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Research article

Dynamic Analysis of Axially Loaded Beams Partially Supported by an Elastic Foundation Under a Moving Harmonic Load

Bui Van Tuyen*

ABSTRACT

In this paper, a first-order shear deformable beam element is formulated and employed to study the dynamic behavior of beams partially resting on a Pasternak elastic foundation subjected to an axial force and a moving harmonic load. The beam element is derived by using the solution of differential equilibrium equations of a beam segment to interpolate the transverse displacement and rotation. The dynamic response of the beams is computed with the aid of the Newmark method. A parametric study is carried out to highlight the effects of the moving load parameters, the axial force amplitude, and the foundation support on the dynamic characteristics of the beams. The obtained result reveals that the axial force and the moving velocity have an important role in the dynamic behavior of the beams, and the influence of the axial force on the dynamic behavior of the beam is dependent on the excitation frequency.

Key words: Axially loaded beam, partial foundation support, moving harmonic load, dynamic response, finite element method

INTRODUCTION

It is well known that displacements and stresses of a structure subjected to moving loads are quite different from those obtained by a static analysis of the structure subjected to the same loads. The displacements and stresses of the structure in a dynamic analysis are dependent not only on the magnitude of external loads but also on the velocity and frequency of the loads.

The dynamic analysis of beams traversed by moving loads has an important role in railway and bridge engineering, and this topic has been investigated for many years. In [1], Timoshenko et al. employed the mode superposition method to obtain the dynamic deflection of a Bernoulli beam subjected to a moving harmonic force. A number of closed-form solutions based on Fourier and Laplace transform methods for beams under various types of moving loads are given in the well-known monograph of Fryba [2]. Thambiratnam and Zhuge [3] used a simple two-node Bernoulli beam element to compute the dynamic response of beams on a Winkler elastic foundation subjected to moving loads. The dynamic stiffness matrix was employed by Chen et al. [4] in studying the dynamic behavior of an infinite Timoshenko beam on a viscoelastic foundation to a moving harmonic load. Dugush and Eisenberger [5] employed modal and direct integration methods to evaluate the natural frequencies and mode shapes of Bernoulli-type beams excited by moving loads with variable velocity. Using the Fourier transform method, Kim [6] obtained the steady-state response to moving loads of axially loaded beams resting on a Winkler elastic foundation. Kocatürk and Şimşek [7] investigated the vibration of viscoelastic beams subjected to a moving harmonic force by approximating the displacements by polynomials.

The objective of the present paper is to investigate the dynamic behavior of beams partially resting on an elastic foundation under a moving harmonic load. The beams are initially loaded by an axial force and then subjected to a harmonic load moving with a constant velocity from the left end to the right end. The foundation is considered herein as the Pasternak foundation, which is represented by two parameters [8,9]. A first-order shear deformable beam element is derived and used in combination with the Newmark method to compute the dynamic response of the beams. To avoid shear locking and improve the efficiency of the beam element, the solution of the differential equilibrium equations of a beam segment is employed to interpolate the displacement field. The effect of the moving load parameters, the axial load and the foundation support on the dynamic characteristics are studied and highlighted. The influence of the deceleration and acceleration of the moving load is also examined and discussed.

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MATHEMATICAL MODEL

A simply supported beam partially resting on an elastic foundation, as shown in Figure 1, is considered. The beam is initially loaded by an axial force Q, and it is then subjected to a harmonic load $F = P\cos(\Omega t)$, moving from the left end to the right end. Denoting L, EI and GA are the length, bending and shear rigidities of the beam, respectively.

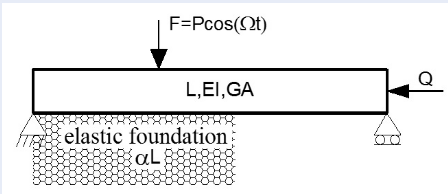

Figure 1: An axially loaded beam partially supported by an elastic foundation under a moving harmonic load.

