

Application of the *hp*-finite element method to modeling thermal fields of high voltage underground cables buried in multi-layer soil

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(Manuscript Received on September 30th, 2013, Manuscript Revised October 28th, 2013)

ABSTRACT:

In this paper, we investigate the application of the adaptive higher-order Finite Element Method (hp-FEM) to heat transfer problems in electrical engineering. The proposed method is developed based on the combination of the Delaunay mesh and higher-order interpolation functions. In which the Delaunay algorithm based on the distance function is used for creating the adaptive mesh in the whole solution domain and the higher-order polynomials (up to 9th

order) are applied for increasing the accuracy of solution. To evaluate the applicability and effectiveness of this new approach, we applied the proposed method to solve a benchmark heat problem and to calculate the temperature distribution of some typical models of buried double- and single -circuit power cables in the homogenous and multi-layer soils, respectively.

Keywords: *Underground cables, Adaptive higher-order finite element method (hp-FEM), temperature distribution.*

INTRODUCTION

The underground cables system, which is one of main transmission and distribution systems of power systems, is used by the power companies and industry in densely populated cities instead of overhead lines even its installation and maintenance are more expensive and complicated. Moreover, the stability and safety operations of buried power cables are the expectation of power utilities.

The important characteristics in operation and design of the underground power cables are the current-carrying capacity and usable working life. These values very much depend on the maximum operating temperature and the ability to transfer the cables-generated heat to the surrounding soil domain. Therefore, the thermal field computation of buried power cables is a very important task of many power engineers, researchers and manufactures all over the world.

In general, this problem is solved by using the analytical and/or numerical methods. In particular, the results of developments and applications of the numerical methods to engineering problems have gained much attention in recent years.

The numerical methods, such as Finite Difference Method (FDM), Boundary Element Method (BEM) and Finite Element Method (FEM) and Meshfree methods, with their advantage is to provide more accurate simulating than the analytical method in complex geometries have been applying to calculate several practical heat transfer problems in electrical engineering such as the thermal field distribution of underground cables [1]-[4], heat simulation for MEMS design [5] and thermal field of transformer [6]. In recent years, the *hp*-FEM has been strongly developed [7]-[8] and successfully applied to many problems of civil, mechanical and electrical engineering [9] due to this method has given the very high accurate solutions. Besides, the adaptive Delaunay mesh that is still used in the application to FE - [10] creates the flexible FE mesh in the whole solving region. Which means that the small-size elements are much more efficient and distributed in domain where the solution has important features. Thus this algorithm can decrease CPU times but still ensure the high accuracy of numerical solution.

Unfortunately, the application of the *hp*-FEM to heat transfer problems is very rare. For this reason, we have proposed an approach of the *hp*-FEM that is the combination of the adaptive Delaunay mesh and higher-order interpolation functions. The advantages of this approach are that it strongly increases the accuracy of solution and can decrease the CPU times compared with the uniform mesh and/or lower-order FEM. In order to demonstrate the advantage and applicability of this method, we have used it to

test on the benchmark heat problem and to calculate the steady-state thermal distribution of some typical power cable systems buried in the homogenous and multi-layer soils.

THE *hp*-FEM FOR THERMAL TRANSFER PROBLEMS

In general, the Poisson equation describes the steady state heat transfer in homogeneous medium can be written as

$$\nabla \cdot (k \nabla T) + Q = 0 \quad (1)$$

where k (°Cm/W) is the thermal resistivity of medium. Q (W/m) is the heat generation rate in the heat source and T (°C) is the unknown temperature.

In the two-dimensional medium, the unknown temperature function $T^e(x, y)$ in (1) can be approximated in per element by terms of the p^{th} -order polynomial as follows

$$T^e(x, y) = \sum_{i=1}^n a_i^e q_i(x, y) = a^T q(x, y) \quad (2)$$

where $n = \frac{(p+1)(p+2)}{2}$ is the total number of nodes in per element and $q(x, y) = [1, x, y, x^2, xy, y^2, \dots, y^p]$.

