ON THE APPROXIMATION OF BAYESIAN ESTIMATORS IN FUNCTIONAL SPACES

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ABSTRACT: *In this paper we investigate problem of finding the approximation of Bayesian estimators by the polinomial functions.*
Keywords: *Bayesian estimators, Bayesian risk function, location parameter, variance parameter, compound parameter.*

1. Introduction and Notations:

The article present some results on the Bayesian estimators in multidimensional nonlinear regression models.

First of all, we give some notations (see [1],[2],[3]):

$M(n\times q), M(p\times r), M(s\times s)$: spaces of all $n\times q$-matrices, $p\times r$-matrices, $s\times s$-matrices.

$M^{\geq}(s\times s)$: space of all non-negative definite $s\times s$-matrices

$\mathcal{B}(n\times q), \mathcal{B}(p\times r), \mathcal{B}(s\times s)$: Borel σ_ algebras in $M(n\times q), M(p\times r), M(s\times s)$

Θ : compact subset of $M(p\times r)$

Let us consider the following multidimensional nonliner regression models:

$$X = \varphi(\theta) + \varepsilon$$

Where,

X is a observed random variable, taking the values in $M(n\times q), EX = \varphi(\theta)$

ε is a random error variable, taking the values in $M(n\times q), E(\varepsilon) = 0$.

θ is an unknown location parameter, $\theta \in \Theta$

φ is a known non-linear function, $\varphi : \Theta \to M(n\times q)$

<u>Definition 1.1:</u> A Borel function $h : (M(n\times q), \mathcal{B}(n\times q)) \to ((M(p\times r), \mathcal{B}(p\times r))$ is called an estimator for the location parameter $\theta \in \Theta$.

Let $B := B(M(n\times q), M(p\times r))$ denote the space of all bounded Borel functions from $M(n\times q)$ to $M(p\times r)$. Clearly, it forms a class of estimators for the location parameter $\theta \in \Theta$ and it is a Banach space with the norm

$$\|h\|_B = \sup_{x\in M(n\times q)} |h(x)| \qquad \text{(see [1],[2])}$$

The composed function defined by L(h(.), .): $M(n\times q)\times\Theta \to \overline{R^+}$ is called a loss function (see [1],[2]).

<u>Definition 1.2:</u> A functional Ψ: B $\to$ $\overline{R^+}$ is said to be a Bayesian risk funtion with a priori distribution τ if

$$\Psi(h) = \int_{\Theta} \int_{M(n\times q)} L(h(x),\theta) f_\theta(x)\mu(dx)\tau(d\theta)$$

<u>Definition 1.3:</u> An estimator $\hat{h} \in$ B said to be a Bayesian estimator with a priori distribution τ if

$$\Psi(\hat{h}) = \inf_{h\in B} \Psi(h)$$

2. On the approximation for the location parameter and variance parameter:

First, in this section we will find an approximation polinomial function of Bayesian estimator for the location parameter $\theta \in \Theta$, where Θ is a compact subset of R. Let I be the range of the observed random variable X, where I be a compact subset of R. Let B(I) be the class of all bounded estimators of the location parameter $\theta \in \Theta$. Let C(I) be the space of all continuous functions on I. Clearly, B(I) and C(I) are Banach spaces and C(I) $\subset$ B(I).

Theorem 2.1: Let K be a class of all estimators of the location parameter $\theta \in \Theta$ satisfying the conditions of the theorem 2.1 in [1]. Suppose that there exists $C' > 0$ such that:

$$|f_\theta(x)| \le C', \forall x \in I, \forall \theta \in \Theta$$

Then $\overline{K} \subset$ B(I) and the Bayesian estimator $\hat{h} \in \overline{K}$ can be arbi trarily closely approximated by a polinomial function.

Proof: First, take any $h \in \overline{K}$ and any $\varepsilon > 0$. Then by the Lusin theorem, there are continuous function $g \in C(I)$ and $C'' > 0$ such that: $\mu\ \{x \in I : h(x) \neq g(x)\} < \frac{\varepsilon}{4.C.C'.C''}$, where, $|h(x)| < C''$, $|g(x)| < C''$., μ is the Lebesgue measure on R and C defined as in theorem 2.1 in [1].

