APPLICATION OF THE NEW GENERATION WAVELET TRANSFORM IN IMAGE COMPRESSION

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ABSTRACT : Applications of the first generation of wavelets(such as wavelets B-97, Daubechies, B-spline...) in image compression and noisy filter have gained the good results comparing to those of the previous methods which are based on Fourier and DCT transforms [1],[2],[3]. Since 1996 until now, the mathematicians and the signal processing experts develop a new wavelet generation which is the so-called LIFTING transform. Because of the simple algorithm of Lifting, we can design the wavelets which adapt themselves to the signals with variable statistics models. Those signals are popular in real applications, especially such as signals of image or video.

In this paper we only study the efficiency of Lifting transforms in image compression, analyse and compare them to the results of the first generation wavelets which were introduced in the previous conferences [1],[2],[3]. Contrary to the first generation wavelets designed especially for image compression, the normal Lifting transforms are used in our experiments.

Index Terms: DCT (discrete cosine transform), DWT *(discrete wavelet transform),* JPEG *(Joint Photographic Expert Group), Lifting wavelet transforms.*

1. INTRODUCTION

Today, research in wavelet image coding continues to grow at a rapid pace. In 1996, Wim. Sweldens [5]&[6] suggested the Lifting transforms which were built from the prediction and update and didn't base on Fourier transform like the first generation wavelets. In our paper, DWT stands for the first generation wavelets, and Lifting for the second generation .

The main motivation behind the development of wavelets was the search for fast algorithms to compute compact representations of functions and data sets. So, what about is the ability of Lifting in image compression? There has been so far no comprehensive and comparative study of the performance of various bases of DWT and Lifting. So, it is difficult to find an optimum basis for compression. We were thus motivated to perform a comparative study of the bases of DWT and Lifting, in order to provide a general and evaluating view on the advantages and the drawbacks of different kinds of wavelets and of different generations..

The rest of paper is organized as follows: Section 2 reviews DWT and introduces the Lifting transform. Section 3 compares the basis functions of DWTs with Liftings and presents experimental results. Section 4 concludes the paper.

2. DWT AND LIFTING TRANSFORM :

2.1. DWT representations :

Wavelets are basis functions that satisfy certain mathematical requirements for using in representing data or other functions [4]. The main idea of using wavelets is to analyze data according to scale. Dilations and translations of the mother function Φ, or analyzing wavelet, define an orthogonal wavelet basis :

$$\Phi_{(s,\ell)}(x) = 2^{-(\frac{s}{2})}\Phi(2^{-s}x - \ell)$$

The variables s and ℓ are integers that scale and dilate the mother function Φ to generate wavelets. The scale index indicates the wavelet width, and the location index ℓ gives its position. Notice that the wavelet width are rescaled , or dilated by powers of 2, and translated by integers. The self-similarity of wavelet bases caused by the scales and dilations. Once we know about the mother function, we know everything about the wavelet bases.

To span our data domain at different resolutions, the analyzing wavelet is used in a scaling equation :

$$W(x) = \sum_{k=-1}^{N-2} (-1)^k c_{k+1} \Phi(2x + k)$$

where W(x) is the scaling function for the mother function Φ, and c_k are the wavelet coefficients.

The wavelet expansion means that the coefficients are used in a linear combination of the wavelet functions. One thing to remember is that the wavelet transforms do not have a single set of basis functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. In addition, there are lots of the types of wavelet bases, or wavelet families, such as Haar, Daubechies, Coiflets, Symlets ...

2.2. Lifting transform representations :

Lifting, a space-domain construction of biortho-gonal wavelets, consists of the iteration of the following three basic operations (Fig 1) [5],[6]:

- Split: Divide the original data into two disjoint subsets. For instance, we will split the original data set x[n] into:
  $x_e[n]$ = x[2n] (the even indexed points), and $x_o[n]$ = x[2n+1] (the odd indexed points).
- Predict: Generate the wavelet coefficients d[n] as the error in predicting $x_0[n]$ from $x_e[n]$ using prediction operator P : $d[n] = x_o[n] - P(x_e[n])$ (1)
- Update: Combine $x_e[n]$ and d[n] to obtain scaling coefficients c[n] that represent a coarse approximation to the original signal x[n]. This is accomplished by applying an update operator U to the wavelet coefficients and adding to $x_e[n]$:
  $c[n] = x_e[n] + U(d[n])$ (2)

These three steps form a Lifting stage (Fig 1) .

