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Research Article

Cominimax modules and generalized local cohomology modules

Bui Thi Hong Cam, Nguyen Minh Tri*

ABSTRACT
The local cohomology theory plays an important role in commutative algebra and algebraic geometry. The I-cofiniteness of local cohomology modules is one of interesting properties which has been studied by many mathematicians. The I-cominimax modules is an extension of I-cofinite modules which was introduced by Hartshorne. An R -module M is I-cominimax if $\mathrm{Supp}_R M \subseteq V(I)$ and $\mathrm{Ext}^i_R(R/I,M)$ is minimax for all $i \geq 0$. The aim of this paper is to show some conditions such that the generalized local cohomology module $H^i_I(M,N)$ is I-cominimax for all $i \geq 0$. We prove that $H^i_I(M,K)$ if is I-cofinite for all $i \geq 0$. We prove that if $H^i_I(M,K)$ is I-cofinite for all $i < t$ and all finitely generated R-module K, then $H^i_I(M,N)$ is I-cominimax for all $i < t$ and all minimax R-module N. If M is a finitely generated R-module, N is a minimax R-module and t is a non-negative integer such that $\dim \mathrm{Supp}_R H^i_I(M,N) \leq 1$ for all $i < t$, then $H^i_I(M,N)$ is I-cominimax for all i < t . When $\dim R/I \leq 1$ and $H^i_I(N)$ is I-cominimax for all $i \geq 0$, then $H^i_I(M,N)$ is I-cominimax for all $i \geq 0$.
Key words: Generalized local cohomology, I-cominimax

INTRODUCTION

Let R be a local Noetherian ring, I an ideal of R and M a finitely generated R -module. It is well known that the local cohomology modules $H^i_I(M)$ are not generally finitely generated for i > 0. In a 1970 paper Hartshorne[1] gave the concept of I-cofinite modules. An R-module K to be I-cofinite if $\mathrm{Supp}_R K \subseteq V(I)$ and $\mathrm{Ext}^j_R(R/I,K)$ is finitely generated for all $j \geq 0$. Hartshorne asked which rings R and ideals I the modules $H^i_I(M)$ were I-cofinite for all i and all finitely generated modules M.

In 1, if (R , m) is a complete regular local ring and M is a finitely generated R-module, then $H^i_I(M)$ is I -cofinite in two cases:

- I is a nonzero principal ideal, or
- I is a prime ideal with $\dim R/I = 1$

In 1991, Huneke and Koh[2] proved that if R is a complete local Gorenstein domain, I is a one dimension ideal of R and M is a finitely generated R-module, then $H^i_I(M)$ is I-cofinite for all i. In 1997, Yoshida in[3] or Delfino and Marley in[4] extended (b) to all one dimension ideals I of an arbitrary local ring R. In 1998, Kawasaki[5] proved (a) in an arbitrary commutative Noetherian ring. The local condition in (b) has been removed by Bahmanpour and Naghipour in[6].

In[7], Herzog gave a generalizations of the local cohomology theory. Let j be a non-negative integer and M a finitely generated R-module an N an R-module. The j-th generalized local cohomology module of M and N with respect to I is defined by

$$H^j_I(M,N) \cong \varinjlim_{n} \left(Ext^j_R(M/I^nM,N) \right)$$

We see that if $M = R$, then $H^j_I(M,N) = H^j_I(N)$ the usual local cohomology module of Grothendieck[8]. Another similar question is: When is the module $H^j_I(M,N)$ I-cofinite for all $j \geq 0$?

In 2001, Yassemi [[9], Theorem 2.8] showed that in a Gorenstein ring, $H^j_I(M,N)$ is I -cofinite for all $j \geq 0$ where I is non-zero principal ideal. In 2004, Divaani-Aazar and Sazeedeh [[10], Theorem 2.8 and Theorem 2.9] have eliminated the Gorenstein hypothesis and showed that if either

1. I is principal, or
2. R is complete local and I is a prime ideal with $\dim R/I = 1$, then $H^j_I(M,N)$ is I-cofinite for all $j \geq 0$.

