AN APPROACH TO THREE CLASSICAL TESTS OF THE GENERAL THEORY OF RELATIVITY IN THE VECTOR MODEL FOR GRAVITATIONAL FIELD

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***ABSTRACT**: In this paper, based on the vector model for gravitational field we have found a metric tensor of the space –time that in the first order approximate it lead to the Schwarzschild metric tensor in the General Theory of Relativity(GTR). Thus, we have also obtained 3 classical tests of GTR in the Vector Model for Gravitational Field.*

1.INTRODUCTION

We have known that when using the equivalence principle in a very direct way, Einstein made the first derivation of the red shift which appropriated good with experiments and he also predicted the deflection of the light rays in the gravitational field of the Sun but it only approached half of the experimental value.

Some authors as R. Adler , M. Bazin and M. Schiffer[1]; Frank W.K.Firk[2] obtained the Schwarzschild metric tensor in the context of special relativity by using the equivalence principle , however these approaches were difficult to understand.

In this paper, based on the vector model for gravitational field and the Special relativity, we have found the metric tensor of the space –time that in the first order approximate it leads to the Schwarzschild metric tensor in GTR. This approach is a clear deduction. Thus, we have also obtained 3 classical tests of GTR in the Vector Model for Gravitational Field.

2. SPACE AND TIME IN NON-INERTIAL REFERENCE FRAMES AND IN GRAVITATIONAL FIELD

It is known that [3, 4] time is uniform and space is both uniform and isotropic in inertial frames of reference. The geometrical properties of uniform and isotropic space can be described by Euclidean geometry.

In uniform and isotropic space, the length of line segments do not depend upon the region of space they are in. We divide the axes of coordinates into equal segments $\Delta x = \Delta y = l$ and draw straight lines, parallel to the axes, through the points of division. Plane xy is thus divided into unit cells (fig.1, fig.2) in the form of equal squares.

In exactly the same way, owing to the uniformity of time in an inertial reference frame, the interval of time Δt between two events is independent to the point of space at which these events occur.

Space is non-uniform in a non- inertial reference frame. Indeed, we know that the length of a line segment is less in a moving frame than in one in which the line segment is at rest:

$$\Delta x' = \Delta x\sqrt{1-\frac{v^2}{c^2}} \qquad (1)$$

But in motion with constant acceleration $v^2 = 2ax$, where a is the acceleration of the non-inertial frame of reference , we obtain[3] :

$$\Delta x' = \Delta x\sqrt{1-\frac{2ax}{c^2}} \tag{2}$$

We see that in non-inertial frames of reference, the length of a line segment depends upon the region of space it is located in. The length of the same line segment differs at points with different abscissae.

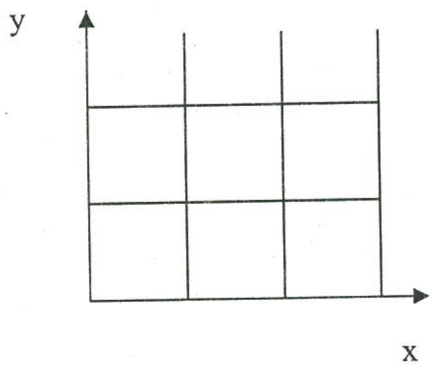

Fig.1

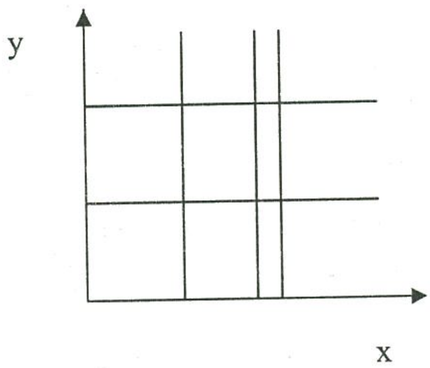

Fig.2

But a line segment along the axis y retains unchanged length because (in our case) there is no motion along this axis: $\Delta y' = \Delta y$

Space is not only non-uniform in a non-inertial reference frame but is anisotropic.