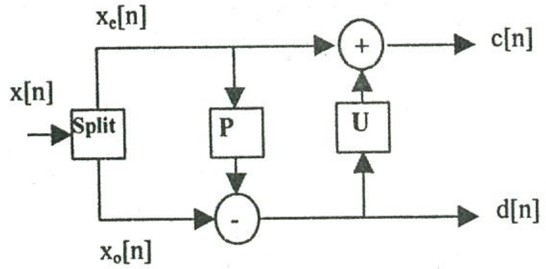

Fig 1: The Forward Lifting transform.

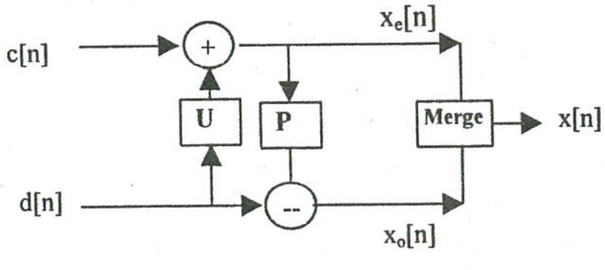

Fig 2: The Inverse Lifting transform.

Iteration of the Lifting stage on the output c[n] creates the complete set of scaling and wavelet coefficients $c^j[n]$ and $d^j[n]$. (Here, j is the levels of wavelet decomposition).

The Lifting steps are easily inverted as well. Rearranging (1) and (2), we have :

$$x_e[n] = c[n] - U(d[n]) \quad (3)$$

$$x_o[n] = d[n] + P(x_e[n]) \quad (4)$$

Then, we merge $x_e[n]$ and $x_o[n]$ to get the original data x[n]. These three rearranged steps form an

Inverse Lifting stage (Fig 2). Iteration of the Inverse Lifting stage on the sets of scaling and wavelet coefficients $c^j[n]$ and $d^j[n]$ will output $c^{j-1}[n]$. (Here, j is the levels of wavelet reconstruction). In the last stage, we will get the x[n].

3. Comparisions of DWT and Lifting in image compression :

We will compare them from 2 aspects: the mathematical characteristics of image compression and the direct results in real experiments.

3.1. Comparisions of the basis functions about the mathematical characteristics :

The different wavelet basis families make different trade-offs between how compactly the basis functions are localized in space and how smooth they are. Only some features of their bases are important in the image compression.

These features depend on the properties of the basis functions as follows .

a. Linear phase

The FIR filter banks of orthogonal wavelet bases are asymmetry [4]. This property translates into nonlinear phase in the associated FIR filters. This may cause artifacts at the borders of the wavelet subbands. These artifacts can be avoided if we use linear phase wavelet filter. If we want both symmetry (linear phase) and compact support(or perfect reconstruction) in wavelets, we are led to biorthogonal wavelets. The bases of Lifting transforms are only biorthogonal so far.

b. Vanishing moment

Wavelets are classified within a family most often used by the number of vanishing moments (zero moments). The importance of zero moments comes from the following fact. The wavelets with the vanishing moments of degree (k+1) can suppress a k-degree polynomial part of the signal [4]. This feature of wavelets is important in approximation of smooth functions. It also increases the coding gain.

c. Regularity

An orthogonal filter with a certain number of zeros at the aliasing frequency (π in the two channel case) is called regular, if its iteration tends to a continuous function [4]. The higher regularity can improve the coding gain and artifacts might be less visible. The importance of this property is potentially twofold when the decomposition is iterated in coding scheme such as the wavelet coder. When the decomposition is not iterated, regularity is of little concern.

d. Filter length

The length of wavelet bases are important in practice. From a view point of image compression, the higher the length of wavelet filter increases, the higher the degree of smoothness will get and the better the coding performance will become. However, if the length of wavelet filter increases, the reconstructed images will be contained more ringing artifacts and not worth the computational cost.

All the important properties of the bases almost depend on their filter length . We can consider that characteristic from Table 1.

e. Computational costs

One important consideration in application is the computational complexity or computational cost, which directly relates to its implementational efficiency in real time. The computational cost is governed by the number of multiplications and additions involved for calculating a wavelet coefficient, with the former being the dominating factor. We will use these operation counts as a measure of the computational cost of using a wavelet filter. In the context of wavelet decomposition(or reconstruction), these counts are generally directly proportional to the sum of the lengths of low-pass and high-pass filters. However, it is possible to reduce further the multiplication counts if we can exploit the symmetry of the filters.

A comparison of computational complexity between various popular DWTs and Liftings is provided by [6], in Table 2.

In the Table 2, please note that the fast wavelet algorithm are not applied to DWTs and Liftings. Liftings are deduced from DWTs by the factorization method of Daubechies and Sweldens [6].

From Table 2, Liftings are faster because DWTs require much larger operation counts of multiplications and additions.