The unknown coefficients of a_i^e in (2) can be determined by enforcing (2) at n nodes and then they are substituted back into (2). Thus (2) can be written as follows

$$T^e(x, y) = \sum_{i=1}^n \Phi_i^e(x, y) T_i^e \quad (3)$$

where Φ_i^e is the interpolation function is given by

$$\Phi_i^e = P_I^e(L_1^e) P_J^e(L_2^e) P_K^e(L_3^e), \quad I + J + K = n \quad (4)$$

In which [7] we have

$$P_\alpha^e(L_1^e) = \prod_{m=0}^{\alpha-1} \frac{pL_1^e - m}{\alpha - m} = \frac{1}{\alpha!} \prod_{m=0}^{\alpha-1} (pL_1^e - m) \quad \text{with } \alpha = I, J, K > 0 \quad (5)$$

with L_1^e, L_2^e, L_3^e are the area coordinates of e^{th} element.

In the *hp*-FE procedure, the whole solving domain is subdivided by the triangular elements, where the total number of nodes in per element depends on the order of interpolation polynomial.

The *hp*-FE solution will be obtained by minimizing the numerical integrations of piecewise polynomials. Finally, we have the matrix equation as follows

$$\mathbf{AT} = \mathbf{b} \quad (6)$$

where \mathbf{T} is a column vector of temperatures at finite element nodes, \mathbf{A} is the heat conductivity matrix with

$$A_{ij}^e = \iint_{\Omega} \left(k_x \frac{\partial \Phi_i^e}{\partial x} \frac{\partial \Phi_j^e}{\partial x} + k_y \frac{\partial \Phi_i^e}{\partial y} \frac{\partial \Phi_j^e}{\partial y} \right) dx dy \quad i, j = 1, 2, \dots, n \quad (7a)$$

and \mathbf{b} is a vector is given by

$$b_i^e = \iint_{\Omega} Q \Phi_i^e dx dy \quad i = 1, 2, \dots, n \quad (7b)$$

NUMERICAL RESULTS

Benchmark heat problem

In order to demonstrate the advantage of the the *hp*-FEM, we now apply it to the benchmark heat problem which has the following equation

$$k \nabla^2 T(x, y) + Q = 0, \quad x, y \in \Omega = [0, 1]^2 \quad (8)$$

where one assumes that $k=1$ and without internal heat source - [5], and the function values on boundaries are as

$$\begin{aligned} T(x, y) &= 0 & x, y \in \Gamma_1, \\ \frac{\partial T(x, y)}{\partial x} &= 0 & x, y \in \Gamma_2, \\ T(x, y) &= \sin(\pi x) & x, y \in \Gamma_3, \\ \frac{\partial T(x, y)}{\partial x} &= 0 & x, y \in \Gamma_4, \end{aligned} \quad (9)$$

Table I. Error of HP-FE Solutions Of Benchmark Problem

Methods	Number of nodes	Number of elements	L_{∞}
1 st -order FEM	400	722	2.1465e-004
2 nd - order FEM	1521	722	1.3601e-007
3 rd - order FEM	3364	722	2.6232e-008
4 th - order FEM	5929	722	8.8536e-012
5 th - order FEM	9216	722	3.9838e-013
6 th - order FEM	13225	722	3.4861e-014

Where

$$\Gamma_1 = \{(x, y) : 0 \leq x \leq 1, y = 0\} \quad \Gamma_2 = \{(x, y) : 0 \leq y \leq 1, x = 1\},$$

$$\Gamma_3 = \{(x, y) : 0 \leq x \leq 1, y = 1\} \quad \Gamma_4 = \{(x, y) : 0 \leq y \leq 1, x = 0\}$$

The analytical solution of this benchmark problem is given by

$$T(x, y) = \frac{2}{\pi} y - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n\pi x) \sinh(2n\pi y)}{(4n^2 - 1) \sinh(2n\pi)} \quad (10)$$

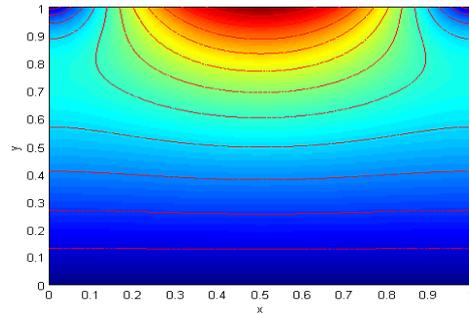


Fig. 1. Contour plot of benchmark problem.