Based on the first-order shear deformation theory, the axial and transverse displacements of a point in the beam are given by

$$\begin{cases} u_1(x,z,t) = -z\theta(x,t) \\ u_3(x,z,t) = w(x,t) \end{cases} \quad (1)$$

where $\theta(x,t)$ and $w(x,t)$ are the rotation of the cross section and transverse displacement of a point on the themed plane, respectively.

The axial and shear strains resulting from the displacement field in (1) are of the form

$$\begin{cases} \varepsilon_{xx} = -z\dfrac{\partial\theta}{\partial x} \\ y_{xz} = \left(\dfrac{\partial w}{\partial x} - \theta\right) \end{cases} \quad (2)$$

The constitutive equation based on the linear behavior for the beam material is as follows:

$$\sigma_{xx} = E\varepsilon_{xx},\ \tau_{xz} = G\gamma_{xz} \quad (3)$$

with E and G, respectively, the Young's modulus and shear modulus.

From Eqs. (2) and (3), one can write the strain energy of the beam in the form

$$U = \frac{1}{2}\int_0^L \left[EI\left(\frac{\partial\theta}{\partial x}\right)^2 + \overline{GA}\left(\frac{\partial w}{\partial x} - \theta\right)^2\right]dx \quad (4)$$

where $\overline{GA} = \psi GA$ with ψ is the correction factor.

The potential energy of the axial load Q is given by

$$V_Q = \frac{1}{2}\int_0^L Q\left(\frac{\partial w}{\partial x}\right)^2 dx \quad (5)$$

The Parsternak foundation type in which the interaction between the springs of the traditional Winkler foundation is adopted herein. The Pasternak foundation is represented by two parameters, k_W and k_G, and the stiffness of the Winkler springs and the shear layer, respectively, is adopted herein. In this regard, the strain energy stored in the foundation deformation is from [8]

$$U_F = \frac{1}{2}\int_0^{Lf}\left[k_W w^2 + k_G\left(\frac{\partial w}{\partial x}\right)^2\right]dx \quad (6)$$

where L_f is the foundation support length.

The potential energy due to the harmonic load F is given by

$$V_Q = \int_0^L P\cos\Omega t\,\delta(x - vt)\,dx \quad (7)$$

in which v is the velocity of the load F; $\delta(.)$ is the Diract delta function, and x is the abscissa measured from the current position of the load F to the left end of the beam.

The kinetic energy for the beam resulting from the displacements (1) is of the form

$$T = \frac{1}{2}\int_0^l \rho A\dot{w}^2 dx + \frac{1}{2}\int_0^l \rho I\dot{\theta}^2 dx \quad (8)$$

where ρ is the mass density, and $\dot{w} = \partial w/\partial t$, $\dot{\theta} = \partial\theta/\partial t$.

By applying Hamilton's principle to Eqs. (4)-(7), one can obtain the differential equations of motion for the beam. However, due to the nonuniform rigidities resulting from the partial foundation support, a closed-form solution for such equations is rarely obtained. The finite element formulation is derived in the next section to constract the discrete equation of motion and to compute the response of the beam.

FINITE ELEMENT FORMULATION

Consider a two-node uniform beam element with length l. The element is supported by the elastic foundation and stressed by the axial force Q At each node, the element has two degrees of freedom, namely, a lateral translation and a rotation about an axis normal to the plane of the paper. Thus, the vector of nodal displacements contains four components as

$$d = \left\{w_i \quad \theta_i \quad w_j \quad \theta_j\right\}^T \quad (9)$$

where the superscript 'T' refers to the transpose of a vector or a matrix. To derive the stiffness and mass matrix for the finite element analysis, we need to employ an interpolation scheme. Simple linear functions for both the lateral displacement w and rotation q can be adopted. However, the element formulated on such linear interpolation is slow convergence and suffers from the shear locking problem. As demonstrated by Luo in [10], a first-order shear deformation beam element derived in the context of the so-called field consistent approach possesses many advantages, including the high accuracy and lack of shear locking. In this regard, the present work looks for the interpolation functions by solving the following homogenous equilibrium equations of a beam segment

$$\begin{cases} EI\dfrac{\partial^2\theta}{\partial x^2}+\overline{GA}\left(\dfrac{\partial w}{\partial x}-\theta\right)=0 \\ \overline{GA}\left(\dfrac{\partial^2 w}{\partial x^2}-\dfrac{\partial\theta}{\partial x}\right)=0 \end{cases} \tag{10}$$

