Control of active suspension system using H_∞ and adaptive robust controls

- **Trong Hieu Bui**
- **Quoc Toan Truong**

University of Technology, VNU- HCM

ABSTRACT:

This paper presents a control of active suspension system for quarter-car model with two-degree-of-freedom using H_∞ and nonlinear adaptive robust control method. Suspension dynamics is linear and treated by H_∞ method which guarantees the robustness of closed loop system under the presence of uncertainties and minimizes the effect of road disturbance to system. An Adaptive Robust Control (ARC) technique is used to design a force controller such that it is robust against actuator uncertainties. Simulation results are given for both frequency and time domains to verify the effectiveness of the designed controllers.

Keywords: *Active suspension, Hydraulic actuator, H_∞ control, Adaptive robust control.*

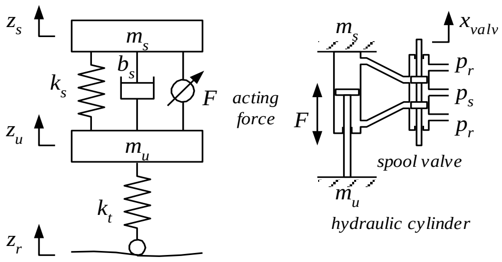
1. INTRODUCTION

Automotive suspension systems have been developed from the begin time of car industrial with a simple passive mechanism to the present with a very high level of sophistication. Suspensions incorporating active components are studied to improve the overall ride performances of automotive vehicle in recent years. Active suspension must provide a trade-off between several competing objectives: passenger comfort, small suspension stroke for packing and small tire deflection for vehicle handling. In the early studies, linear model of suspension are used with the assumption of ideal force actuator. The most applicable force actuator using in practice is hydraulic actuator that has a high non-linearity characteristic. Hence to solve completely problem, recently studies consider to the dynamics and the non-linearity of hydraulic actuator [2,7,9].

This paper presents a control of active suspension system for quarter-car model with two-degree-of-freedom by using H_∞ and nonlinear adaptive robust control method. The system is divided into two parts: the linear part is whole system except actuator and nonlinear part is hydraulic actuator. The linear part is treated using H_∞ control method that guarantees the robustness of closed loop system under the presence of uncertainties and minimizes the effect of disturbance. The variations of system parameters are solved by multiplicative uncertainty model. In hydraulic actuator, there are some unknown factors such as bulk modulus of hydraulic fluid that has strong effect to actuator dynamics. Hence, the nonlinear adaptive control is suitable for designing actuator controller. This paper applied the ARC technique to design a the controller robust against actuator uncertainties[3,4]. The error between desired acting force calculated from H_∞ controller and actual force generated by hydraulic actuator is considered as the disturbance to the linear system. Simulations have been done in both frequency and time domains to verify the effectiveness of the designed controllers.

2. SYSTEM MODELING

The scheme of suspension system and hydraulic actuator used in this paper is described in Fig. 1.



a. Quarter-car model   b. Hydraulic actuator

Fig.1 Suspension system and actuator

Define parameters as the follows

m_s : sprung mass

m_u : unsprung mass

b_s : damping coefficient

k_s : spring stiffness coefficient

k_t : tire stiffness coefficient

F : active force

z_s : displacement of the car body

z_u : displacement of wheel

z_r : displacement of road

Assume that the spring stiffness coefficient and tire stiffness coefficient are linear in their operation range; the tire does not leave the ground; and z_s and z_u are measured from the static equilibrium point. From the scheme of the system model in the Fig. 1, the state variables are chosen as follows

$x_1 = z_s - z_u$: suspension deflection

$x_2 = \dot{z}_s$: velocity of car body

$x_3 = z_u - z_r$: tire deflection

$x_4 = \dot{z}_u$: velocity of wheel

$x_5 = F$: active force

$x_6 = x_{valve}$: position of valve from its closed position.