When I is a principal ideal, Cuong, Goto and Hoang [[11], Theorem 1.1] gave another proof for $H^j_I(M,N)$ is I-cofinite for all j. They also showed that if $\dim M \leq 2$ or $\dim N \leq 2$, then $H^j_I(M,N)$ is I-cofinite for all j.

An extension of I-cofinite modules is I-cominimax modules which was introduced in 2009[12]. An R-module M is called I-comininax if $\mathrm{Supp}_R M \subseteq V(I)$ and $\mathrm{Ext}^i_R(R/I,M)$ is minimax for all $i \geq 0$ (see [2, 3.1 and 2.2(ii)]). Naturally, we have a question:

Department of Natural Science Education, Dong Nai University, Dong Nai, Vietnam

Correspondence

Nguyen Minh Tri, Department of Natural Science Education, Dong Nai University, Dong Nai, Vietnam

Email: nguyenminhtri@dnpu.edu.vn

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Question: When are the modules $H_I^i(N)$ or $H_I^i(M,N)$ I-cominimax for all $i \geq 0$?
In [2, 3.10], we see that if N is an I-minimax R-module and I is a principal ideal, then $H_I^i(N)$ is I-cominimax for all $i \geq 0$. In 2011, Mafi[13] proved that if N is a minimax R-module, then $H_I^i(N)$ is I-cominimax for all $i \geq 0$ when one of the following cases holds:

1. $\dim R/I \leq 1$;
2. $\mathrm{cd}(I) = 1$;
3. $\dim R \leq 2$.

In[14], the authors showed that, if M is a minimax R-module with $\mathrm{Supp}_R\left(H_I^i(M)\right) \leq 1$ for all $i \geq 0$ and N is a finitely generated R-module with $\mathrm{Supp}_R N \subseteq V(I)$, then $\mathrm{Ext}_R^j\left(N, H_I^i(M)\right)$ is minimax for all $i \geq 0$.
In [18], the authors proved that in a local ring, if M is a finitely generated R-module and N, L are two minimax R-modules with $\mathrm{Supp}_R L \subseteq V(I)$, then $\mathrm{Ext}_R^j\left(L, H_I^i(M,N)\right)$ is minimax for all i and j when one of the following cases holds:

1. $\dim R/I \leq 1$;
2. $\mathrm{cd}(I) = 1$;
3. $\dim R \leq 2$.

The aim of this paper is to study the I-cominimaxness of $H_I^i(M,N)$. Theorem 2.2 shows that if $H_I^i(M,K)$ is I-cofinite for all $i < t$ and all finitely generated R-module K, then $H_I^i(M,N)$ is I-cominimax for all $i < t$ and all minimax R-module N. We will see in Theorem 2.4 that if M is a finitely generated R-module, N is a minimax R-module and t is a non-negative integer such that $\dim \mathrm{Supp}_R H_I^i(M,N) \leq 1$ for all $i < t$, then $H_I^i(M,N)$ is I- cominimax for all $i < t$. When $\dim R/I \leq 1$, Theorem 2.9 shows that if $H_I^i(N)$ is I-cominimax for all $i \geq 0$, then $H_I^i(M,N)$ is I-cominimax for all $i \geq 0$.

MAIN RESULTS

In[15], Zöschinger introduced the class of minimax modules. An R-module K is said to be a minimax module, if there is a finitely generated submodule T of K, such that K/T is Artinian.
Remark 2.1 There are some elementary properties of minimax modules:

1. The class of minimax modules contains all finitely generated modules and all Artinian modules.
2. Let $0 \to L \to M \to N \to 0$ be an exact sequence of R-modules. Then, M is minimax if and only if L and N are both minimax. Thus, any submodule and quotient of a minimax module is minimax. Moreover, if N is finitely generated and M is minimax, then $\mathrm{Ext}_R^j(N,M)$ and $\mathrm{Tor}_j^R(N,M)$ are minimax for all $j \geq 0$.
3. The set of associated primes of any minimax R-module is finite.
4. If M is a minimax R-module and p is a non-maximal prime ideal of R , then M_p is a finitely generated R_p -module.

