}.$$

Ngoài ra, chúng tôi cũng thảo luận tính nửa liên tục dưới, nửa liên tục trên và tính liên tục của hàm gap tham số cho bài toán này. Theo những hiểu biết của mình, chúng tôi cho rằng tới nay chưa từng có bài báo nào nghiên cứu tính nửa liên tục dưới, tính liên tục của hàm gap mà không cần sự trợ giúp của hàm phi tuyến vô hướng đối với bài toán bất đẳng thức tựa biến phân véc tơ tham số hỗn hợp mạnh trong không gian tô pô véc tơ Hausdorff. Do đó những kết quả được trình bày trong bài báo này (Định lý 1.3 và Định lý 1.4) là mới và khác biệt so với một số kết quả chính trong tài liệu tham khảo

***Từ khóa* - Bài toán bất đẳng thức tựa biến phân véctơ; hàm gap tham số; tính nửa liên tục dưới; tính nửa liên tục trên, tính liên tục.**