Introducing a dimensionless parameter

$$\lambda=\frac{1}{l^2}\frac{EI}{(GA)} \tag{11}$$

we can rewrite Eqs. (10) in the form

$$\begin{cases} EI\dfrac{\partial^2\theta}{\partial x^2}+\dfrac{1}{\lambda l^2}EI\left(\dfrac{\partial w}{\partial x}-\theta\right)=0 \\ \dfrac{1}{\lambda l^2}EI\left(\dfrac{\partial^2 w}{\partial x^2}-\dfrac{\partial\theta}{\partial x}\right)=0 \end{cases} \tag{12}$$

Using the command 'dsolve' in the symbolic software Maple [11], we can easily obtain the general solutions for the system of equations (4), which have the forms

$$\begin{cases} w(x)=\dfrac{1}{6}C_1x^3+\dfrac{1}{2}C_2x^2+C_3x+C_4 \\ \theta(x)=C_1\lambda l^2+\dfrac{1}{2}C_1x^2+C_2x+C_3 \end{cases} \tag{13}$$

where the constant $C_1,\ldots,C_4$ is determined from the end conditions of the element.

$$\begin{cases} w_{|x=0}=w_i;\ \theta_{|x=0}=\theta_i \\ w_{|x=l}=w_j;\ \theta_{|x=l}=\theta_j \end{cases} \tag{14}$$

Expressing the displacement and rotation in the forms

$$\begin{aligned} w(x)&=N_{w1}w_i+N_{w2}\theta_i+N_{w3}w_j+N_{w4}\theta_j \\ \theta(x)&=N_{\theta1}w_i+N_{\theta2}\theta_i+N_{\theta3}w_j+N_{\theta4}\theta_j \end{aligned} \tag{15}$$

or $w(x)=N_w d$ and $\theta(x)=N_\theta d$ with $N_w=\left\{N_{w1}\quad N_{w2}\quad N_{w3}\quad N_{w4}\right\}^T$, $N_\theta=\left\{N_{\theta1}\quad N_{\theta2}\quad N_{\theta3}\quad N_{\theta4}\right\}^T$ are thematrices of interpolation functions for w and θ, respectively. From Eqs. (12)-(15), we can obtain the expressions for N_{wi} and $N_{\theta i}$, (i = 1.4) as follows:

$$\begin{aligned} N_{w1}&=\tfrac{1}{1+12\lambda}\left(2\tfrac{x^3}{l^3}-3\tfrac{x^2}{l^2}-12\lambda\tfrac{x}{l}+1\right) \\ N_{w2}&=\tfrac{1}{1+12\lambda}\left[\tfrac{x^3}{l^2}-(2+6\lambda)\tfrac{x^2}{l}+(1+6\lambda)x\right] \\ N_{w3}&=\tfrac{1}{1+12\lambda}\left(-2\tfrac{x^3}{l^3}+3\tfrac{x^2}{l^2}+12\lambda\tfrac{x}{l}\right) \\ N_{w4}&=\tfrac{1}{1+12\lambda}\left[\tfrac{x^3}{l^2}-(1-6\lambda)\tfrac{x^2}{l}-6\lambda x\right] \end{aligned}$$

and

$$\begin{aligned} N_{\theta1}&=\tfrac{6}{1+12\lambda}\left(\tfrac{x^2}{l^3}-\tfrac{x}{l^2}\right) \\ N_{\theta2}&=\tfrac{1}{1+12\lambda}\left[3\tfrac{x^2}{l^2}-(4+3\lambda)\tfrac{x}{l}+(1+12\lambda)\right] \\ N_{\theta3}&=\tfrac{6}{1+12\lambda}\left(-\tfrac{x^2}{l^3}+\tfrac{x}{l^2}\right) \\ N_{\theta2}&=\tfrac{1}{1+12\lambda}\left[3\tfrac{x^2}{l^2}-2(1-6\lambda)\tfrac{x}{l}\right] \end{aligned}$$