Definition 2.1 (Azami, Naghipour and Vakili) An R-module M is I-cominimax if $\mathrm{Supp}_R M \subseteq V(I)$ and $\mathrm{Ext}_R^i(R/I,M)$ is minimax for all $i \geq 0$
The following result is a generalization of [14, 2.3].
Theorem 2.2 *Let t be a non-negative integer. Assume that $H_I^i(M,K)$ is I-cofinite for all $i < t$ and all finitely generated R-module K. Then $H_I^i(M,N)$ is I-cominimax for all $i < t$ and all minimax R-module N.*
Proof. Since N is a minimax R-module, there is a finitely generated R-module K of such that N/K is artinian. From the short exact sequence $0 \to K \to N \to N/K \to 0$ we get the following exact sequence

$$\cdots \to H_I^i(M,K) \xrightarrow{f_i} H_I^i(M,N) \xrightarrow{g_i} H_I^i(M,N/K) \xrightarrow{h_i} H_I^{i+1}(M,K) \to \cdots$$

Now, the short exact sequence

$$0 \to \mathrm{Im}\, f_i \to H_I^i(M,N) \to \mathrm{Im}\, g_i \to 0$$

induces a long exact sequence

$$\cdots \to \mathrm{Ext}_R^j(R/I, \mathrm{Im}\, f_i) \to \mathrm{Ext}_R^j\left(R/I, H_I^i(M,N)\right) \to \mathrm{Ext}_R^j(R/I, \mathrm{Im}\, g_i) \to \mathrm{Ext}_R^{j+1}(R/I, \mathrm{Im}\, f_i) \to \cdots$$

gives rise to a long exact sequence I -cofinite

$$\cdots \to \mathrm{Ext}_R^j(R/I, \mathrm{Im}\, h_{i-1}) \to \mathrm{Ext}_R^j\left(R/I, H_I^i(M,K)\right) \to \mathrm{Ext}_R^j(R/I, \mathrm{Im}\, f_i) \to \mathrm{Ext}_R^{j+1}(R/I, \mathrm{Im}\, h_{i-1}) \to \cdots$$

By the hypothesis, $H_I^i(M,K)$ is I-cofinite for all $i \geq 0$. Hence $\mathrm{Ext}_R^j\left(R/I, H_I^i(M,K)\right)$ is finitely generated for all $i < t, j \geq 0$. Since N/K is artinian, it follows from [16, 2.6] that $H_I^i(M,N/K)$ is artinian for all $i \geq 0$. It is easy to see that $\mathrm{Ext}_R^j(R/I, \mathrm{Im}\, h_{i-1})$ is Artinian for all $i, j \geq 0$. Consequently, $\mathrm{Ext}_R^j(R/I, \mathrm{Im}\, f_i)$ is minimax for all $i < t, j \geq 0$. Since $\mathrm{Im}\, g_i$ is a submodule of $H_I^i(M,N/K)$, it follows that $\mathrm{Ext}_R^j(R/I, \mathrm{Img}_i)$ is artinian for all $i, j \geq 0$. Thus $\mathrm{Ext}_R^j\left(R/I, H_I^i(M,N)\right)$ is minimax for all $i < t, j \geq 0$.
Before showing a consequence of Theorem 2.2, we recall the concept of the local cohomology dimension of an ideal.
Definition 2.3 The cohomological dimension of I in R, denoted by $cd(I)$ is the smallest integer n such that the local cohomology modules $H_I^i(M) = 0$ for all R-modules M, and for all $i > n$.
We show some conditions such that the module $H_I^i(M,N)$ is I-cominimax for all $i \geq 0$.
Corollary 2.4 *Let M be a finitely generated R-module and N a minimax R-module. If either*

1. *I is principal, or*
2. $\dim M \leq 2$*, or*
3. $\dim N \leq 2$*, or*
4. *cd(I)* = 1,

then $H_I^i(M,\ N)$ is I-cominimax for all $i\ \geq\ 0$.

Proof. (1), (2) and (3), Combining heorem 2.2 with [11, 1.1] or [11 , 1.3], it follows that $H_I{}^i(M, N)$ is I-cominimax for all $i\ \geq\ 0$.

4. follows from [16, 2.2] and Theorem 2.2.