Let us define

$$A = \{x \in I : h(x) \neq g(x)\}$$

Then we have,

$$|\Psi(h) - \Psi(g)| \le \int_\Theta \int_I |L(h(x),\theta) - L(g(x),\theta)|\ f_\theta(x)\mu(dx)\tau(d\theta)$$

$$\le \int_\Theta \int_I C|h(x) - g(x)|\ f_\theta(x)\mu(dx)\tau(d\vartheta)$$

$$\le \int_\Theta \int_A C|h(x) - g(x)|\ f_\theta(x)\mu(dx)\tau(d\theta) + \int_\Theta \int_{I-A} C|h(x) - g(x)|\ f_\theta(x)\mu(dx)\tau(d\theta)$$

$$= \int_\Theta \int_A C|h(x) - g(x)|\ f_\theta(x)\mu(dx)\tau(d\theta)$$

$$\le 2.C.C'.C''.\mu(A) \le \frac{\varepsilon}{2}$$

On the other hand, for $\varepsilon > 0$, using the Weierstrass approximation theorem, we obtain a polinomial function $P_{n,a} \in C(I)$ such that

$$\|g - P_{n,a}\|_{C(I)} < \frac{\varepsilon}{2.C.C'.\mu(I)}$$

Where , n = $n(\varepsilon)$, a = ($a_0, a_1, a_{ao}, \ldots, a_n$) $\in R^{n+1}$

This implies that

$$|\Psi(g) - \Psi(P_{n,a})| \le \int_\Theta \int_I C\|g - P_{n,a}\| \cdot f_\theta(x)\mu(dx)\tau(d\theta) < \frac{\varepsilon}{2}$$

Consequently,

$$|\Psi(h) - \Psi(P_{n,a})| < \varepsilon$$

Thus we obtain an approximation polinomial function $P_{n,a}$ with the coeficient $a \in R^{n+1}$ and the proof of the theorem is completed ■.

Next, note that for every $h \in \overline{K}$, there is a n = n(h, ε). By the definition of the function Ψ, for every $a \in R^{n+1}$, there exist an unique $\Psi(P_{n,a}) \in \overline{R}^+$. This means that, there exists a function of several variables F: $R^{n+1} \to \overline{R}^+$ defined by F(a) = $\Psi(P_{n,a})$.

Now, let us define,

$$A_{\varepsilon,h} = \{a \in R^{n+1} : |\Psi(h) - F(a)| < \varepsilon\}$$
$$A_\varepsilon = \bigcup_{h \in \overline{K}} A_{\varepsilon,h}$$

Continuously, consider an Bayesian estimator $\hat{h} \in \overline{K}$, i.e $\Psi(\hat{h}) = \inf_{h \in \overline{K}} \Psi(h)$

By the above consideration, there exists a polinomial function $P_{n,\hat{a}}$ with the $\hat{a} \in R^{n+1}$ such that

$$|F(\hat{a}) - \Psi(\hat{h})| < \varepsilon \qquad (1)$$

It follows that $\hat{a} \in A_\varepsilon$

Suppose that there exists an $a^* \in A_\varepsilon$, such that,

$$F(a^*) = \inf_{a \in A_\varepsilon} F(a)$$

Consequently,

$$F(a^*) - F(\hat{a}) \le 0 \qquad (2)$$

On the other hand, we have

$$F(\hat{a}) - \Psi(\hat{h}) < \varepsilon$$
$$\Psi(\hat{h}) - \Psi(h^*) < \varepsilon$$
$$\Psi(h^*) - F(a^*) \le \varepsilon$$

It follows that,

$$F(a^*) - F(\hat{a}) > -3.\varepsilon \qquad (3)$$

From (2) and (3), we obtain that,

$$|F(a^*) - F(\hat{a})| < 3.\varepsilon \qquad (4)$$

Finally, from (1) and (4), we have that

$$|F(a^*) - \Psi(\hat{h})| < 4.\varepsilon$$

This means that, there exists a polonomial function P_{n,a^*} such that

$$|\Psi(\hat{h}) - \Psi(P_{n,a^*})| < 4.\varepsilon$$

Consequently, P_{n,a^*} is the required polinomial function ■.