In this section, we have used the higher-order (up to 6th-order) FEM to solve this benchmark problem. The contour plot of isothermal lines and the error comparison between the higher-order FE solutions are illustrated in Fig.1. and TABLE.I., respectively. It has shown that the solutions of the higher-order FEM are much more accurate than the one of the 1st-order FEM.

Temperature distribution of three-phase cable system buried in homogenous soil

In this section, we present the application of the *hp*-FEM to calculate the temperature distribution of high voltage underground cables buried in rectangular homogenous soil as in

Fig.2. Besides, the following assumptions are also given as

The effect of radiation and convection at the ground surface are neglected, Thermal resistivity of soil is constant, The length of cable is more larger than its buried depth.

Table II. Parameters specification – [1]

Thermal conductivity of soil	1.05	W/°Cm
Heat source generated by each cable	43.5	W/m
Diameter of cables	0.03155	m
Distance between two phases	0.15	m
The depth of cables buried in soil	1	m
Temperature of soil surface	10	°C

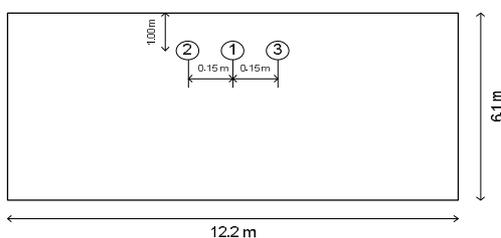


Fig. 2. Model of the buried three-phase cable system in rectangular soil.

As above introduced, the first step of the adaptive *hp*-FE procedure is to subdivide the solving domain by mean of adaptive triangular elements corresponding to nodes. In this work, we use the adaptive Delaunay algorithm [10] to create the adaptive elements in the whole solving domain, and then we apply the higher-order interpolation polynomial to each element.

Finally, the *hp*-FE solutions are obtained by solving (6). In this approach, each conductor of cable system is assumed to be a heat source and data of this problem given by TABLE.II. The results are illustrated in Figs. 3.-7. The comparison between the *hp*-FE solutions and those of FDM, BEM and COMSOL software is presented in TABLE.III.

Table III. Comparison of Numerical Solutions of Cable Temperatures

Methods	Nodes	Elements	Average Temperature (°C)		
			Cable 2	Cable 1	Cable 3
1 st -order FEM	259	465	68.0953	72.7356	68.1626
2 nd - order FEM	985	465	70.7892	75.4573	70.7851
3 rd - order FEM	2176	465	70.8615	75.5333	70.8616

4 th - order FEM	3832	465	70.8775	75.5491	70.8776
5 th - order FEM	5953	465	70.8780	75.5499	70.8781
6 th - order FEM	8539	465	70.8810	75.5528	70.8810
7 th - order FEM	11590	465	70.8809	75.5528	70.8810
8 th - order FEM	15106	465	70.8814	75.5534	70.8815
9 th - order FEM	19087	465	70.8812	75.5532	70.8813
COMSOL	1683	3237	70.7690	75.4940	70.9480
FDM	-	-	68.7690	72.3170	68.7690
BEM - [1]	-	-	67.0400	71.5500	67.0400

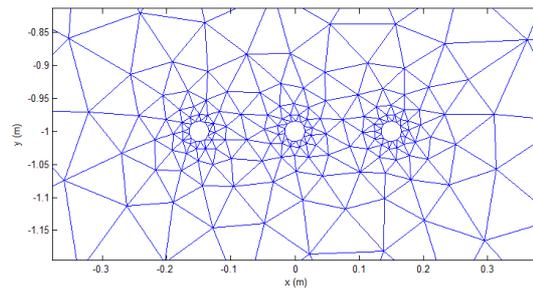


Fig. 3. Zoom solving domain surrounding cables is discretised by using the adaptive 1st-order elements.