The governing dynamic equations of suspension system including hydraulic actuator can be presented as the following[9]

$$\dot{x}_1 = x_2 - x_4 \tag{1}$$

$$\dot{x}_2 = \frac{1}{m_s}\left(-k_s x_1 - b_s(x_2 - x_4) + x_5\right) \tag{2}$$

$$\dot{x}_3 = x_4 - \dot{z}_r \tag{3}$$

$$\dot{x}_4 = \frac{1}{m_u}\left(k_s x_1 + b_s(x_2 - x_4) - k_t x_3 - x_5\right) \tag{4}$$

$$\dot{x}_5 = -\beta x_5 - \alpha_f A^2 (x_2 - x_4) + \gamma\sqrt{A}\sqrt{P_s A - \mathrm{sgn}(x_6) x_5}\, x_6 \tag{5}$$

$$\dot{x}_6 = \frac{1}{\tau}(-x_6 + u) \tag{6}$$

where,

$\gamma \equiv \alpha_f C_d w_f \sqrt{1/\rho}$

$\beta \equiv \alpha_f C_{tm}$

$\alpha_f \equiv 4\beta_e / V_t$

A : piston area

P_s : supply pressure of the fluid

C_d : discharge coefficient

w_f : spool valve area gradient

ρ : hydraulic fluid density

C_{tm} : total leakage coefficient of the piston

β_e : effective bulk modulus

V_t : total actuator volume

τ : time constant

u : input to servo-valve

Equations (1)-(4) represent the quarter-car dynamics and equations (5)-(6) drive the hydraulic actuator dynamics.

3. H_∞ CONTROL OF LINEAR PART

Let's define the force error

$$e = x_5 - x_5^{\,d} \qquad (7)$$

where x_5 is actual control force generated from actuator and x_5^d is the desired control force which is calculated from H_∞ controller. Consider x_5 as the control input, the systems (1)-(4) can be rewritten in the form

$$\dot{x}_p = A_p x_p + B_p x_5 + \Gamma \begin{bmatrix} \dot{z}_r \\ e \end{bmatrix} \qquad (8)$$

and the measured output is the velocity of car body

$$y_p = C_p x_p \qquad (9)$$

where

$$x_p = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix},$$

$$A_p = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\dfrac{k_s}{m_s} & -\dfrac{b_s}{m_s} & 0 & \dfrac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{b_s}{m_u} & -\dfrac{k_t}{m_u} & -\dfrac{b_s}{m_u} \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 \\ \dfrac{1}{m_s} \\ 0 \\ -\dfrac{1}{m_u} \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{m_s} \\ -1 & 0 \\ 0 & -\dfrac{1}{m_u} \end{bmatrix},$$

$$C_p^T = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Three interest performance variables are: body vibration isolation, measured by the sprung mass acceleration $\ddot{z}_s$; suspension travel, measured by the deflection of suspension $z_s - z_u$; and tire load constancy, measured by the tire deflection $z_u - z_r$. Then three considered transfer functions from disturbance $\dot{z}_r$ to the acceleration of the sprung mass $H_A(s)$, to the suspension deflection $H_{SD}(s)$, and to the tire deflection $H_{TD}(s)$ can be derived as the following

$$H_A(s) = \frac{\ddot{Z}_s(s)}{\dot{Z}_r(s)} = \frac{\dot{X}_2(s)}{\dot{Z}_r(s)} \qquad (10)$$

$$H_{SD}(s) = \frac{Z_s(s) - Z_u(s)}{\dot{Z}_r(s)} = \frac{X_1(s)}{\dot{Z}_r(s)} \qquad (11)$$

$$H_{TD}(s) = \frac{Z_u(s) - Z_r(s)}{\dot{Z}_r(s)} = \frac{X_3(s)}{\dot{Z}_r(s)} \qquad (12)$$

The augmented system $G(s)$ for H_∞ control problem is given in the Fig. 2.

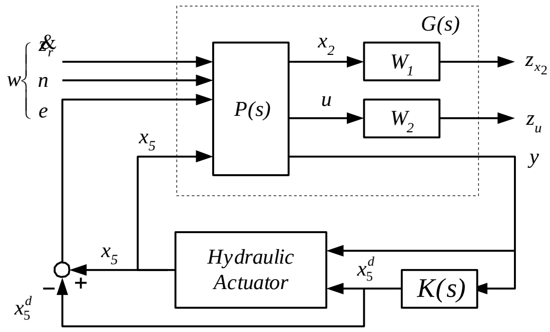


Fig. 2. Configuration of control system

The state space expression of the plant $P(s)$ with adding measurement noise n can be written in the following form