Example 2.1: Suppose that the conditional regular distribution $Q_\theta, \theta \in \Theta$ is an uniform distribution with the following density function:

$$f_\theta(x) = \frac{1}{\theta} . 1_{(0 \le x \le \theta)}$$

Where, $\Theta = [1,2], x \in I = [0,2]$

Assume that, the priori distribution τ on compact parameter space $(\Theta, \mathcal{B}(\Theta))$ is an uniform distribution with the density function : $g(\theta) = 1_{(1 \le \theta \le 2)}$

Let us consider the loss function

$$L(h(x),\theta) = (h(x)-\theta)^2$$

Then we obtain the following Bayesian risk function with a priori distribution τ,

$$\Psi(P_{n,a}) = \int_{\Theta}\int L(P_{n,a},\theta).f_\theta(x)\mu(dx)\tau(d\theta)$$

$$= \int_{\Theta}\int_{I}(P_{n,a}(x)-\theta)^2.f_\theta(x)\mu(dx)\tau(d\theta)$$

$$= \int_1^2\int_0^\theta\left(\sum_{i=0}^n a_i.x^i-\theta\right)^2\frac{1}{\theta}.dxd\theta$$

$$= \sum_{i=0}^n\sum_{j=0}^n a_i a_j\frac{2^{i+j+1}-1}{(i+j+1)^2}+\frac{7}{3}\quad -2\sum_{i=0}^n a_i\frac{2^{i+2}-1}{(i+1)(i+2)}$$

Consequently, we obtain the following function F of several variables:

$$F(a) = \Psi(P_{n,a})$$

Now, considering the spesial case n = 1, we have

$$\text{F(a)} = F(a_0,a_1) = a_0^2+\left(\frac{7}{9}\right).a_{aù}^2+\left(\frac{3}{2}\right).a_0.a_1-3.a_0-\left(\frac{7}{3}\right).a_1+\frac{7}{3}$$

Clearly, $F(a_0,a_1)$ has a local minimum at a = (a_0^*,a_1^*), where $a_0^* = 1{,}645; a_1^* = -0{,}194$

Therefore, we have following the approximation polinomial function

$$P_1(x) = 1{,}645-0{,}194.x$$

If n =2, then $P_2(x) = 1{,}48-0{,}25.x+0{,}27.x^2$

And if n =3, then $P_3(x) = 1{,}47-0{,}177.x+0{,}142.x^2+0{,}055.x^3$

Next, by a similar argument as in the theorem 2.1, we obtain the followimg theorem on the approximation of Bayesian estimator for the variance parameter $\sigma^2 \in R^+$ (see [2]).

Theorem 2.2: Let K be a class of all estimators of the variance parameter $\sigma^2 \in R^+$, satisfying the conditions of the theorem 3.1 in [2]. Suppose that, there exists $C' > 0$ such that:

$$|f_\sigma(x)| < C', \forall x \in I, \forall\sigma^2 \in R^+$$

Then $\overline{K} \subset B(I)$ and the Bayesian estimator $\hat{h} \in \overline{K}$ can be arbitrarily closely approximated by a polinomial function.

Now, we consider the space of all essentially bounded Borel functions from I to R, denoting by $L^\infty(I) = L^\infty(\mu, I, R)$, where I is a compact subset of R. Clearly L^∞ (I) forms a class of the location parameter $\theta \in\in \Theta$, where Θ is a compact subset of R, and $L^\infty(I)$ is an Banach space with the norm:

$$|h|_\infty = \inf_{\mu(B)=0}\ \sup_{x\in I-B}|h(x)|$$

Theorem2.3: Let K be a class of the location parameter $\theta \in \Theta$ satisfying the conditions of theorem 2.2 in [2]. Suppose that there exists C' such that: $|f_\theta(x)| < C'(\text{mod}\,\mu), \forall\theta \in \Theta$

Then $\overline{K} \subset L^\infty(I)$ and the Bayesian estimator $\hat{h} \in \overline{K}$ can be arbitrarily closely approximated by a polinomial function.