Next, we will show some results concerning to small dimensions and R is an arbitrary (not local) commutative Noetherian ring.

Theorem 2.4 Let M be a finitely generated R-module, N a minimax R-module and t a non-negative integer such that $\dim \operatorname{Supp}_R\left(H_I^i(M,N)\right) \leq 1$ for all $i < t$. Then $H_I^i(M,N)$ is I-cominimax for all i < t and $\operatorname{Hom}\left(R/I, H_I^t(M,N)\right)$ is minimax.

Proof. Since N is minimax, there is a finitely generated R-module K of such that N/K is artinian. The short exact sequence $0 \to K \to N \to N/K \to 0$ gives rise to a long exact sequence.

$$\cdots \to H_I^i(M,K) \xrightarrow{f_i} H_I^i(M,N) \xrightarrow{g_i} H_I^i(M,N/K) \xrightarrow{h_i} H_I^{i+1}(M,K) \to \cdots$$

Since N/K is artinian, it follows from [17, 2.6] that $H_I^i(M,N/K)$ is artinian for all $i \geq 0$. By the assumption, we induce $\dim \operatorname{Supp}_R\left(H_I^i(M,K)\right) \leq 1$ for all $i < t$. It follows from [11, 1.2] that $H_I^i(M,K)$ is I-cofinite for all $i < t$. Now, the short exact sequence

$$0 \to \operatorname{lm} f_i \to H_I^i(M,N) \to \operatorname{Im} g_i \to 0$$

induces a long exact sequence

$$\cdots \to \operatorname{Ext}_R^j(R/I, \operatorname{Im} f_i) \to \operatorname{Ext}_R^j\left(R/I, H_I^i(M,N)\right) \to \operatorname{Ext}_R^j\left(R/I, \operatorname{lm} g_j\right) \to \operatorname{Ext}_R^{j+1}(R/I, \operatorname{Im} f_i) \to \cdots$$

Note that $\operatorname{Ext}_R^j(R/I, \operatorname{Img}_i)$ is artinian for all $i, j \geq 0$. Let $i < t$, the short exact sequence

$$0 \to \operatorname{lm} h_{i-1} \to H_I^i(M,K) \to \operatorname{Im} f_i \to 0$$

induces a long exact sequence

$$\cdots \to \operatorname{Ext}_R^j(R/I, \operatorname{Im} h_{i-1}) \to \operatorname{Ext}_R^j\left(R/I, H_I^i(M,K)\right) \to \operatorname{Ext}_R^j(R/I, \operatorname{Im} f_i) \to \operatorname{Ext}_R^{j+1}(R/I, \operatorname{Im} h_{i-1}) \to \cdots.$$

Since $H_I^i(M,K)$ is I-cofinite, it follows that $\operatorname{Ext}_R^j\left(R/I, H_I^i(M,K)\right)$ is finitely generated for all $j \geq 0$. By the artinianness of $\operatorname{Ext}_R^j(R/I, \operatorname{Im} h_{i-1})$, we can conclude that $\operatorname{Ext}_R^j(\mathrm{R/I}, \operatorname{Im} f_i)$ is minimax for all $j \geq 0$. Therefore $\operatorname{Ext}_R^j\left(R/I, H_I^i(M,N)\right)$ is minimax for all $j \geq 0$. We have two following exact sequences

$$0 \to \operatorname{Hom}_R(R/I, \operatorname{Im} f_t) \to \operatorname{Hom}_R\left(R/I, H_I^t(M,N)\right) \to \operatorname{Hom}_R(R/I, \operatorname{Im} g_t)$$

and

$$\operatorname{Hom}_R\left(R/I, H_I^t(M,K)\right) \to \operatorname{Hom}_R(R/I, \operatorname{Im} f_t) \to \operatorname{Ext}_R^1(R/I, \operatorname{Im} h_{t-1}) \to \cdots$$

Since $\dim \operatorname{Supp}_R\left(H_I^i(M,K)\right) \leq 1$ for all $i < t$, it follows from [4, Theorem 1.2] that $\operatorname{Hom}_R\left(R/I, H_I^t(M,K)\right)$ is finitely generated. On the other hand, $\operatorname{Ext}_R^1(R/I, \operatorname{Im} h_{t-1})$ is an artinian R-module. Therefore $\operatorname{Hom}_R(R/I, \operatorname{Im} f_t)$ is minimax. We see that $\operatorname{Hom}_R(R/I, \operatorname{Im} g_t)$ is artinian and then $\operatorname{Hom}_R\left(R/I, H_I^t(M,N)\right)$ is minimax.