$$\dot{x}_p = A_p x_p + B_{p1} w + B_{p2} x_5 \qquad (13)$$

$$z_p = C_{p1} x_p + D_{p11} w + D_{p12} x_5 \qquad (14)$$

$$y_p = C_{p2}x_p + D_{p21}w + D_{p22}x_5 \qquad (15)$$

The state space expression of the plant $G(s)$ can be written as follows

$$\dot{x} = Ax + B_1 w + B_2 x_5 \qquad (16)$$

$$z = C_1 x + D_{11} w + D_{12} x_5 \qquad (17)$$

$$y = C_2 x + D_{21} w + D_{22} x_5 \qquad (18)$$

where,

$$x = \begin{bmatrix} x_p \\ x_w \end{bmatrix},\quad z = z_p,\quad y = y_p,$$

$$A = \begin{bmatrix} A_p & 0 \\ B_w C_{p11} & A_w \end{bmatrix}$$

$$B_1 = \begin{bmatrix} B_{p1} \\ B_w D_{p111} \end{bmatrix},\quad B_2 = \begin{bmatrix} B_{p2} \\ B_w D_{p121} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} D_w C_{p11} & B_w \\ \alpha_w D_{p12} & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} C_{p2} & 0 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} D_w D_{p111} \\ \alpha_w D_{p112} \end{bmatrix},\quad D_{12} = \begin{bmatrix} D_w D_{p121} \\ \alpha_w D_{p122} \end{bmatrix},$$

$$D_{21} = D_{p21},\quad D_{22} = D_{p22},$$

The H_∞ control problem is to find an internal stabilizing controller, $K(s)$, for the augmented system, $G(s)$, such that the inf-norm of the closed loop transfer function, T_{zw}, is below a given positive scalar γ

$$\underset{K(s)stabilizing}{Find} \quad \|T_{zw}\|_\infty \le \gamma \qquad (19)$$

Furthermore, from the small gain theorem the robust stability of the closed loop system under presence of parameter uncertainty is assured if $\gamma < 1$. Here the change of the parameters of the system is treated by multiplicative uncertainty model $\Delta(s)$. It is derived from the nominal plant $P_n(s)$ and the perturbed plant $P_p(s)$ as follows

$$\Delta(s) = \frac{P_p(s)}{P_n(s)} - 1 \qquad (20)$$

The weighting is chosen to satisfy

$$\sigma[\Delta(s)] < |W_1(s)|,\quad \forall\omega \qquad (21)$$

The transfer function from disturbance to the state of the augmented system is

$$T_{x\dot{x}_5} = \{sI - [A + B_2 K(s) C_2]\}^{-1}[B_1 + B_2 K(s) D_{21}]\begin{bmatrix}1\\0\\0\end{bmatrix} \qquad (22)$$

where $K(s)$ is H_∞ controller. Three transfer functions (10)-(12) become

$$H_{AC}(s) = s[E_2 \quad 0]T_{x\dot{x}_5}$$

$$H_{SD}(s) = [E_1 \quad 0]T_{x\dot{x}_5}$$

$$H_{TD}(s) = [E_3 \quad 0]T_{x\dot{x}_5}$$

where

$$E_1 = [1 \quad 0 \quad 0 \quad 0],$$

$$E_2 = [0 \quad 1 \quad 0 \quad 0],\quad E_3 = [0 \quad 0 \quad 1 \quad 0]$$

4. ADAPTIVE ROBUST CONTROL OF NONLINEAR PART

In this part we will derive the controller for hydraulic actuator used in suspension system. The controller is designed based on adaptive robust control technique proposed by Bin Yao[3]. Consider hydraulic actuator dynamic equations (5)-(6). The parameter is considered as unknown parameter $\alpha_f = 4\beta_e / V_t$. The main reason for choosing α_f as unknown factor is that the bulk modulus of hydraulic fluid is known to change dramatically even when there is a small leakage between piston and cylinder.

The equation (5) can be written in the form

$$\dot{x}_5 = \theta[a_1 x_5 + a_2(x_2 - x_4) + a_3\sqrt{P_s A - \mathrm{sgn}(x_6) x_5}\, x_6] + d \qquad (23)$$

where $a_1 = -C_{tm}$; $a_2 = -A^2$; $a_3 = C_d w_f \sqrt{A/\rho}$; and θ is unknown parameter; d denotes disturbances and their extents are known

$$\theta \in \Omega_\theta \overset{\Delta}{=} \{\theta : \theta_{\min} < \theta < \theta_{\max}\}$$

$$|d| < d_M$$

The adaptive control law can be obtained as the following steps.