Theorem 2.4: Let K be a class estimatorsm of the variance $\sigma^2 \in R^+$, satisfying the conditions of the conditions the theorem 3.2 in [2]. Suppose that there exists $C' > 0$ such that:

$$|f_\sigma(x)| < C'(\text{mod}\,\mu),\ \forall\sigma^2 \in R^+.$$

Then $\overline{K} \subset L^\infty(I)$ and the Bayesian estimator $\hat{h} \in \overline{K}$ can be arbitraeily closely approximated by a polinom function.

3. On the approximation for the compound parameter:

First, in this section we will consider the existence of Bayesian estimator for the compound parameter $\lambda = (\theta, \sigma), \theta \in \Theta, \sigma^2 \in M^{>}(s \times s)$, where Θ : is a compact subset in $M(p \times r)$ $M^{\geq}(s \times s)$ is a set of all non-negative definite $s \times s - matrices.$ in the space $M(s \times s)$ (see [2],[3]).

Definition 3.1: A Borel function
h: $(M(n \times q), \mathcal{B}(n \times q)) \rightarrow (M(p \times r) \times M(s \times s), \mathcal{B}(p \times r) \times \mathcal{B}(s \times s))$ is called an estimator for the compound parameter $\lambda = (\theta, \sigma^2)$ Let B =B $(M(n \times q), M(p \times r) \times M(s \times s))$ denote the space of all bounded Borel functions from M($n \times q)toM(p \times r) \times M(s \times s)$. Clearly B-space form a class of estimators of the compound parameter λ and it is a Banach space with the norm:

$$\|h\|_B = \sup_{x \in M(n \times q)} \|h(x)\|_{M(p \times r) \times M(s \times s)}$$

Definition 3.2: A functional $\Psi : B \rightarrow \overline{R}^+$, defined by

$$\Psi(h) = \int_{\Theta \times M^{\geq}(s \times s)} \int_{M(n \times q)} L(h(x), \lambda) f_\lambda(x) \mu(dx) \eta(d\lambda)$$

is called a Bayesian risk function with a priori distribution η, where $\eta = \tau \times \nu$ is the product of the measures τ and ν:

τ is a priori distribution of the location parameter θ

ν is a priori distribution of the variance parameter σ^2

Definition 3.3: An estimator $\hat{h} \in B - space$ is said to be a Bayesian estimator with a prior distrbution η if

$$\Psi(\hat{h}) = \inf_{h \in B} \Psi(h)$$

Theorem 3.1: Let K be a class of all estimators for the compound parameter λ satisfying the following conditions:

(i) $h(M(n \times q)) \subset \Theta \times M^{\geq}(s \times s), \forall h \in K$

(ii) $\forall \varepsilon > 0, \exists$ finite partition $\{E_i\}_{i=1}^m \subset M(n \times q)$ and points $x_i \in E_i$,I = 1,2,…,m such that:

$$\sup_{x \in E_i} \|h(x) - h(x_i)\| < \varepsilon, \forall h \in K, \forall i = 1,2,\ldots,m$$

(iii) There exists C > 0 such that:

$$|L(y,\lambda) - L(y',\lambda)| \leq C\|y - y'\|, \forall y, y' \in M(p \times r) \times M(s \times s), \forall \lambda \in \Theta \times M^{\geq}(s \times s).$$

Then $\overline{K}$ is a compact subset of B-space and in $\overline{K}$ there exists a Bayesian estimator.

Proof: By a similar argument of the theorem 2.1 in [1], it can be seen that K is a relatively compact subset of B-space.

Continuously, we will prove that $h(M(n \times q)) \subset \Theta \times M^{\geq}(s \times s), \forall h \in \overline{K}$

Indeed, take any $h \in \overline{K}$. Then, there exists a sequence $(h_n) \subset K$ such that

$$\|h_n - h\|_B \rightarrow 0, \text{ as } n \rightarrow 0$$

This implies that

$$\|h_n(x)-h(x)\|_{M(p\times r)\times M(s\times s)} \to 0, \text{ as } n\to\infty$$

Consequently: $\|h_n'(x)-h'(x)\|_{M(p\times r)} \to 0$ and $\|h_n''(x)-h''(x)\|_{M(s\times s)} \to 0$ as $n\to\infty.$, where $h=(h',h'')$.