It can be seen from the above two equations that in the limit, $\overline{GA}\to\infty$, the interpolation functions N_{wi} return to the Hermitial polynomials, which are often employed in developing the traditional Bernoulli beam element. In addition, in this case, the interpolation function $N_{\theta i}$ adds to the derivative of the functions N_{wi} with respect to x. The element formulated from the above interpolation functions is thus free from shear locking.

Having the interpolation functions derived, on can write the expression for the strain energy in Eq. (4) in the following matrix forms:

$$U=\frac{1}{2}\sum^{NE}d^Tkd \tag{16}$$

where NE is the total number of elements used to discretize the beam, and **k** is the element stiffness matrix with the following form:

$$\begin{aligned} k=&\textstyle\int_0^l[EIN^T_{\theta,x}N_{\theta,x} \\ &+\overline{GA}\left(\dfrac{\partial N_w}{\partial x}-N_\theta\right)^T\left(\dfrac{\partial N_w}{\partial x}-N_\theta\right)]dx \end{aligned} \tag{17}$$

The strain energy stored in the elastic foundation is now of the form

$$U_F=\frac{1}{2}\Sigma^{NE}d^Tk_Fd \tag{18}$$

where $\mathbf{k}_F$ is the element foundation stiffness with the following form:

$$k_F=\textstyle\int_0^l[k_wN_w^TN_w+k_G\left(\dfrac{\partial N_w}{\partial x}\right)^T\dfrac{\partial N_w}{\partial x}]dx \tag{19}$$

The potential energy of the axial force Q can also be written in the forms

$$V_Q=\frac{1}{2}\Sigma^{NE}d^Tk_Qd \tag{20}$$

with

$$k_Q=\textstyle\int_0^l QN^T_{w,x}N_{w,x}dx \tag{21}$$

Finally, the potential of the moving load F is of the form

$$V_P=\Sigma^{NE}d^Tf_{ex} \tag{22}$$

in which f_{ex} is the element nodal force vector with the following form:

$$f_{ex}=P\cos\omega t\left\{N_{w1}\quad N_{w2}\quad N_{w3}\quad N_{w4}\right\}^T \tag{23}$$

Finally, the kinetic energy of the beam in Eq. (8) can also be written in the form

$$T = \frac{1}{2}\sum^{NE} \dot{d}^{T} m\dot{d} \quad (24)$$

where **m** is the element mass matrix with the following form:

$$m = \int_0^l \rho A N_w^T N_w dx + \int_0^l \rho I N_\theta^T N_\theta d \quad (25)$$

where A is the cross-sectional area and I is the moment of inertia of the beam cross section.

GOVERNING EQUATIONS

Having the element stiffness matrices and nodal force vector derived, one can construct the discrete equation of motion for the beam in case of neglecting the damping effect as follows:

$$M\dot{D} + KD = F = P\cos(\Omega t) N \quad (26)$$

where M and K are the structural mass and stiffness matrices, respectively, obtained by assembling the element matrices m and ke = k + k_F + k_Q formulated in Section 3 in the standard way of the finite element method; D and $\dot{D}$ are the vectors of structural nodal displacements and accelerations, respectively; and is the vector of shape functions for the beam, which has the form

$$N\{0 \ldots N_{w1}\ N_{w2}\ N_{w3}\ N_{w4} \ldots 0\}^T$$

where N_{w1}, N_{w2}, N_{w3}, N_{w4} interpolation function for w, in which the abscissa x is measured from the left-hand node of the current loading element.

Eq. (26) is solved herein by the direct integration Newmark method using the average constant acceleration formula, which ensures an unconditional numerical stability [12].