Step 1: Let's define

$$b = a_3\sqrt{P_s A - \mathrm{sgn}(x_6)x_5} \quad (24)$$

Equation (23) becomes

$$\dot{x}_5 = \theta[a_1 x_5 + a_2(x_2 - x_4) + b x_6] + d \quad (25)$$

Define the error variable:

$$z_1 \equiv x_5 - x_5^d \quad (26)$$

To find a virtual control law α for x_6 such that x_5 tracks its desired value x_5^d using the procedure suggested in [3]. The term b, representing the nonlinear static gain between the flow rate and the valve opening x_6, is a function of x_6 and also is non-smooth since x_6 appears through a discontinuous function $\mathrm{sgn}(x_6)$. So a smooth modification is needed[3].

Define the smooth projection $\pi(\hat{\theta})$:

$$\pi(\hat{\theta}) = \begin{cases} \theta_{\max} + \varepsilon_\theta\left\{1 - \exp\left[-\dfrac{1}{\varepsilon_\theta}(\hat{\theta} - \theta_{\max})\right]\right\} & (\hat{\theta} > \theta_{\max}) \\ \hat{\theta} & (\hat{\theta} \in [\theta_{\min}, \theta_{\max}]) \\ \theta_{\min} + \varepsilon_\theta\left\{1 - \exp\left[\dfrac{1}{\varepsilon_\theta}(\hat{\theta} - \theta_{\min})\right]\right\} & (\hat{\theta} < \theta_{\min}) \end{cases}$$

The control law α is given by

$$\alpha = \alpha_a + \alpha_r \quad (27)$$

The adaptive part α_a and the robust control part α_r are calculated as follows

$$\alpha_a = \frac{1}{a_3}\left[-a_1 x_5 - a_2(x_2 - x_4) + \frac{1}{\hat{\theta}_\pi}(\dot{x}_{5d} - k_1 z_1)\right] \quad (28)$$

$$\alpha_r = -\frac{1}{4\theta_{\min} a_3} z_1 \left[\frac{1}{\varepsilon_{11}}\theta_M^2 (a_1 x_5 + a_2(x_2 - x_4) + a_3\alpha_a)^2 + \frac{1}{\varepsilon_{12}} d_M^2\right] \quad (29)$$

where

k_1 : tunable parameter

$$\hat{\theta}_\pi = \pi(\hat{\theta})$$

$$\theta_M = \theta_{\max} - \theta_{\min} + \varepsilon_\theta$$

θ is estimated by $\hat{\theta}$ using the following adaptation law

$$\dot{\hat{\theta}} = \gamma_1 z_1 [a_1 x_5 + a_2(x_2 - x_4) + a_3\alpha_a],$$

$$\gamma_1 > 0 \quad (30)$$

ε_θ is a known arbitrary small positive number and ε_{11}, ε_{12} are adjustable small positive numbers.

Step 2: To find an actual control law for u such that x_6 tracks the desired control function α synthesized in step 1 with a guaranteed transient performance.

Define the error variable

$$z_2 \equiv x_6 - \alpha \quad (31)$$

Adaptive robust control law consists of two parts: an adaptive part and a robust control part

$$u = u_a + u_r \quad (32)$$

The adaptive part and robust control part are calculated as follows

$$u_a = \frac{\tau}{b}\left[-k_2 z_2 - p_e - \frac{\partial \alpha}{\partial \hat{\theta}}\gamma_1 \tau_{2c}\right] \quad (33)$$

$$u_r = -\frac{\tau}{4b} z_2 h_2 \tag{34}$$

where

$$p_e = \hat{\theta}_\pi \frac{w_1}{w_2} z_1 - \frac{1}{\tau} b x_6 + \frac{\partial b}{\partial x_5} \dot{x}_5 - \dot{\alpha}_c \tag{35}$$

$$\dot{x}_5 = \hat{\theta}_\pi [a_1 x_5 + a_2 (x_2 - x_4) + b x_6] \tag{36}$$

$$\dot{\alpha}_c = \frac{\partial \alpha}{\partial x_5} \dot{x}_5 + \frac{\partial \alpha}{\partial t} \tag{37}$$

$$\tau_{2c} = \tau_{1c} - w_2 z_2 \Phi \tag{38}$$

$$\tau_{1c} = -w_1 z_1 [a_1 x_5 + a_2 (x_2 - x_4) + a_3 \alpha] \tag{39}$$

$$\Phi = \frac{w_1}{w_2} z_1 + \left[\frac{\partial b}{\partial x_5} - \frac{\partial \alpha}{\partial x_5}\right] [a_1 x_5 + a_2 (x_2 - x_4) + a_3 \alpha] \tag{40}$$

$$h_2 = \frac{1}{\varepsilon_2} \theta_M^2 \Phi^2 \tag{41}$$

k_2, w_1, w_2 and ε_2 are arbitrary positive numbers.