On the other hand: $h_n(M(n\times q))\subset \Theta\times M^2(s\times s), \forall n\in N.$

This means that: $h_n'(x)\in\Theta$ and $h_n''(x)\in M^2(s\times s), \forall n\in N.$

It follows that

$$h(M(n\times q)\subset \Theta\times M^2(s\times s), \forall h\in \overline{K},$$

as to be shown.

Finally, let us consider the Bayesian risk function with the a priori distribution η:

$$\Psi(h)=\int_{\Theta\times M^2(s\times s)}\int_{M(n\times q)} L(h(x),\lambda)f_\lambda(x)\mu(dx)\eta(d\lambda)$$

Clearly, Ψ is a continuous function on $\overline{K}$ and in $\overline{K}$ there exists a Bayesian estimator. Theorem is proved■.

Now, we will investigate problem of finding an approach of Bayesian estimator for the compuond parameter.

Theorem 3.2: Let K be a class of all estimators for the compound parameter $\lambda=(\theta,\sigma^2)$, satisfying the conditions of the theorem 3.1, where Θ is a compact subset of R and $\sigma^2\in\overline{R}^+=[0,\infty)$. Suppose that, there exists $C'>0$ such that: $|f_\lambda(x)|\le C', \forall\lambda\in\Theta\times\overline{R}^+, \forall x\in I$, where I is a compact subset of R.

Then $\overline{K}\subset B(I,R^2)$ and the Bayesian estimator $\hat{h}\in\overline{K}$ can be arbitrarily closely approximated by a polinomial function.

Proof: First, take any $h\in\overline{K}$. Then there exist $C''>0$ such that: $|h(x)|\le C'', \forall x\in I.$, $h(x)=(h'(x),h''(x)), where, h'\in B(I,R), h''\in B(I,R)$. Since h',h'' are bounded Borel functions, by the $Lu\sin's$ theorem there are continuous functions $g',g''\in C(I)$ such that

$$\mu\{x\in I: h'(x)\ne g'(x)\} < \frac{\varepsilon}{8.C.C'.C''}$$

$$\mu\{x\in I: h''(x)\ne g''(x)\} < \frac{\varepsilon}{8.C.C'.C''}$$

Let us define $A'=\{x\in I: h'(x)\ne g'(x)\}, A''=\{x\in I: h''(x)\ne g''(x)\}$, $A=\{x\in I: h(x)\ne g(x)\}$ and note that $A=A'\cup A''$

This implies that: $\mu(A)\le\mu(A')+\mu(A'')<\dfrac{\varepsilon}{4.C.C'.C''}$

Consequently:

$$|\Psi(h)-\Psi(g)|\le \int_{\Theta\times R^+}\int_I |L(h(x),\lambda)-L(g(x),\lambda)|f_\lambda(x)\mu(dx)\eta(d\lambda)$$

$$\le \int_{\Theta\times R^+}\int_I C.\|h(x)-g(x)\|f_\lambda(x)\mu(dx)\eta(d\lambda)$$

$$\le \int_{\Theta\times R^+}\int_A C.\|h(x)-g(x)\|f_\lambda(x)\mu(dx)\eta(d\lambda)+\int_{\Theta\times R^+}\int_{I-A} C\|h(x)-g(x)\|f_\lambda(x)\mu(dx)\eta(d\lambda) < \frac{\varepsilon}{2}$$