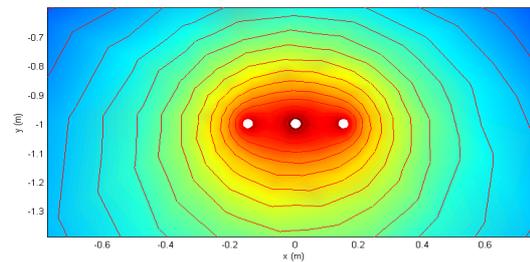


Fig. 4. Contour plot of isothermal lines surrounding cables is solved by the adaptive 1st-oder FEM.

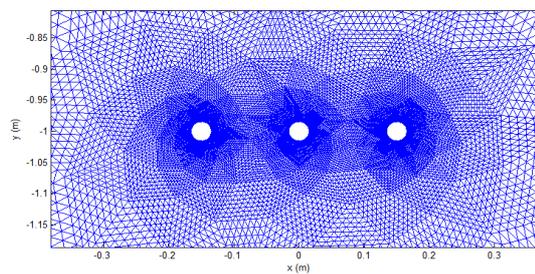


Fig. 5. Zoom solving domain surrounding cables is discretised by using the adaptive 9th-order elements.

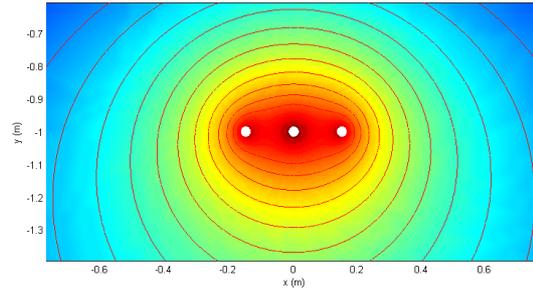


Fig. 6. Contour plot of isothermal lines surrounding cables obtained by using the adaptive 9th-order FEM.

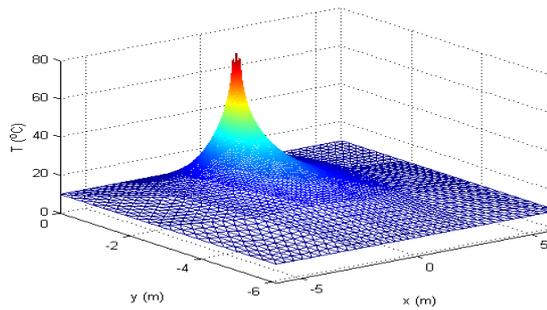


Fig. 6. Temperature of underground cable system is obtained by using the adaptive 9th-order FEM.

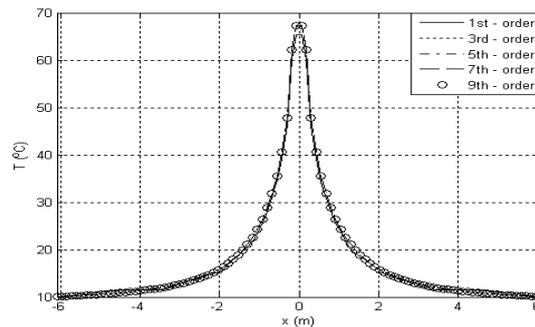


Fig. 7. Comparison of temperature solutions of underground cable system are solved by the adaptive *hp*-FEM.

Temperature distribution of three-phase cable system buried in two-layer soil

In this case, we study the thermal behavior of cables in two-layer medium. We use the cables in the case of double circuit and cables are directly buried at the depth of 1.9m in native soil. The boundary between two layer at the depth of 3.0m.

The values of thermal conductivity of upper and under layer are 1.00 (W/°Cm), 1.30 (W/°Cm), respectively. The ground surface is represented by convective boundary (Cauchy condition) with convection loss coefficient of 5 (W/°Cm²). The phase spacing of each cable circuit is 0.25m and cables still loaded 662A and generate heat rate of 32.029 (W/m).