In [18, 3.1 and 3.2], the authors showed that $H_I^i(M,N)$ is I-cominimax for all $i \geq 0$ when $\dim R/I \leq 1$ where R is a local ring. Now we consider that R is not a local ring.

Corollary 2.5 *Let M be a finitely generated R-module, N a minimax R-module and t a non-negative integer. Assume that* $\dim M/IM \leq 1$ *or* $\dim N \leq 1$ *or* $\dim R/I \leq 1$*. Then* $H_I^i(M,N)$ *is I-cominimax for all* $i \geq 0$.

Corollary 2.6 *Let M be a finitely generated R-module, N a minimax R-module and t a non-negative integer. Assume that* $\operatorname{Supp}_R\left(H_I^i(M,N)\right)$ *is finite for all* $i < t$*. Then* $H_I^i(M,N)$ *is I-cominimax for all* $i < t$ *and* $\operatorname{Hom}_R\left(R/I, H_I^t(M,N)\right)$ *is minimax. In particular,* $\operatorname{Ass}\left(H_I^t(M,N)\right)$ *is a finite set.*

Proof. Since $\operatorname{Supp}_R\left(H_I^i(M,N)\right)$ is a finite set, we can conclude that $\dim \operatorname{Supp}_R\left(H_I^i(M,N)\right) \leq 1$. It follows from Theorem 2.4 that $H_I^i(M,N)$ is I-cominimax for all $i < t$ and $\operatorname{Hom}_R\left(R/I, H_I^t(M,N)\right)$ is minimax. Moreover, we have

$$\operatorname{Ass}\left(H_I^t(M,N)\right) = \operatorname{Ass}\left(\operatorname{Hom}_R\left(R/I, H_I^t(M,N)\right)\right)$$

By Remark 2.1.3, $\operatorname{Ass}\left(H_I^t(M,N)\right)$ is a finite set.

Corollary 2.7 *Let N be a non-zero minimax R-module and I an ideal of R. Let t be a non-negative integer such that* $\dim \operatorname{Supp}_R H_I^i(N) \leq 1$ *for all* $i < t$ *. Then the following statements hold:*

1. *the R-modules* $H_I^i(N)$ *are I-cominimax for all* $i < t$;
2. *the R-module* $\operatorname{Hom}_R\left(R/I, H_I^t(N)\right)$ *is minimax.*

Lemma 2.8 *Let M be a finitely generated R-module such that* $\operatorname{Supp}_R M \subseteq V(I)$ *and N an I-cominimax R-module. Then* $\operatorname{Ext}_R^i(M,N)$ *is minimax for all* $i \geq 0$.

Proof. The proof is by induction on i. Since N is an I-cominimax R-module, the module $\mathrm{Ext}^i_R(R/I,N)$ is minimax for all $i \geq 0$. By Gruson's theorem, there is a chain of submodules of M.

$$0 = M_0 \subseteq M_1 \subseteq \ldots \subseteq M_k = M$$

such that M_j/M_{j-1} is a homomorphic image of $(R/I)^t$ for some positive integer t . We consider short exact sequences

$$0 \to K \to (R/I)^m \to M_1 \to 0$$

and

$$0 \to M_{j-1} \to M_i \to M_j/M_{j-1} \to 0$$

The first exact sequence induces a long exact sequence

$$0 \to \mathrm{Hom}_R(M_1,N) \to \mathrm{Hom}_R((R/I)^m,N) \to \mathrm{Hom}_R(K,N) \to \cdots$$