On the other hand, since $g', g'' \in C(I)$, by the Weaerstrass polinomial approximation theorem, there are polinomial functions $P_{n,a}, P_{n,b} \in C(I)$ such that:

$$\|g' - P_{n,a}\|_{C(I)} < \frac{\varepsilon}{4.C.C'.\mu(I)}$$

$$\|g'' - P_{n,b}\|_{C(I)} < \frac{\varepsilon}{4.C.C.\mu(I)}$$

Let us denote $P_{n,a,b} = (P_{n,a}, P_{n,b})$. Then, we obtain:

$$|\Psi(g) - \Psi(P_{n,a,b})| \leq \int_{\Theta\times R^+}\int_I C.\|g - P_{n,a,b}\|_{C(I)}.f_\lambda(x)\mu(dx)\eta(d\lambda)$$

$$\leq \int_{\Theta\times R^+}\int_I C.\|g' - P_{n,a}\|_{C(I)} f_\lambda(x)\mu(dx)\eta(d\lambda) + \int_{\Theta\times R^+}\int_I C\|g'' - P_{n,b}\|_{C(I)} f_\lambda(dx)\eta(d\lambda) < \frac{\varepsilon}{2}$$

It follows that: $|\Psi(h) - \Psi(P_{n,a,b})| < \varepsilon$ and the proof of the theorem is completed■.

Next, note that for every $h \in \overline{K}$, there is an $n = n(h,\varepsilon)$. By the definition of the functional Ψ, for every a $\in R^{n+1}$ and every b $\in R^{n+1}$, there exists an unique $\Psi(P_{n,a,b}) \in \overline{R}^+$. This mean that, there exists a function of several variables F : $R^{n+1} \times R^{n+1} \to \overline{R}^+$, derfined by F((a,b)) = $\Psi(P_{n,a,b})$

Now, let us define

$$A_{\varepsilon,h} = \{(a,b) \in R^{n+1} \times R^{n+1} : |\Psi(h) - F((a,b))| < \varepsilon\},$$

$$A_\varepsilon = \bigcup_{h\in\overline{K}} A_{\varepsilon,h}$$

Continuously, consider an Bayesian estimator $\hat{h} \in \overline{K}$, i.e.

$$\Psi(\hat{h}) = \inf_{h\in\overline{K}} \Psi(h)$$

By the above consideration, there exists a polinomial function $P_{n,\hat{a},\hat{b}} = (P_{n,\hat{a}}, P_{n,\hat{b}})$ with the $(\hat{a},\hat{b}) \in R^{n+1} \times R^{n+1}$ such that

$$|F((\hat{a},\hat{b})) - \Psi(\hat{h})| < \varepsilon \qquad (1)$$

It follows that $(\hat{a},\hat{b}) \in A_\varepsilon$

Suppose that, there exists an $(a^*,b^*) \in A_\varepsilon$ such that: $F((a^*,b^*)) = \inf_{(a,b)\in A_\varepsilon} F((a,b))$

Consequently

$$F((a^*,b^*)) - F((\hat{a},\hat{b})) < 0 \qquad (2)$$

On the other hand, we have,

$$F((\hat{a},\hat{b})) - \Psi(\hat{h}) < \varepsilon$$

$$\Psi(\hat{h}) - \Psi(h^*) < \varepsilon$$

$$\Psi(h^*) - F((a^*,b^*)) < \varepsilon$$

It follows that

$$F((a^*,b^*)) - F((\hat{a},\hat{b})) > -3.\varepsilon \qquad (3)$$

From (2) and (3), we obtain that,

$$\left|F((a^*,b^*)) - F((\hat{a},\hat{b}))\right| < 3.\varepsilon \quad (4)$$

Finally, from (1) and (4), we have

$$\left|F((a^*,b^*)) - \Psi(\hat{h})\right| < 4.\varepsilon$$

This means that, there exists a polinomial function P_{n,a^*,b^*} such that

$$\left|\Psi(\hat{h}) - \Psi(P_{n,a^*,b^*})\right| < 4.\varepsilon$$

Therefore, P_{n,a^*,b^*} is a required polinomial function ■.