5. SIMULATION RESULTS

The numerical values using in this simulation are given in the Table 1[9].

The weighting function is chosen as

$$W(s) = \begin{bmatrix} W_1(s) & 0 \\ 0 & \alpha_W \end{bmatrix} = \begin{bmatrix} \dfrac{3.135s + 9.2625}{0.93s + 29} & 0 \\ 0 & 3.5 \times 10^{-4} \end{bmatrix}$$

The controller is calculated with the value of $\gamma = 0.99$. The road velocity disturbance is assumed to be from road displacement $r = 0.1 \sin 2\pi f t$. The parameters of ARC controller are chosen to be $\gamma_1 = 5e6$, $k_1 = 150$, $k_2 = 10$, $\varepsilon_\theta = 0.001$, $\varepsilon_{11} = 5$, $\varepsilon_{12} = 2$, $\varepsilon_2 = 5$ and $d_M = 2$.

Table 1. Numerical values for simulation

Parameters	Values	Units
m_s	*290*	*kg*
m_u	*59*	*kg*
b_s	*1000*	*Ns/m*
k_s	*16812*	*N/m*
k_t	*190000*	*N/m*
α_f	*4.515e13*	*N/m*^{5}
β	*1.00*	
γ	*1.545e9*	*N/(m*$^{5/2}$*kg*$^{1/2}$*)*
A	*3.35e-4*	*m*^{2}
P_s	*10342500*	*N/m*^{2}

Frequency domain

The plot of uncertainties and weighting functions are given in Fig. 3. Figures (4)-(6) show the gain plots for three transfer functions (10)-(12) in cases of passive system, active system with desired force and actual force input. As shown in the figures, the designed nonlinear ARC controller can treat the nonlinearity and keep the H_∞ frequency performance well.