NUMERICAL RESULTS

Using the formulated finite beam element and the stated numerical algorithm, a computer code was developed and used in the dynamic analysis. To investigate the dynamic response, the beam with the following geometry and material data is adopted herewith [7] L=20 m, I=0.08824 m^4, ρA=1000 kg/m, E=2.1x10^{11} N/m^2, and υ=0.3, where in addition to the previous notations, υ denotes Poisson's ratio. The amplitude of the moving load is taken as P=100kN.

Two types of boundary conditions, namely, simply supported (SS) and clamped at one end and simply supported at the other (CS), are considered. The effect of the partial support by the elastic foundation is examined by assuming that the beams to be supported on the part αL, with $0 \le \alpha \le 1$, from the left-hand end. For the convenience of discussion, α is named the supporting parameter below. A mesh of 20 equal elements and a correction factor $\psi = \frac{10(1+\nu)}{(12+11\nu)}$ are used in the analysis. To facilitate the discussion, the following dimensionless parameters are introduced for the foundation stiffness [13]

$$k_1 = \frac{L^4}{EI}k_W;\ k_2 = \frac{L^2}{\pi^2 EI}k_G \quad (27)$$

Additionally, the axial force is

$$r = \frac{Q}{Q_b} = \frac{L^2}{EI}Q \quad (28)$$

where Q_b is the buckling load of the SS beam without the foundation support.

The dimensionless parameter for the fundamental frequency is defined as

$$\mu = \left(\frac{\rho A L^4}{EI}\omega_1^2\right)^{1/4} \quad (29)$$

where ω_1 is the fundamental frequency.

Formulation verification

The derived formulation is first verified. To this end, Table 1 lists the frequency parameter of the SS beam with a slenderness ratio L/h=20 fully supported by the elastic foundation at various values of the compressive axial force and the foundation parameters, where the result obtained by Naidu and Rao in Ref. [13] is also given. It can be seen from the table that the present frequency parameters are in very good agreement with those of Ref. [13], regardless of the foundation stiffness and the axial force. The result of Ref. [13] was obtained by using an Euler-Bernoulli beam element. Note that the results in Table 1 were achieved by using 18 elements.

To verify the derived formulation in evaluating the dynamic response of the beam, Figure 1 shows the deflection under the loaded point of the simply supported beam under a moving load with a velocity v=15 m/s for two cases of the excitation frequency, namely, Ω=0 and Ω=40 rad/s. The result obtained by Timoshenko et al. in Ref. [1] using the mode superposition method is also displayed in the figure. As seen from the figure, the deflection curves obtained by the present finite element formulation are in good agreement with those of Ref. [1]. The result in Figure 2 was also obtained by 18 elements. More elements have been employed in computing the frequency and dynamic response of the beam, but no improvement has been seen. Due to this convergence result, 18 elements are used in all computations reported below.

Effect of axial force

The effect of the compressive axial force on the frequency and dynamic response is discussed in this subsection. To this end, Table 2 and Table 3 list the frequency parameters of the SS and CS beams partially supported by the elastic foundation for different values of the axial force parameter. As seen from the tables, the axial force parameter *r* has a significant influence on the frequency of both the SS and CS beams. The frequency parameter of both beams clearly decreases with increasing compressive axial force, regardless of the foundation stiffness. The tables also show the role of the foundation stiffness on the frequency parameters of the beams. As expected, the foundation stiffness has a positive influence on the frequency, and the frequency is increased by the increase in the foundation stiffness, regardless of the axial compressive force.

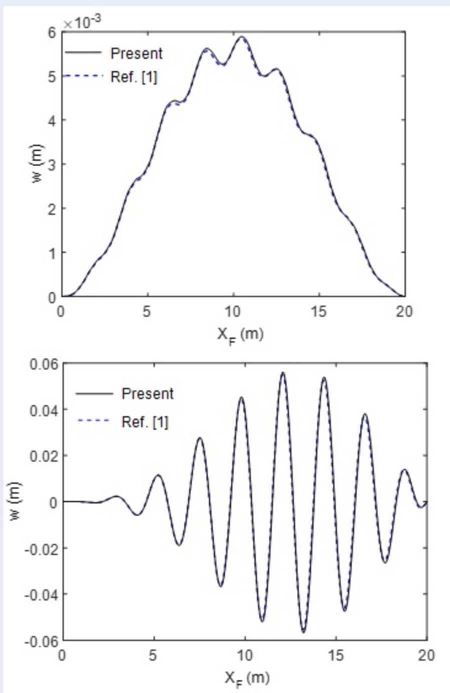

Figure 2: Deflection under a moving load of the beam without foundation support for v=15/s and (a) Ω=0, (b) Ω=40 rad/s