Finally, we will consider the Bayesian estimators in the Banach space
$L^\infty := L^\infty(\mu, M(n\times q), M(p\times r)\times M(s\times s))$

Theorem 3.3: Let K be a class of all estimators of the compound parameter $\lambda = (\theta,\sigma^2)$ satisfying the following conditions:

(i) $h(M(n\times q)) \subset \Theta\times M^{\geq}(s\times s)(\text{mod}\,\mu), \forall h\in K$

(ii) $\forall\varepsilon>0, \exists$ finite partition $\{E_i\}_{i=1}^m \subset M(n\times q)$ and points $x_i\in E_i, i=1,2,...,m$

such that:

(a) $\exists C_1 : \|h(x_i)\| \leq C_1, \forall h\in K, \forall i=1,2,...,m$

(b) $\forall h\in K, \exists B\in\mathcal{B}(n\times q), \mu(B)=0$ such that

$$\sup_{x\in E_i\setminus B}\|h(x)-h(x_i)\|_{M(p\times r)\times M(s\times s)} < \varepsilon, \forall i=1,2,3,...,m$$

(iii) There exists C > 0 such that

$$|L(y,\lambda)-L(y',\lambda)| \leq C\|y-y'\|, \forall y,y'\in M(p\times r)\times M(s\times s), \forall\lambda\in\Theta\times M^{\geq}(s\times s)$$

Then K is a relatively compact subset of L^∞ and in the class $\overline{K}$ there exists a Bayesian estimator.

Now, let $L^\infty(I)$ be the class of all essentially bounded estimators of the compuond parameter $\lambda=(\theta,\sigma^2), \theta\in\Theta, \sigma^2\in R^+ = [0,\infty)$, where I is compact subset of R and Θ is a compact subset of R.

Theorem 3.4: Let K be a class of the compound parameter $\lambda=(\theta,\sigma^2)$ satisfying the conditions of the theorem 3.3. Suppose that there exists $C'>0$ such that

$$|f_\lambda(x)| \leq C'(\text{mod}\,\mu), \forall\lambda\in\Theta\times M^{\geq}(s\times s)$$

Then $\overline{K}\subset L^\infty(I,R^{\alpha})$ and the Bayesian estimator $\hat{h}\in\overline{K}$ can be arbitrarily closely approximated by a polinomial function.

4. Concluding remarks:

In this section we will summarize the results on the Bayesian estimators in multidimensional nonlinear regression models with compact parameter space.

In[1] we considered the existence of Bayesian estimators for the locantion parameter in the Banach spaces $B(R^n,R^p)$ and $L^\infty(\mu,R^n,R^p)$

In [2] we considered the existence of Bayesian estimators for the location parameter and variance parameter in the Banach spaces $B(M(n\times q),M(p\times r))$ and $L^\infty(\mu,M(n\times q),M(p\times r))$

In [3] we investigated Bayesian estimators for the locantion parameter , variance parameter and compound parameter in the Banach spaces

$L^1(\mu, M(n\times q), M(p\times r))$, $L^1(\mu, M(n\times q), M(s\times s))$, $L^1(\mu, M(n\times q), M(p\times r)\times M(s\times s))$

In this paper we investigated the approximation of Bayesian estimators for the location parameter, variance parameter and compound parameter in the Banach spaces $B(I, R^{\alpha})$ and $L^{\infty}(I, R^{\alpha})$.

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VỀ XẤP XỈ ƯỚC LƯỢNG BAYES TRONG CÁC KHÔNG GIAN HÀM

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TÓM TẮT: *Bài báo đưa ra các xấp xỉ cho ước lượng Bayes của tham ẩn định vị, tham ẩn phương sai và tham ẩn hỗn hợp trong mô hình hồi qui phi tuyến nhiều chiều với không gian tham compăc bằng kĩ thuật giải tích hàm.*

Từ khoá: *Ước lượng Bayes, hàm mạo hiểm Bayes, tham ẩn định vị, tham ẩn phương sai, tham ẩn hỗn hợp.*

REFERENCES

[1]. Ung Ngoc Quang, On the existence of Bayesian estimates in nonlinear statistical models with compact parameter space, Acta Math. Vietnamica, Vol.19, No.2 (1994), 149-160.

[2]. Ung Ngoc Quang, On the existence of Bayesian estimates in multidimensional nonlinear statistical models with compact parameter space, Vietnam Journal of Mathematics, Vol. 23, No.2 (1995), 229-240.

[3]. Ung Ngoc Quang, On the Bayesian estimators in multidimensional nonlinear regression models, Journal Science and Technology developmet, Vietnam National University - Ho Chi Minh City, Vol.4, No.7 (2001), 23-29.