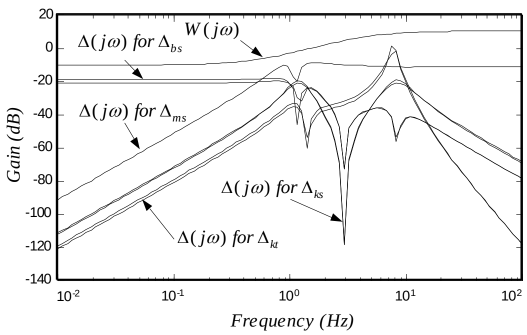


Fig. 3. Plots of uncertainties and weighting function

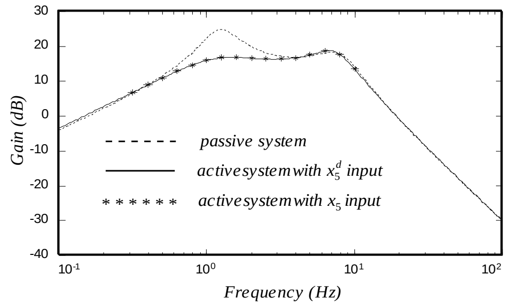


Fig. 4. Gain plots for body acceleration transfer function

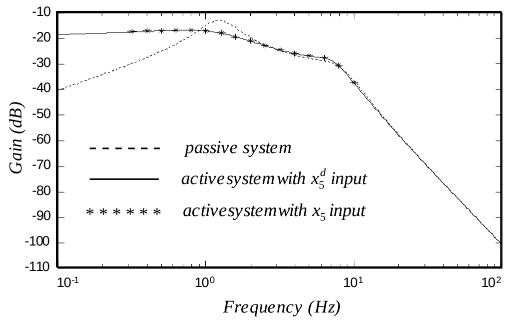


Fig. 5. Gain plots for suspension deflection transfer function

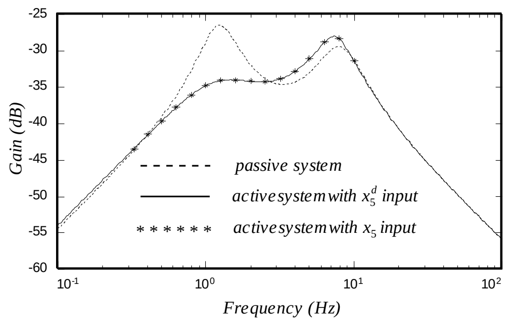


Fig. 6. Gain plots for tire deflection transfer function

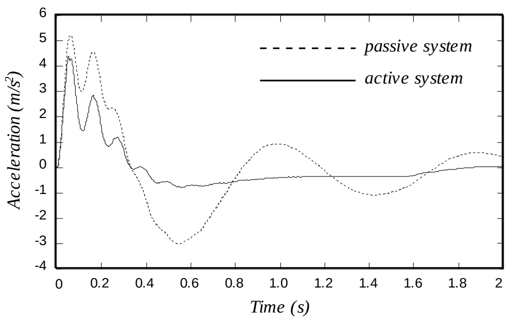


Fig. 7. Acceleration with step disturbance

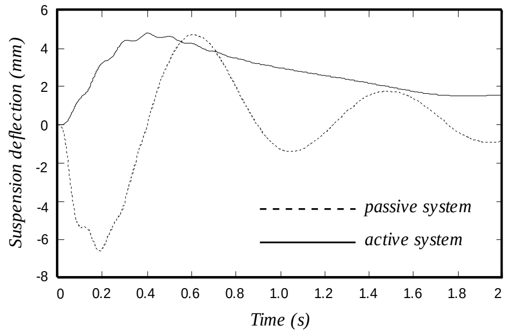


Fig. 8. Suspension deflection with step disturbance

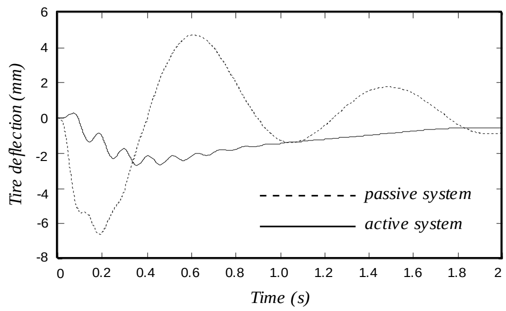


Fig. 9. Tire deflection with step disturbance

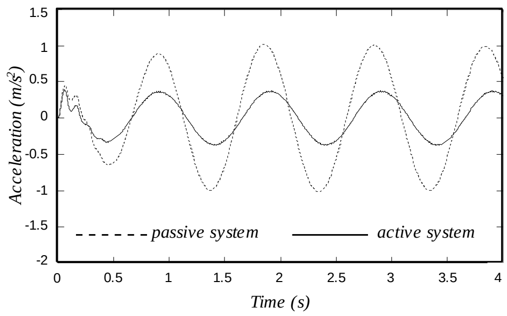


Fig. 10. Acceleration with sine disturbance

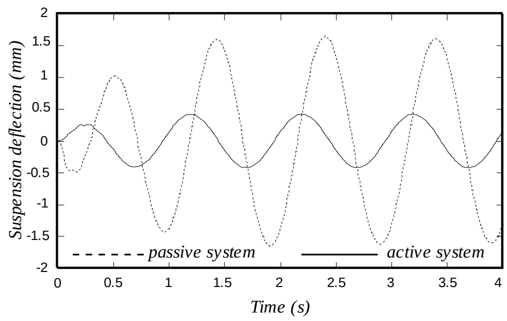


Fig. 11. Suspension deflection with sine disturbance

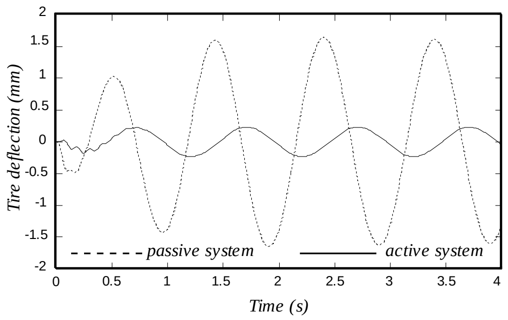


Fig. 12. Tire deflection with sine disturbance

Time domain

The responses of the system with step and sine wave disturbances are considered. Responses of the system in case of step disturbance are given in Figs. (7)-(9). The step road velocity is of 0.1 m/s. Body acceleration and tire deflection are much reduced but the suspension deflection is higher. Responses of the system in case of sine wave disturbance are given in Figs. (10)-(12). The road amplitude is assumed to be $0.1m$ with frequency of $1Hz$. At this frequency, active system reduces considerably the effects of disturbance.