Indeed, the two directions are not equivalent along the axis of abscissas. In our example, elements of length decrease along the positive direction of axis and increase along the negative direction.

It should be noted that the laws of conservation of linear and angular momentum do not hold in non-inertial reference frames due to the non-uniformity and anisotropy of space in these frames.

Finally, it can also be shown that time is non-uniform in non-inertial reference frames, as a result, the law of conservation of energy does not hold for such frames.

In a moving reference frame, an interval of time between two events occurring at the same point is :

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3}$$

Making use, as above, of the expression $v^2 = 2ax$, we obtain[3]:

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{2ax}{c^2}}} \tag{4}$$

Because of the non-uniformity and anisotropy of space in non-inertial reference frames, it is necessary to describe properties of such space by means of a non-Euclidean geometry.

From the previous papers[5,6], we see that inertial force field is just gravitational force field, therefore space –time in a gravitational field (as in a field of inertial forces) is non- Euclidean.

3. MOTION OF BODIES IN A GRAVITATIONAL FIELD

Consider a particle moving freely under the influence of purely gravitational forces. Because inertial forces are just gravitational forces in non-inertial reference frames [5,6], there is a freely falling coordinate system ξ^α in which equation of motion of this particle is that of straight line in space – time[4]. This means:

$$\frac{d^2\xi^\alpha}{d\tau^2} = 0 \tag{5}$$

Where $d\tau$ is the proper time. With:

$$d\tau^2 = -\eta_{\alpha\beta} d\xi^\alpha d\xi^\beta \tag{6}$$

Where:

$$\eta_{\alpha\beta} = \begin{cases} +1 \text{ if } \alpha = \beta = 1,2,3 \\ -1 \text{ if } \alpha = \beta = 0 \\ 0 \text{ if } \alpha \neq \beta \end{cases}$$

If we now use any other coordinate system x^μ , which can be a Cartesian coordinate system at rest in the laboratory, but also may be curvilinear, accelerated, rotating ,... The freely falling coordinates ξ^α are functions of the x^μ , and Eq.(5) becomes:

$$0 = \frac{d}{d\tau}\left(\frac{\partial\xi^\alpha}{\partial x^\mu}\frac{dx^\mu}{d\tau}\right) = \frac{\partial\xi^\alpha}{\partial x^\mu}\frac{d^2x^\mu}{d\tau^2} + \frac{\partial^2\xi^\alpha}{\partial x^\mu \partial x^\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$$

Multiply this by $\frac{\partial x^\lambda}{\partial \xi^\alpha}$, and use the familiar product rule:

$$\frac{\partial\xi^\alpha}{\partial x^\mu}\frac{\partial x^\lambda}{\partial \xi^\alpha} = \delta^\lambda_\mu$$

This gives the equation of motion:

$$0 = \frac{d^2x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \tag{7}$$

Where $\Gamma^\lambda_{\mu\nu}$ is the *affine connection,* defined by:

$$\Gamma^\lambda_{\mu\nu} \equiv \frac{\partial x^\lambda}{\partial \xi^\alpha}\frac{\partial^2\xi^\alpha}{\partial x^\mu \partial x^\nu} \tag{8}$$

The proper time (6) may also be expressed in an arbitrary coordinate system:

$$d\tau^2 = \eta_{\alpha\beta}\frac{\partial\xi^\alpha}{\partial x^\mu}dx^\mu\frac{\partial\xi^\beta}{\partial x^\nu}dx^\nu \text{ or } d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu \qquad (9)$$

Where $g_{\mu\nu}$ is the metric tensor, defined by:

$$g_{\mu\nu} \equiv \frac{\partial\xi^\alpha}{\partial x^\mu}\frac{\partial\xi^\beta}{\partial x^\nu}\eta_{\alpha\beta} \qquad (10)$$