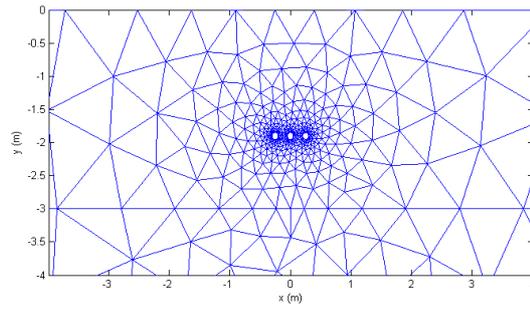


Fig. 8. Domain surrounding cables buried in two-layer soil is discretised by using the adaptive 1st-order elements.

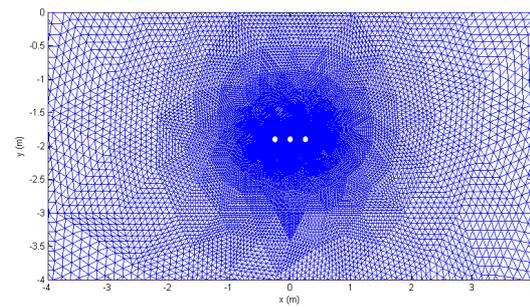


Fig. 9. Domain surrounding cables buried in two-layer soil is discretised by using the adaptive 9th-order elements.

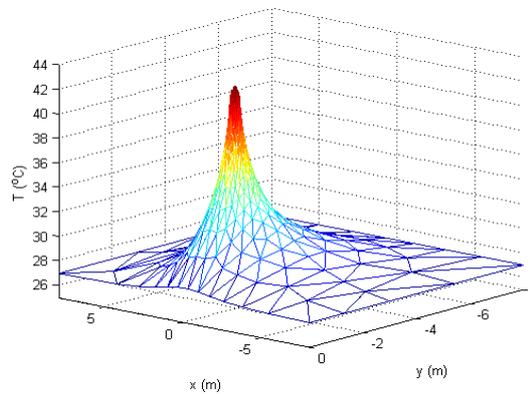


Fig. 10. Temperature of underground cable system buried in two-layer soil is obtained by using the adaptive 1st-order FEM.

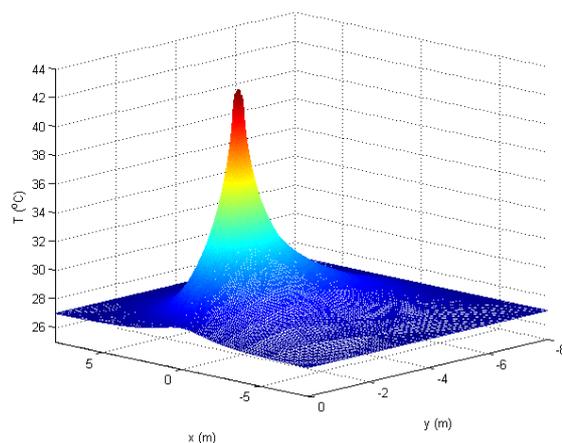


Fig. 11. Temperature of underground cable system buried in two-layer soil is obtained by using the adaptive 9th-order FEM.

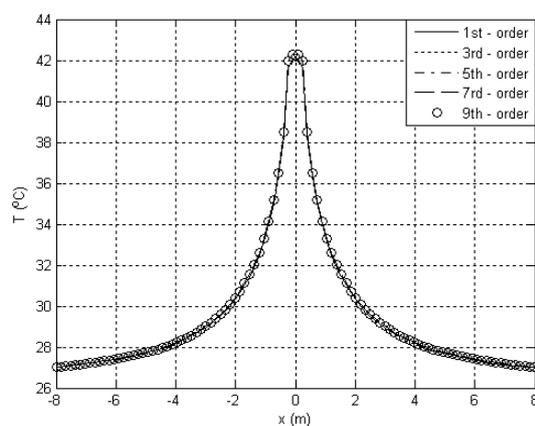


Fig. 12. Comparison of temperature solutions of underground cable system buried in two-layer soil are solved by the adaptive *hp*-FEM.