Table 1 introduces the properties of DWTs, such as Haar, Daubechies, Symlets, B-spline Coiflets [8] and Liftings .

Table 1 : Comparision of features of wavelet mothers (N= 1,2,3,4...)

Wavelets	Vanishing moments	Regularity	Filter length	Filter characteristic
Haar	1	Not continuous	2	Symmetry
Daubechies (Db-N)	N (psi functions)	About 0,2N for large N	2N	Asymmetry
Sym-N (N=2,3...8)	N	----	2N	Near from symmetry
Coif-N (N= 1,2,...5)	2N (psi functions) 2N-1(phi functions)	----	6N	Near from symmetry
Bior-Nr.Nd (B-spline)	Nr – 1	Nr-1 , Nr-2 (psi rec. Functions)	Max(2Nr, 2Nd) + 2	Symmetry
Lift-Nr.Nd	Nr or Nd	-----	Nr or Nd	Symmetry

Table 2 : Comparision of computational cost between Lifting and DWT [6].

Wavelet type	DWT	Lifting	% improved speed
Haar	3	3	0%
D4	14	9	56%
D6	22	14	57%
B(9,7)	23	14	64%
B-Spline (4,2)	17	10	70%

Vanishing moments of Coiflets are the highest but their filters have long lengths and asymmetry phase. Biorthogonal wavelets- Bior Nr.Nd and Lift Nr.Nd-satisfy the requirement of image compression which are linear phase filter, high regularity, quite short filter length, high vanishing moments.The parameters in the Table 1 show that Liftings can be applied in image compression like DWTs.

We can consider the experimental schemes to evaluate which wavelets are better. Furthermore, these results will persuade us of the ability of image compression using Lifting transforms. Some experimental parameters to evaluate the performance of DWTs and Liftings will be introduced in the next part.

3.2. Comparisions from experiments :

We suggest two experimental schemes in Fig.3 and Fig.4. In Fig 3, the procedure follows the steps :

1- Dividing an image into 8x8 blocks.

2- Using 2D-DWT or 2D-Lifting on these 8x8 blocks. The decomposition levels are depend on the size of testing images. We choose 4 for the levels.

3- We calculate the variance and histogram for every subband.

4- Using IDWT or the inverse-Lifting to reconstruct the image.

The entropy, variance and histogram are the statistical specific characteristics of compression. If the variance is high, we need more bits to code data . So, the code gain will decrease. If lots of data found from histogram are zero, we can believe in the efficiency of coders behind the wavelet transforms.

In Fig 4, the procedure is similar to Fig 3, as followed :

1- Using 2D-DWT or 2D-Lifting on these 8x8 blocks.

2- Rounding the real wavelet coefficients.

3- Coding the integer wavelet coefficients with the Shapiro's EZW algorithm [8].

4- Calculating the following parameters :

* The number of POS, NEG, IZ, ZTR [8].
* The size of file after having coded by EZW.

5- Using IDWT or the inverse-Lifting to reconstruct the image.

6- Calculating the PSNR, MSE to evaluate the losses of information from rounding the real data.

The Shapiro's EZW algorithm exploits the zerotree structure of wavelet coefficients and attains the coding gain which is better than that of the zero run length coding. Therefore, the more ZTR and the less POS, NEG we get, the better coding gain will be attained. For this reason, we use EZW to indirectly evaluate the ability of the transforms.We have performed many sets of experiments using three different types of wavelets and the images, "Lena" 256x256 and 512x512. The three wavelet families are :

* Daubechies (Db2, Db3, Db4).
* B-spline (Bior 2.2, Bior 4.4, Bior 3.5) [8].
* Lifting (Lift 2.2, Lift 4.2, Lift 4.4).

Figs 5a,6a,7a show the histograms of the DWT and Lifting wavelet coefficients with the same filter length. Figs 5b,6b,7b are like Figs 5a,6a,7a but the vertical axes are limited to [-80,80]. Fig's 8 show the LL4 variances of the DWT and Lifting with the same filter length. The Fig's 9 introduces the LL4 variances of the Lifting with the different filter lengths. From Table 3, we can deduce the ability of generation of zero-tree structure data. Table 4a) indicates the loss of information by rounding the real wavelet coefficients. Table 4b) indicates the loss of information by rounding the coefficients of Lift(4,4) on some different types of images such as Lena 256x256, Wbarb 256x256 [9] and Sinsin128x128 [9] .