For mass-less particles the right-hand side of (6) vanishes. Instead of τ we can use $\sigma \equiv \xi^0$, so that (7) and (9) become:

$$0 = \frac{d^2x^\lambda}{d\sigma^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma} \qquad (11)$$

$$0 = g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma} \qquad (12)$$

We also recall the relation between $g_{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu}$ as follows [1,4,7]:

$$\Gamma^\sigma_{\lambda\mu} = \frac{1}{2}g^{\nu\sigma}\{\frac{\partial g_{\mu\nu}}{\partial x^\lambda} + \frac{\partial g_{\lambda\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\lambda}}{\partial x^\nu}\} \qquad (13)$$

We also see the case of a particle moving slowly in a weak stationary gravitational field, (7) becomes the corresponding Newtonian equation:

$$\frac{d^2x}{dt^2} = -\nabla\phi \qquad (14)$$

If we have: g_{00} =-(1+ 2ϕ). We also note that equation (7) just is the equation of the geodesic in the curved space-time with the metric tensor $g_{\mu\nu}$.

4. AN APPROACH TO THE METRIC TENSOR OF SPACE-TIME WITH THE PRESENT OF THE GRAVITATIONAL FIELD

In this section, we introduce an approach to the Schwarzschild metric tensor, it is similar to an approach from the equivalence principle of different authors [1,2].

Suppose that we want to seek the interval of events attached to a particle which moves in a gravitational field of M_g. To realize this, we should displace a standard ruler and a standard clock along the orbit of the particle.But when the ruler and clock move along the orbit in the gravitational field, the length of ruler and rate of the clock vary at each point. Because of this reason, the measured results at each point in the gravitational field can not be compared together if we do not know any relation between the etalons at each point. Seeking this relation between etalons, then we convert measured results at each point with its etalons into measured results with the standard etalon is the synchronizing of the rulers and the clocks as the synchronizing of clocks in the special theory of relativity.

Firstly, we seek the relation between etalons at each point then convert the measured results.

We choose the length of a ruler and rate of a clock at a point O which is very distance from the field source, say l_0 and τ_0, as a standard etalon. The gravitational potential is the cosmic background potential φ_{g0} at this point.

We suppose that there is every locally Minkowskian coordinate system at each point in a gravitational field. This hypothesis agrees the axiom which Gauss took as the basis of non-Euclidean geometry. Gauss assumed that at any point on a curved surface we may erect a locally Cartesian coordinate system in which distances obey the law of Pythagoras. This hypothesis also is a part of the equivalence principle [4].

We find the interval of two events near a point N(r) in the gravitational field with the local etalon at N(r) as follows:

$$ds^2 = c^2dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{15}$$

Now we seek the expression of the interval (15) if it is measured by the standard etalon l_0, τ_0. Firstly, we seek the relation between etalons (l_0, τ_0) and etalon (l, τ). Consider two observers A and B, their etalons are the same. Observer A is rest with respect to the cosmological background potential φ_{g0}. Observer B moves with respect to it with velocity of v= at in x direction (the vertical direction). We have the following relations from the special theory of relativity:

$$\varphi_g = \frac{\varphi_{g0}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{16}$$

Where φ_g is the gravitational potential on the system of B.

$$l_B(vertical) = \sqrt{1 - \frac{v^2}{c^2}}\, l_A(vertical) \tag{17}$$

$$l_B(horizontal) = l_A(horizontal) \tag{18}$$

$$\tau_B = \frac{\tau_A}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19}$$

From (16), (17) and (19) we have :

$$l_B(vertical) = \frac{\varphi_{g0}}{\varphi_g} l_A(vertical) \tag{20}$$

$$\tau_B = \frac{\varphi_g}{\varphi_{g0}} \tau_A \tag{21}$$

Where φ_{g0} is the gravitational potential in the system A; φ_g is the gravitational potential in the system B.