where K is a submodule of $(R/I)^m$ for some positive integer number m. Since $\mathrm{Hom}_R((R/I)^m,N) \cong \mathrm{Hom}_R(R/I,N)^m$, it follows that $\mathrm{Hom}_R(M_1,N)$ is minimax. By similar arguments, we also get that $\mathrm{Hom}_R\left(M_j/M_{j-1},N\right)$ is minimax for all $1 \leq i \leq k$. Now, the exact sequence

$$0 \to \mathrm{Hom}_R\left(M_j/M_{j-1},N\right) \to \mathrm{Hom}_R\left(M_j,N\right) \to \mathrm{Hom}_R\left(M_{j-1},N\right) \to \cdots$$

deduces that $\mathrm{Hom}_R\left(M_j,N\right)$ is minimax for all j and then $\mathrm{Hom}_R(M,N)$ is minimax. Therefore, we have the conclusion when $i = 0$.

Let $i > 0$. The short exact sequence

$$0 \to K \to (R/I)^m \to M_1 \to 0$$

gives rise to a long exact sequence

$$\cdots \to \mathrm{Ext}^{i-1}_R(K,N) \to \mathrm{Ext}^i_R(M_1,N) \to \mathrm{Ext}^i_R\left((R/I)^t,N\right) \cdots$$

By the inductive hypothesis, $\mathrm{Ext}^{i-1}_R(K,N)$ is a minimax R-module. Since $\mathrm{Ext}^i_R((R/I)^m,N) \cong \mathrm{Ext}^i_R(R/I,N)^m$, it follows that $\mathrm{Ext}^i_R(M_1,N)$ is minimax. Analysis similar to the above proof, we have $\mathrm{Ext}^i_R(M_k,N)$ is minimax and which completes the proof.

The following result shows a connection on the I-cominimaxness of $H^i_I(N)$ and $H^i_I(M,N)$ when R is not a local ring and N is an arbitrary R-module.

Theorem 2.9 *Let M be a finitely generated R-module with $\mathrm{pd}(M) < \infty$ and N an R- module. Let I be an ideal of R with $\dim R/I = 1$ and t a non-negative integer such that $H^i_I(N)$ is I-cominimax for all $i < t$. Then $H^i_I(M,N)$ is I-cominimax for all $i < t$.*

Proof. We prove by induction on $p = pd(M)$. If $p = 0$, then M is a projective R-module. It follows from [19, 2.5] that $H^i_I(M,N) \cong \mathrm{Hom}_R\left(M,H^i_I(N)\right)$ for all $i \geq 0$. By [20, 10.65], we have

$$\mathrm{Ext}^j_R\left(R/I,\mathrm{Hom}_R\left(M,H^i_I(N)\right)\right) \cong \mathrm{Ext}^j_R\left(M/IM,H^i_I(N)\right)$$

for all $i < t, j \geq 0$. Therefore $\mathrm{Ext}^j_R\left(R/I,H^i_I(M,N)\right) \cong \mathrm{Ext}^j_R\left(M/IM,H^j_I(N)\right)$ where $\mathrm{Ext}^j_R\left(M/IM,H^i_I(N)\right)$ is minimax for all $j \geq 0$ by Lemma 2.8 and then the assertion follows.

Let $p > 0$ and the statement is true for all finitely generated R-module with projective dimension less than p. There is a short exact sequence

$$0 \to K \to P \to M \to 0,$$

where K is finitely generated, P is projective finitely generated. Note that $\mathrm{pd}(K) = p-1$ and then by the inductive hypothesis $H^i_I(K,N)$ is I-cominimax for all $i < t$. On the other hand, there is a long exact sequence $\cdots \to H^i_I(K,N) \to H^{i+1}_I(M,N) \to H^{i+1}_I(P,N) \to \cdots$ in which $H^i_I(K,N)$ and $H^i_I(P,N)$ are I-cominimax for all $i \geq 0$. It follows from [21, 2.6] that $H^i_I(M,N)$ is also I-comiminax for all $i \geq 0$ and the proof is complete.

COMPETING INTERESTS

The authors declare that they have no conflicts of interest.

AUTHOR CONTRIBUTION

Bui Thi Hong Cam has contributed the Theorem 2.2 and has written the manuscript. Nguyen Minh Tri has contributed the Theorem 2.4, 2.9 and revising the manuscript.

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