Temperature distribution of double-circuit three-phase cable system buried in multi-layer soil

In this case, we test the double-circuit three-phase underground cables system buried at the depth of 1.9m in cable bedding in multi-layer soil. The upper layer is trench backfill and surrounding medium is native soil. The values of thermal conductivity of cable bedding, trench backfill and native soil are 1.00 ($W/^\circ C m$), 1.25

($W/^\circ C m$) and 0.80 ($W/^\circ C m$), respectively. The width and height of cable bedding are 1.80m and 0.6m, respectively. The ground surface represented by convective boundary (Cauchy condition) with convection loss coefficient of 5 ($W/^\circ C m^2$). The phase spacing of each cable circuit is 0.25(m) and the nearest distance between two circuits is 0.35(m). Cables still loaded 662A and generated heat rate of 32.029 (W/m). The results are illustrated in Figs. 14.-.16. and TABLE IV.

Table IV. Comparison between HP-FE Solutions of Buried Power cables in Multi-Layer Soil

Order	Number of		Average Temperature (°C)					
	Nodes	Elements	Cable 1	Cable 2	Cable 3	Cable 4	Cable 5	Cable 6
1 st	654	1190	50.3528	52.8046	53.4084	53.4137	52.8085	50.3554
2 nd	2503	1190	50.8008	53.2756	53.8869	53.8875	53.2762	50.8013
3 rd	5542	1190	50.8052	53.2803	53.8917	53.8919	53.2805	50.8054
4 th	9771	1190	50.8080	53.2832	53.8946	53.8947	53.2835	50.8083
5 th	15190	1190	50.8078	53.2830	53.8944	53.8946	53.2832	50.8080
6 th	21779	1190	50.8083	53.2836	53.8950	53.8951	53.2838	50.8085
7 th	29598	1190	50.8258	53.3010	53.9132	53.9118	53.2998	50.8245
8 th	38587	1190	50.8127	53.2879	53.8994	53.8993	53.2878	50.8126
9 th	48766	1190	50.8098	53.2851	53.8966	53.8967	53.2852	50.8098

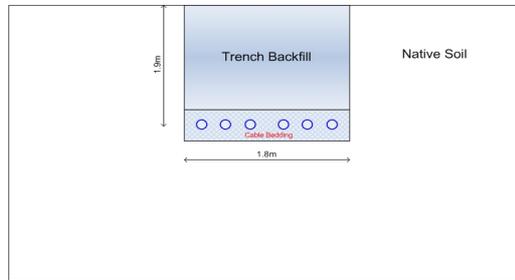


Fig. 13. Double-circuit three-phase cables model in multi-layer soil.

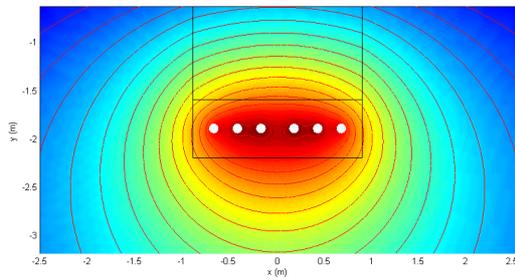


Fig. 14. Contour plot of isothermal lines surrounding double-circuit cable system solved by the 9th-order FEM.

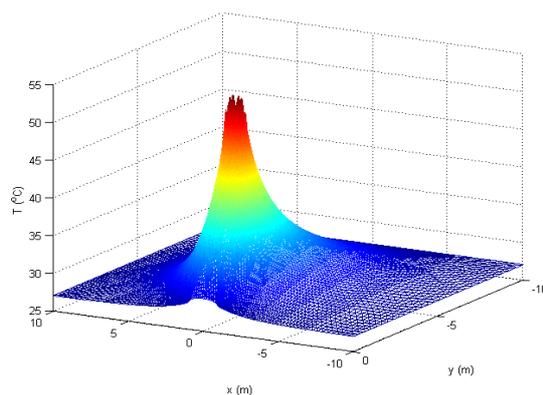


Fig. 15. Temperature of double-circuit cable system solved by the 9th-order FEM.