6. CONCLUSION

This paper presents a control of active suspension system using H_∞ and nonlinear adaptive robust control method. H_∞ controller achieved the robustness with the presence of parameter uncertainties and minimized the effects of disturbance. The nonlinear ARC controller treats well the non-linearity and the parameter uncertainties of hydraulic actuator. Simulation results show that the designed controller can keep the good performance of H_∞ controller in both frequency and time domains.

Điều khiển hệ thống treo chủ động của xe ô tô dùng H_∞ và điều khiển thích nghi bền vững

- **Trong Hieu Bui**
- **Quoc Toan Truong**

Trường Đại học Bách khoa, ĐHQG-HCM

TÓM TẮT:

Điều khiển hệ thống treo chủ động của xe ô tô là một đề tài thú vị trong lĩnh vực nghiên cứu về ô tô. Bài báo này đề xuất phương pháp điều khiển hệ thống treo chủ động bằng lý thuyết H_∞ và điều khiển thích nghi bền vững. Kỹ thuật điều khiển thích nghi bền vững (ARC) được sử dụng để thiết kế bộ điều khiển lực bền vững với các thông số không biết chắc của bộ chấp hành. Kết quả mô phỏng đã thể hiện tính hiệu quả của bộ điều khiển đề nghị.

Từ khóa: : *Hệ thống treo chủ động, Điều khiển H_∞, Điều khiển thích nghi bền vững.*

REFERENCES

[1]. T.T. Nguyen, V.G. Nguyen and S.B. Kim, *Control of Active Suspension System by Using Hinf Theory*, ICASE Transaction on Control, Automation, and System Engineering, Vol. 2, No. 1, pp. 1-6, March, 2000.

[2]. Takanori Fukao, Arika Yamawaki and Norihiko Adachi, *Nonlinear and Hinf Control of Active Suspension Systems with Hydraulic Actuators*, Proceeding of the 38th Conference on Decision and Control, pp. 5125-5128, Phoenix, Arizona USA, December 1999.

[3]. Bin Yao, George T.C. Chiu, John T. Reedy, *Nonlinear Adaptive Robust Control of One-DOF Electro_Hydraulic Servo Systems*, ASME International Mechanical Engineering Congress and Exposition, pp. 191-197, 1997.

[4]. Bin Yao and M. Tomizuka, Adaptive Robust Control of SISO nonlinear systems in a semi-strict feedback form, Automatica, vol. 33, no. 5, pp. 893-900, 1997.

[5]. Supavut Chantranuwathana and Huei Peng, *Adaptive Robust Control for Active Suspensions*, Proceeding of the American Control Conference, pp. 1702-1706, San Diego, California USA, June 1999.

[6]. T.T. Nguyen, *Control of Active Suspension System by Using H_∞ Theory*, MS. Thesis, PKNU, Korea, June 1998.

[7]. Jung-Shan Lin and Ioannis Kanellakopoulos, *Nonlinear Design of Active Suspensions*, IEEE Control Systems Magazine, Vol. 17, No. 3, pp. 45-59, June 1997.

[8]. Kemin Zhou, John C. Doyle and Keith Glover, *Robust and Optimal Control*, Prentice Hall, Inc., 1996.

[9]. Andrew Alleyne and J. Karl Hedrick, *Nonlinear Adaptive Control of Active Suspensions*, IEEE Transaction on Control Systems Technology, Vol. 3, No. 1, pp. 94-101, March 1995.

[10]. M. Krstic, I. Kanellakopoulos and P. Kokotovic, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, Inc., 1995.

[11]. M. Yamashita, K. Fujimori, K. Hayakawa and H. Kimura, *Application of Hinf Control to Active Suspension System*, Automatica, Vol. 30, No. 11, pp. 1717-1729, 1994.

[12]. Jean-Jacques E. Slotine and Weiping Li, *Applied Non-linear Control*, Prentice-Hall, Inc., 1991.