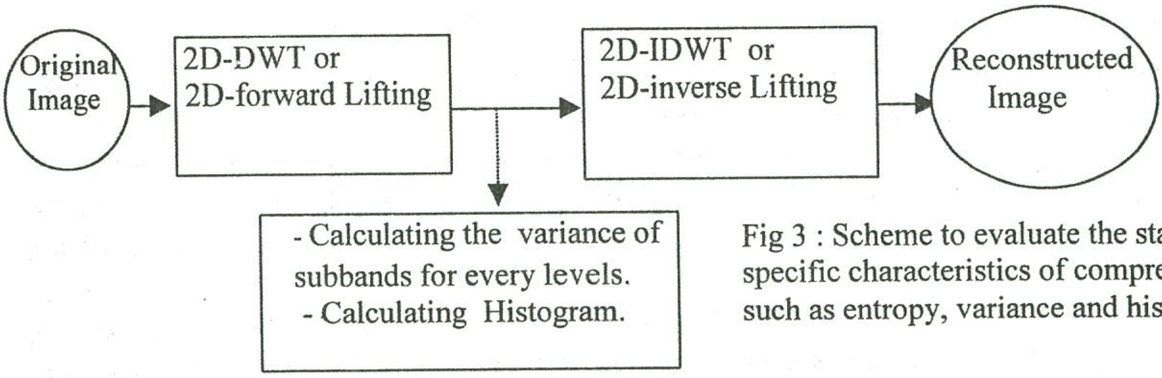

Fig 3 : Scheme to evaluate the statistical specific characteristics of compression such as entropy, variance and histogram

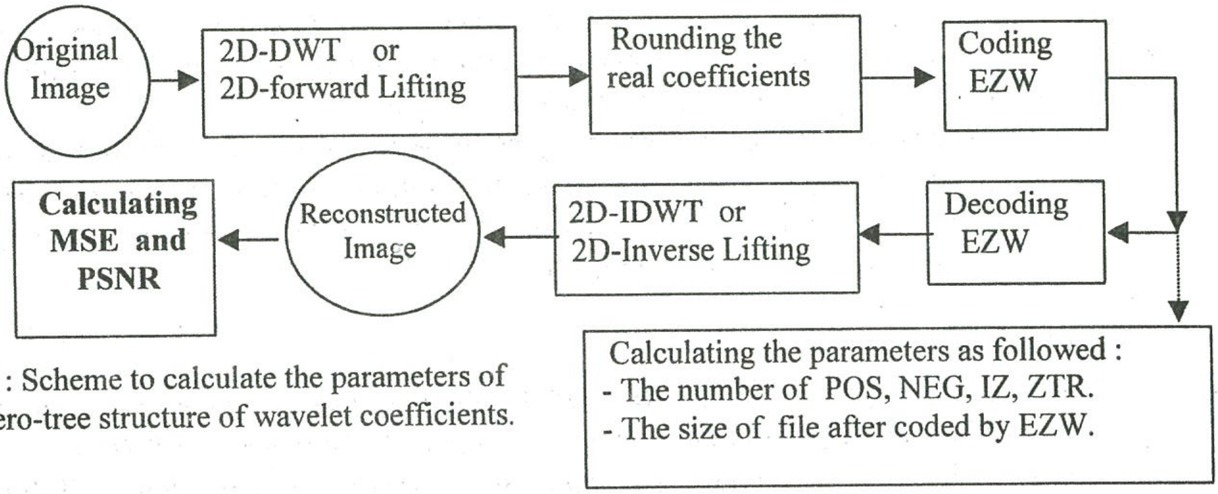

Fig 4 : Scheme to calculate the parameters of the zero-tree structure of wavelet coefficients.

Transforms	The number of POS	The number of NEG	The number of IZ	The number of ZTR
Lift(2,2)	24925	24459	35862	121730
Lift(4,2)	24042	24071	34631	119120
Lift(4,4)	23991	24111	34664	119306
Db2	26760	25324	36376	120760
Db3	26292	25782	36455	120671
Db4	26312	25860	36784	121832
Bior(2,2)	24458	24672	36144	122838
Bior(4,4)	25239	24681	34522	117094
Bior(3,5)	23231	22496	36618	128959

Table 3 : Evaluating the ability of exploitation of zero tree structure

Table 4 a) : Evaluating the losses of information from rounding the real data

Transforms	PSNR (dB)	MSE	The size of EZW file (bytes)
Lift(2,2)	57.660	0.098	58451
Lift(4,2)	57.740	0.096	56914
Lift(4,4)	57.882	0.093	56953
Db2	58.402	0.082	59471
Db3	58.398	0.083	59221
Db4	58.383	0.083	59656
Bior(2,2)	57.690	0.098	58899
Bior(4,4)	58.277	0.085	56859
Bior(3,5)	56.484	0.128	60185