The effects of the axial compressive force on the dynamic response of the beams are shown in Figure 3 and Figure 4 for the SS and CC beams without the foundation support and Figure 5 and Figure 6 for the two beams with the foundation support. As seen from the figures, the axial compressive force has a significant influence on the deflections of the beams. The influence of the axial force, however, also depends on the excitation frequency of the moving load and the boundary condition. For the low excitation frequency, the deflection of the beam increases by increasing the axial compressive force (Figure 3a and Figure 5a). This tendency is not true for the beams under the moving load with the excitation frequency near the fundamental frequencies (Figure 3b, Figure 5b, Figure 6a) and even the opposite (Figure 6b). It is worth noting that the effect of the axial force on the dynamic response of the beams obtained in the present work is different from that reported by Kocatürk and Şimşek in Ref.[7], which concluded that this effect is very small and can be ignored. The reason for this difference may be that the largest axial force employed in[7] is too small, just less than of the buckling load, and the effect is hardly recognized.

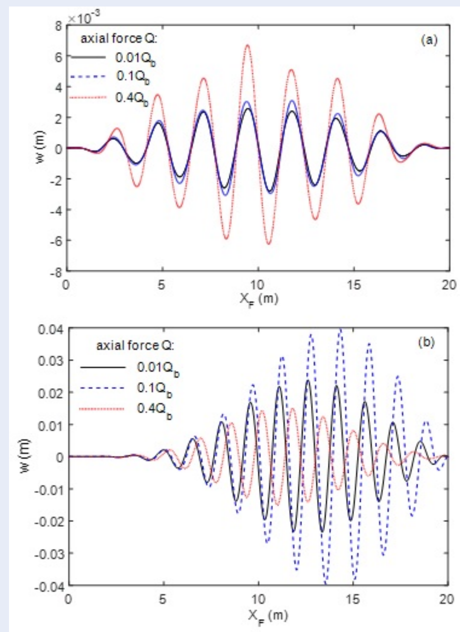

Figure 3: Effect of the compressive axial force on the dynamic response of the SS beam without foundation support for v = 15 m/s and (a) Ω=40rad/s and (b) Ω=60rad/s.

Effect of moving load velocity

The influence of the moving load velocity on the dynamic response of the SS beam partially supported by the elastic foundation with k_1=100 and k_2=1 is shown in Figure 7 for two values of the excitation frequency, Ω=40 rad/s and Ω=40 rad/s. In Figure 8, the influence of the moving load velocity on the CS beam resting on the same foundation stiffness is illustrated for

Table 1: Frequency Parameter Of Ss Beam Fully Supported By Elastic Foundation At Various Values Of Compressive Axial Force And Foundation Stiffness

$(k_1; k_2)$	r	μ	Ref. [18]	$(k_1; k_2)$	r	μ	Ref. [18]
(0,0)	0.0	3.1347	3.1415	(100,0.5)	0.0	3.9561	3.9608
	0.2	2.9646	2.9734		0.2	3.7415	3.7487
	0.4	2.7589	2.7705		0.4	3.4818	3.4928
	0.6	2.4930	2.5097		0.6	3.1462	3.1635
	0.8	2.0963	2.1257		0.8	2.6456	2.6782
(1,0)	0.0	3.1428	3.1496	(100,1)	0.0	4.1392	4.1437
	0.2	2.9723	2.9810		0.2	3.9146	3.9218
	0.4	2.7660	2.7776		0.4	3.6430	3.6541
	0.6	2.4994	2.5161		0.6	3.2918	3.3095
	0.8	2.1017	2.1312		0.8	2.7681	2.8014
(100,0)	0.0	3.7433	3.7483	(100,2.5)	0.0	4.5783	4.5824
	0.2	3.5402	3.5477		0.2	4.3299	4.3370
	0.4	3.2945	3.3055		0.4	4.0294	4.04.8
	0.6	2.9769	2.9940		0.6	3.6410	3.6594
	0.8	2.5033	2.5350		0.8	3.0617	3.0964