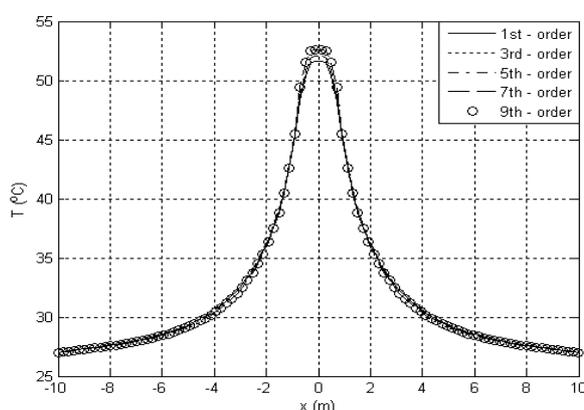


Fig. 16. Comparison of temperature solutions of double-circuit cable system buried in multi-layer soil solved by the hp -FEM.

REMARK

In three tested cases of *A*, *B*, *C* and *D* of Section .III., we have firstly created the adaptive triangular mesh in the whole solving domain, we have then applied the higher-order interpolation and shape functions (up to 9th order) [7]-[8] to each triangular element. Thus the total numbers of triangles were 722 in *case A*, 465 of *case B*, 866 of *case C* and 1190 of *case D* corresponding to the total numbers of nodes have increased from 400 to 13225 in *case A*, from 259 to 19087 in *case B*, from 474 to 35458 in *case C*, and from 654 to 48766 in *case D*. The calculated results have been presented in many Figures and

TABLES. They can give some remarks as follows

TABLE.I. of the benchmark problem has been shown that the hp -FE solutions are in good agreement with the analytical one. Thus the proposed method is very strong and efficient for solving the engineering problems defined by the Poisson equation, including the heat transfer and electromagnetic problems, etc.

Due to the mutual effect of adjacent conductors, the temperature of the mid conductor of any cable system is always highest. Besides, due to the characteristic of adaptive mesh, the total number of nodes on per circle modeled

conductor is equal but their distributed location compared between circles is different, we can see the maximum temperatures of conductor 2 and 3 of *case B* are different while those of FDM and BEM are equal (see TABLE. .III.). This also appears in *case D* (see TABLE.IV.).

The *hp*-FEM is successfully applied to the buried cables in multi-layer soil with the convective boundary. It shows the high applicability and effectiveness of the proposed method in complex engineering problems.

The temperature results obtained by the adaptive *hp*-FEM are more accurate than those of BEM, FDM and COMSOL. Thus they are the

very good datum for design and operation of underground power cables.

CONCLUSION

This paper has applied for the first time the *hp*-FEM to calculating the benchmark heat problem and the temperature distribution of underground cable systems in homogeneous and multi-layer soils. The results of the proposed method are compared to those of other methods, it has been seen that the *hp*-FE solutions are much more accurate and the *hp*-FEM is efficiently applied to complex geometrical problems.

Áp dụng phương pháp phần tử hữu hạn bậc cao thích nghi cho mô phỏng trường nhiệt của cáp ngầm cao thế trong đất nhiều lớp

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TÓM TẮT:

Trong bài báo này, chúng tôi nghiên cứu việc áp dụng phương pháp Phần tử hữu hạn bậc cao thích nghi cho các bài toán truyền nhiệt trong ngành kỹ thuật điện. Phương pháp đề nghị được phát triển dựa trên một sự kết hợp giữa lưới Delaunay và hàm nội suy bậc cao. Trong đó, thuật toán Delaunay đặc cơ sở trên hàm khoảng cách được sử dụng cho việc tạo nên lưới thích nghi trong toàn bộ miền lời giải và các đa thức bậc cao

(đến bậc 9) được áp dụng cho việc làm tăng độ chính xác của lời giải. Nhằm đánh giá khả năng áp dụng và hiệu quả của tiếp cận mới này, chúng tôi đã áp dụng phương pháp đề xuất cho việc giải bài toán nhiệt chuẩn và tính toán phân bố nhiệt độ của một số mô hình đặc trưng của cáp điện lực một mạch và hai mạch được chôn trong đất đồng nhất và nhiều lớp.

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