Table 4b) : Evaluating the losses of information from rounding the real data on some types of images

	PSNR (dB)	MSE
Lena 256x256	57,882	0.093
Wbarb 256x256	55.774	0.098
Sinsin 128x128	50.772	0.071

3.3 Analyzing the results :

- The distribution of wavelet coefficients of Liftings is the same like DWTs (Figs 5,6,7). Lots of zeros concentrate around .
- The ordinate axis of the histograms. Moreover, the number of zeros of Lift(4,4) is more than that of Bior(2,2) and Db2. The LL4 variance (Fig 8) of Lift(4,4) is in the middle of Db2 and Bior(2,2). So, the compressional abilities of Lifting and DWT are nearly the same.
- When the filter lengths increase the coding gain gets more in DWT. Fig 9 proves the similar rule in Lifting transforms. However, if the lengths is too long the variance will increase. It makes go down the coding gain .
- From Table 3, the DWTs generate more ZTR, POS and NEG than Lifts but not much.
- Hence, we compare the size of EZW coded file (Table 4) to evaluate the ability of exploitation of zero tree structure for DWT and Lifting.
- The least MSE (<0.1)and high PSNR (>50dB) will prove that the procedure with rounding(Fig 4a) b)) is almost lossless. It is difficult to find even small loss in performance from the reconstructed images (Fig11a, Fig 11b) . The original images "Lena 256x256" are in Fig10a and Fig10b.

4. CONCLUSION

This paper compares Liftings and DWTs through lots of features. From this comparision, we believe that Liftings can attain the compression efficiency as DWTs.

Because the Lifting wavelet basis functions are constructed in space-domain, Liftings can adapt to the variable features in an image such as smooths or textures. So, Liftings are the good tools to build the adaptive transforms. It promises well a higher efficiency than that of DWTs.

The Lifting transform is a type of the fast wavelets. If we find an fast wavelet algorithm used for DWTs to conform to Liftings, their speed will be improved more. Then we can apply Liftings in video compression. That is our future work.

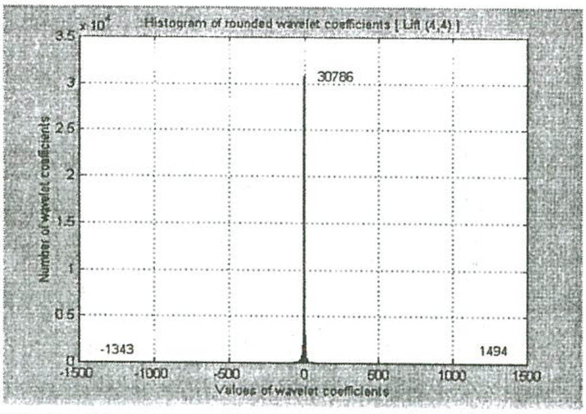

Fig 5a: Histogram of wavelet coefficients Lift(4,4)

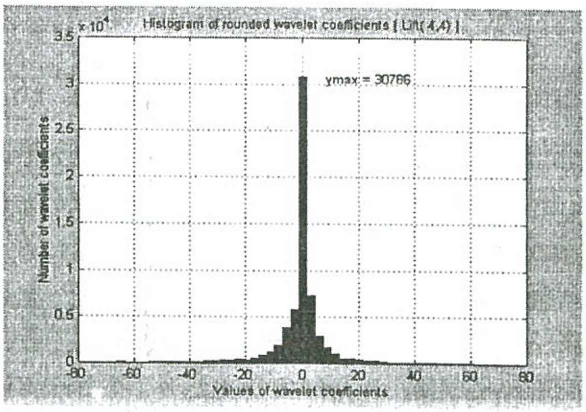

Fig 5b: Histogram of wavelet coefficients Lift(4,4)

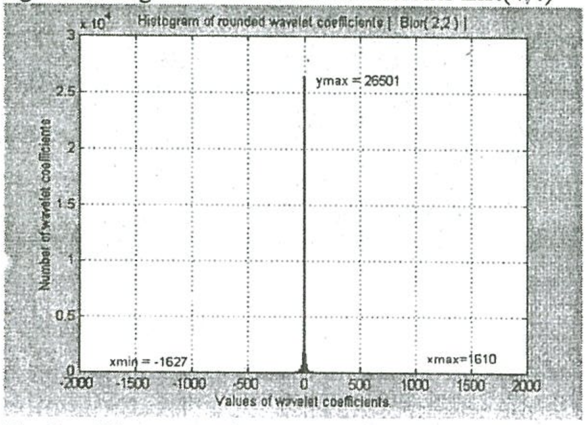

Fig 6a : Histogram of wavelet coefficients [Bior(2,2)]