Table 2: Frequency Parameter Of Ss Beam Partially Supported By Elastic Foundation

$(k_1; k_2)$	r	α	μ	$(k_1; k_2)$	r	α	μ
(1,0)	0.4	0.2	2.7592	(100,1)	0.4	0.2	2.9845
		0.4	2.7623			0.4	3.1817
		0.6	2.7638			0.6	3.3545
		0.8	2.7669			0.8	3.5055
	0.8	0.2	2.0768		0.8	0.2	2.2684
		0.4	2.0980			0.4	2.4192
		0.6	2.1001			0.6	2.5511
		0.8	2.1014			0.8	2.6658
(100,0)	0.4	0.2	2.7924	(100,2.5)	0.4	0.2	3.1830
		0.4	2.9499			0.4	3.4430
		0.6	3.1537			0.6	3.5935
		0.8	3.2737			0.8	3.7696
	0.8	0.2	2.1218		0.8	0.2	2.4221
		0.4	2.2417			0.4	2.6241
		0.6	2.3967			0.6	2.7372
		0.8	2.4875			0.8	2.8745

Table 3: **Frequency Parameter Of Cs Beam Partially Supported By Elastic Foundation**

$(k_1; k_2)$	r	α	μ	$(k_1; k_2)$	r	α	μ
(1,0)	0.4	0.2	3.4470	(100,1)	0.4	0.2	3.4958
		0.4	3.4476			0.4	3.6386
		0.6	3.4491			0.6	3.7801
		0.8	3.4504			0.8	3.9136
	0.8	0.2	2.6274		0.8	0.2	2.6658
		0.4	2.6279			0.4	2.7793
		0.6	2.6291			0.6	2.8899
		0.8	2.6301			0.8	2.9958
(100,0)	0.4	0.2	3.4506	(100,2.5)	0.4	0.2	3.5539
		0.4	3.5061			0.4	3.8062
		0.6	3.6419			0.6	3.9600
		0.8	3.7528			0.8	4.1130
	0.8	0.2	2.6303		0.8	0.2	2.7116
		0.4	2.6741			0.4	2.9147
		0.6	2.7817			0.6	3.0320
		0.8	2.8683			0.8	3.1547

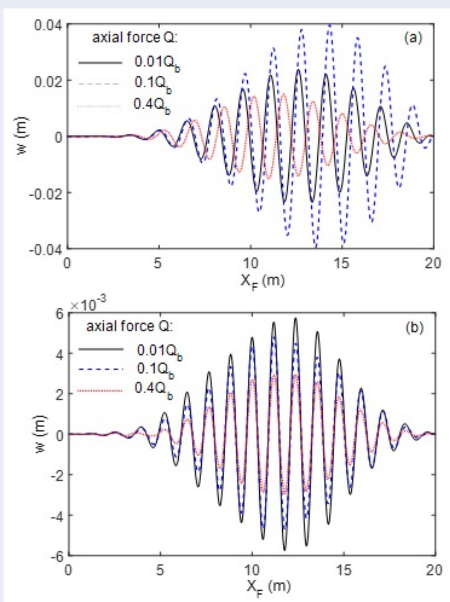

Figure 4: Effect of compressive axial force on dynamic response of CS beam without foundation support for v = 15 m/s and: (a) Ω=60rad/s, (b) Ω=80rad/s.