Now we return the above problem. The gravitational potential at O is the cosmological background potential φ_{g0}, the gravitational potential at N(r) is:

$$\varphi_{gN} = \varphi_{g0} + \varphi_g \tag{22}$$

Where φ_g is the gravitational potential of M_g at N(r).

We see that the potential φ_g causes the gravitational field on the system B and we have the relations (20), (21) between the etalons at A and B. In a similar case, the potential φ_{gN} causes the gravitational field at N so we also have the relations (20) and (21) between the etalons at N and A

From (20), (21) we have the relation between etalons at O and N is:

$$l\ (vertical) = \frac{\varphi_{g0}}{\varphi_{gN}} l_0 (vertical) \tag{23}$$

$$\tau = \frac{\varphi_{gN}}{\varphi_{g0}} \tau_0 \tag{24}$$

From (23) and (24) we have the relations between the interval of length and the interval of time at O and N are:

$$dr = \frac{\varphi_{gN}}{\varphi_{g0}} dr_0 \tag{25}$$

$$dt = \frac{\varphi_{g0}}{\varphi_{gN}} dt_0 \tag{26}$$

Substituting (25), (26) into (15) we obtain:

$$ds^2 = c^2 (\frac{\varphi_{g0}}{\varphi_{gN}})^2 dt_0^2 - (\frac{\varphi_{gN}}{\varphi_{g0}})^2 dr_0^2 - r_0^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \tag{27}$$

Note that we still have $r = r_0$. Noting (22), we have:

$$\frac{\varphi_{gN}}{\varphi_{g0}} = \frac{\varphi_{g0} + \varphi_g}{\varphi_{g0}} = 1 + \frac{\varphi_g}{\varphi_{g0}} = 1 - \frac{\varphi_g}{c^2} \tag{28}$$

We have had $\varphi_{g0} = -c^2$ from the previous paper [6]. Substituting (28) into (27), we obtain:

$$ds^2 = c^2 (1 - \frac{\varphi_g}{c^2})^{-2} dt_0^2 - (1 - \frac{\varphi_g}{c^2})^2 dr_0^2 - r_0^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \tag{29}$$

For $\frac{\varphi_g}{c^2} << 1$, we have:

$$ds^2 = c^2 (1 + 2\frac{\varphi_g}{c^2})\ dt_0^2 - (1 - 2\frac{\varphi_g}{c^2})\ dr_0^2 - r_0^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \tag{30}$$

With $\varphi_g = -\frac{GM_g}{r}$, we obtain from (30):

$$ds^2 = c^2 (1 - 2\frac{GM_g}{rc^2})\ dt_0^2 - (1 + 2\frac{GM_g}{rc^2})\ dr_0^2 - r_0^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \tag{31}$$

(31) is just the Schwarzschild solution in GTR.

From the metric tensor (31) and equations (7),(11),(12) we find the classical three tests of GTR.

5. CONCLUSION

In conclussion, based on the vector model for gravitational field we have obtained the metric tensor of the space-time (29). This metric tensor leads to the Schwarzschild metric tensor in a weak gravitational field, therefore we have also obtained three classical tests of GTR in the Sun system. In a strong gravitational field, the metric tensor (29) is different with the Schwarzschild metric tensor and it gives the small suplementations to these three classical tests.

MỘT TIẾP CẬN ĐẾN 3 HIỆU ỨNG KINH ĐIỂN CỦA THUYẾT TƯƠNG ĐỐI TỔNG QUÁT TRONG MÔ HÌNH VECTƠ CHO TRƯỜNG HẤP DẪN

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***TÓM TẮT**: Trong bài báo này khi dựa trên mô hình véctơ của trường hấp dẫn, chúng tôi thu được một tenxơ mêtric của không thời gian mà trong gần đúng bậc nhất nó dẫn đến tenxơ mêtric Schwarzschild trong Thuyết Tương Đối Tổng Quát (GTR). Như vậy, 3 kiểm tra kinh điển của GTR trong hệ mặt trời cũng tìm lại được trong mô hình hấp dẫn véctơ này.*

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