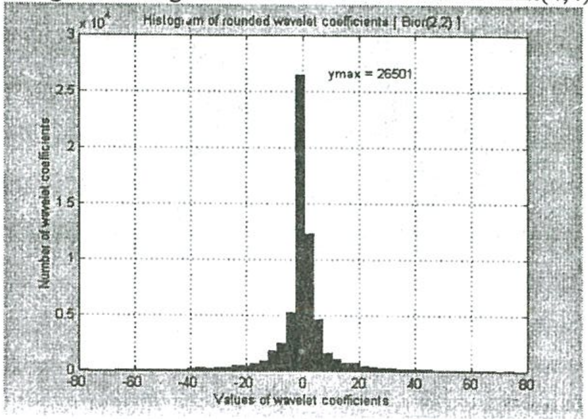

Fig 6b : Histogram of wavelet coefficients [Bior(2,2)]

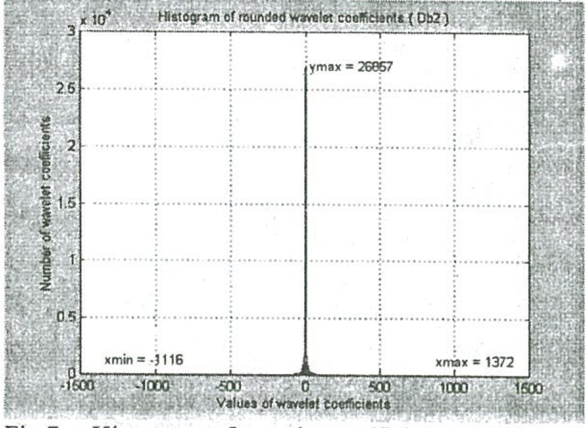

Fig 7a : Histogram of wavelet coefficients Db2

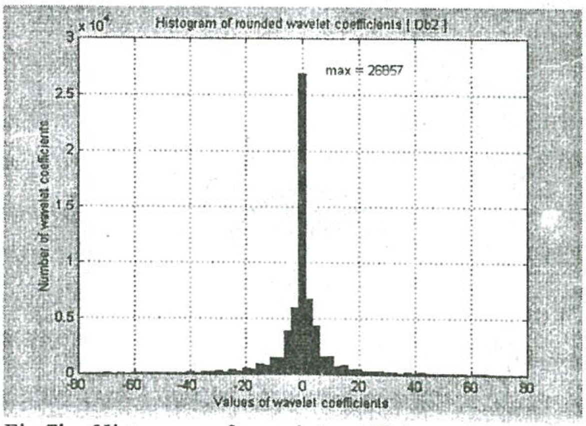

Fig 7b : Histogram of wavelet coefficients Db2

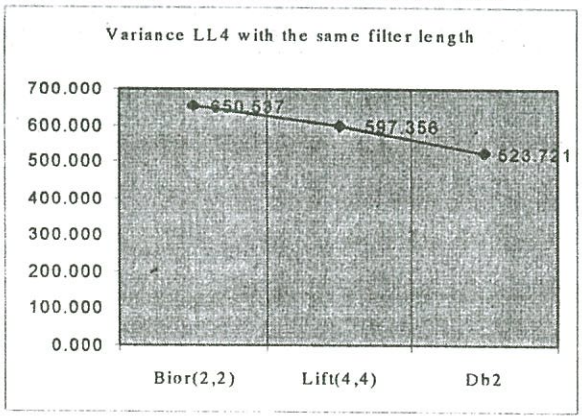

Fig 8 : Variance of subband LL4 of DWT and Lifting with the same filter length(= 4).

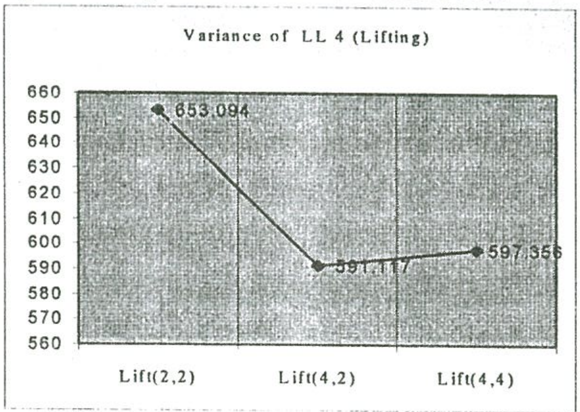

Fig 9 : Variance of subband LL4 of Lifting with the other filter lengths.

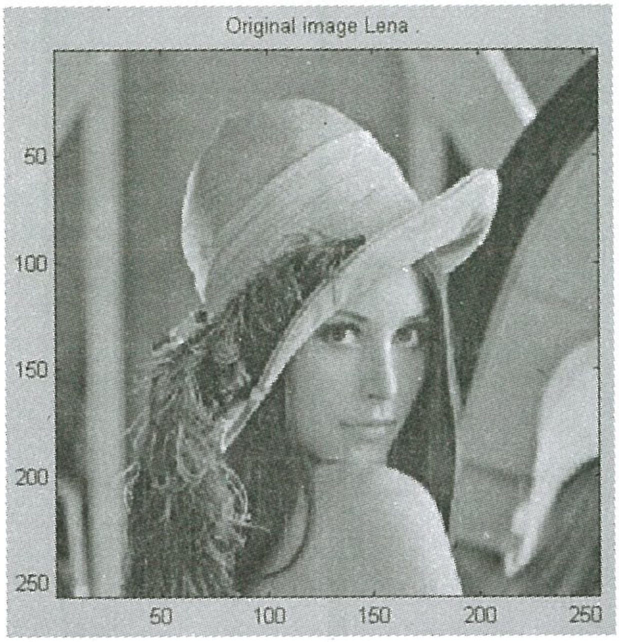

Fig 10a : Original image Lena 256x256

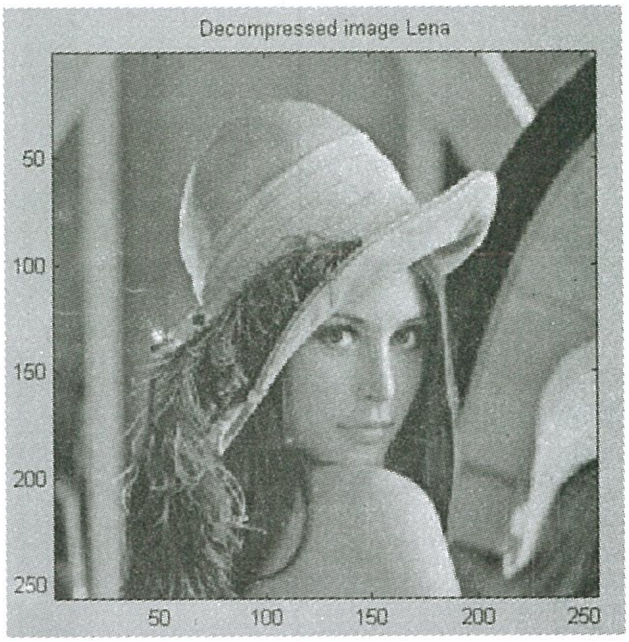

Fig 11a : Decompressed image with Lift(4,4)

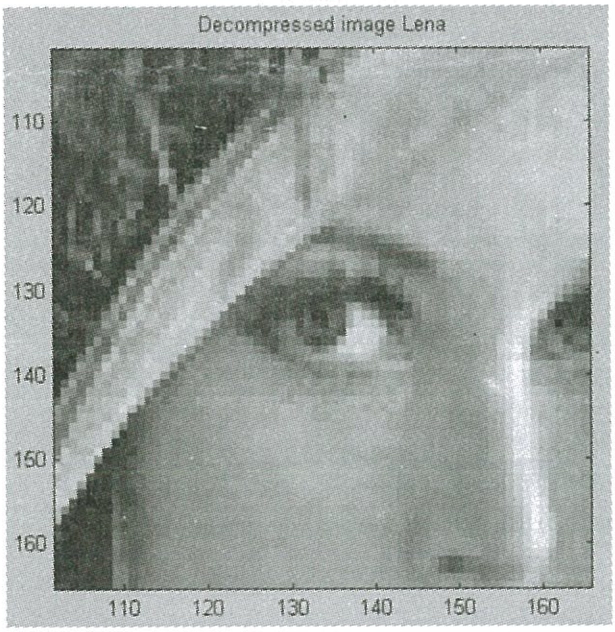

Fig 10b : Original image zoomed in 4 times

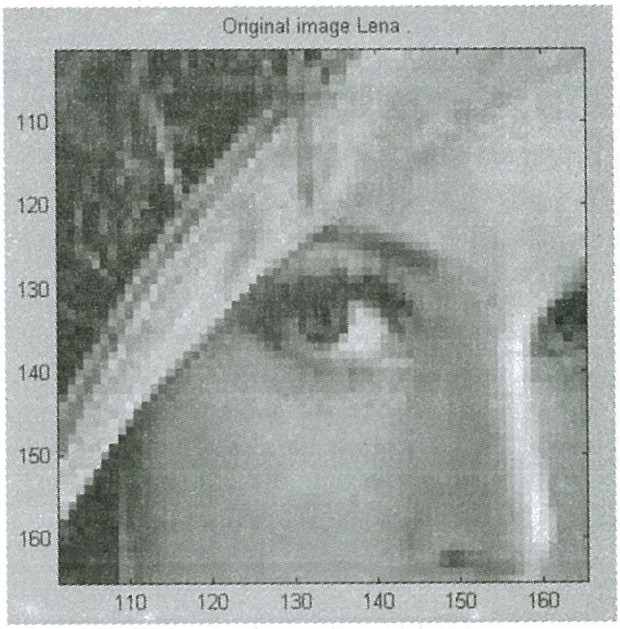

Fig 11b : Decompressed image zoomed in 4 times

ÁP DỤNG BIẾN ĐỔI WAVELET THẾ HỆ MỚI TRONG NÉN ẢNH

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TÓM TẮT: Ứng dụng thế hệ wavelet thứ nhất (như wavelet B97, các họ wavelet Daubechies, họ B-spline...) trong nén ảnh và khử nhiễu tín hiệu, đã cho kết qủa vượt trội so với những phương pháp trước đó(dựa trên biến đổi Fourier và DCT) [1], [2], [3]. Từ khoảng 1996 đến những năm gần đây, các nhà toán học và chuyên gia xử lý tín hiệu một và nhiều chiều, phát triển thế hệ wavelets mới, với tên gọi biến đổi Lifting. Nhờ cấu trúc đơn giản của biến đổi Lifting, người ta có thể thiết kế biến đổi wavelets thích nghi với tín hiệu có mô hình thống kê thay đổi, phù hợp với các áp dụng thực tế, đặc biệt là tín hiệu 2 chiều trở lên.

Bài viết này chỉ giới hạn nghiên cứu hiệu qủa của biến đổi Lifting trong nén ảnh, phân tích so sánh về lý thuyết và thực nghiệm với những kết quả ứng dụng wavelets thế hệ thứ nhất tại các hội nghị trước đây [1], [2], [3]. Để thể hiện rõ kết qủa phân tích lý thuyết, trong thực nghiệm trên ảnh thử chuẩn, chúng tôi sử dụng biến đổi Lifting thông thường (không được thiết kế dành cho nén ảnh) ; ngược lại những wavelet thế hệ thứ nhất được sử dụng là những loại thường được chọn cho nén ảnh.

REFERENCES

[1] Lê Quang Tuấn , Vũ Đình Thành, "Ứng dụng Wavelets trong nén ảnh và khử nhiễu tín hiệu", Tuyển tập của Hội Nghị Vô Tuyến Điện tử Việt Nam lần thứ VII, Hà Nội 12-13/12/1998, trang 219-228.

[2] Lê Quang Tuấn , Vũ Đình Thành, " Nâng cao SNR cho phương pháp khử nhiễu tín hiệu dùng Wavelets ", Tuyển tập của Hội Nghị Khoa Học và Công Nghệ Lần Thứ 7- Đại Học Bách Khoa Tp.HCM 22-23/4/1999, trang 27-36.

[3] Lê Quang Tuấn, Nguyễn Kim Sách, Vũ Đình Thành, "Wavelet Encoding Following the Tree-Like Data Structure of Wavelet Coefficients", Proceedings of 18th Conference of Asean Federation of Engineering Organizations (CAFEO 2000), Ha Noi 22-24 November 2000, PP.112-120.

[4] Martin Vetterli(University of California at Berkeley) and Jelena Kovacevic (AT&T Bell Laboratories),*"Wavelets and Sub-band Coding"*, Prentice Hall ,1995.

[5] W.Sweldens, *"The lifting scheme: A custom-design construction of biorthogonal wavelets"*, Appl.Comput.Harmon. Anal., vol.3, no.2, 1996, pp 186-200.

[6] W.Sweldens, *"The lifting scheme: A construction of second generation wavelets"*, SIAM J.Math. Anal., vol.29, no.2,1997, pp.511-546.

[7] Daubechies, I. and W.Sweldens, *" Factoring wavelet transforms into Lifting steps"*, J. Fourier Anal. Appl., vol 4,Nr.3,1998, preprint.

[8] Shapiro, J.M. *"Embedded Image Coding Using Zerotrees of Wavelet Coefficients"*, IEEE Transactions on Signal Processing, vol.41, no.12, December 1993, pp.3445- 3462.

[9] Michel Misiti, Yves Misiti , Georges Oppenheim, Jean-Michel Poggi, *" Matlab Toolbox User's Guide " version 5.3,* The Math Works Inc. , 1997.