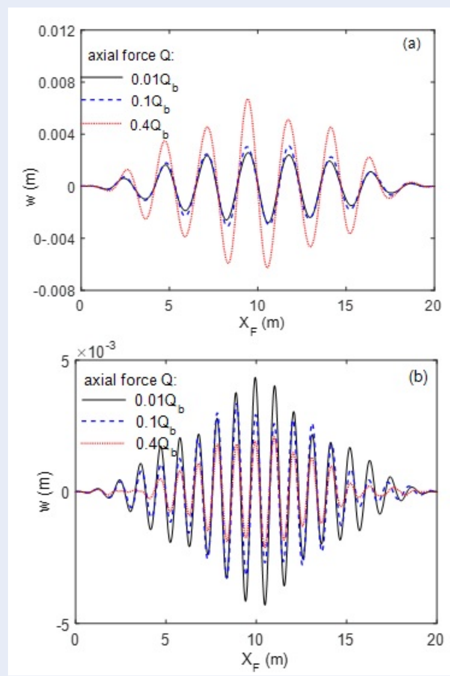

Figure 5: Effect of compressive axial force on dynamic response of SS beam on foundation for v = 15 m/s, (a) Ω=40rad/s, (b) Ω=90rad/s (k_1 = 100, k_2 = 1).

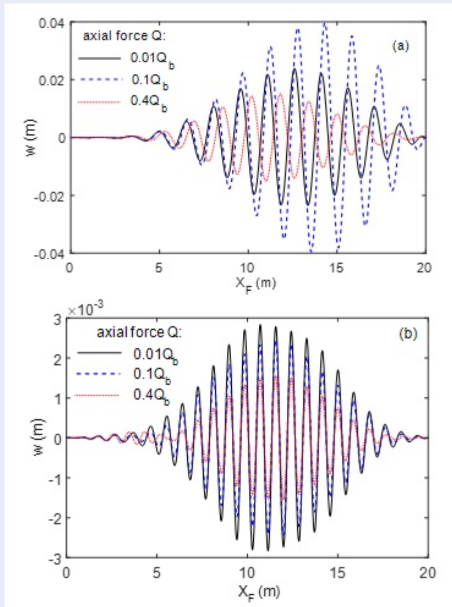

Figure 6: Effect of the compressive axial force on the dynamic response of the CS beam on the elastic foundation for v = 15 m/s, (a) Ω = 60 rad/s,(b)Ω = 110 rad/s (k_1 = 100,k_2 = 1)

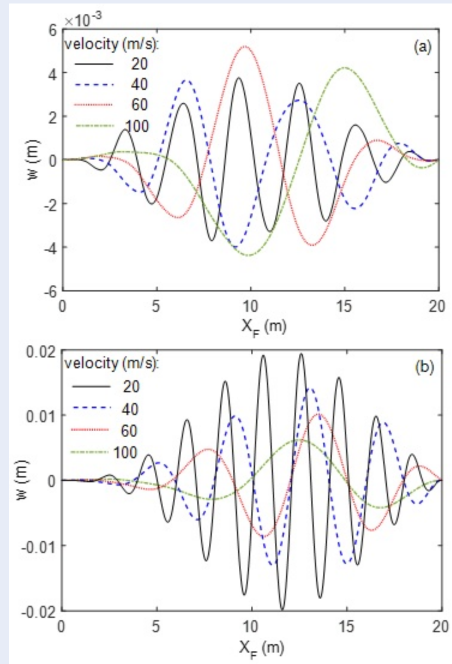

Figure 7: Effect of the moving velocity on the dynamic response of the SS beam on the foundation: (a) Ω = 40rad/s, (b) Ω = 60 rad/s (Q = 0.2Q_b, k_1 = 100, k_2 = 1)

Ω=40 rad/s and Ω=40 rad/s. Both the beams in Figure 7 and Figure 8 are loaded by an axial compressive force with an amplitude of 0.2 Q_b. Again, the effect of the moving velocity on the dynamic response is governed by the excitation frequencies.

For the excitation frequency well separated from the fundamental frequencies, the dynamic deflection of the beams first increases with an increment in the moving velocity and then decreases, regardless of the boundary conditions (Figure 7a and Figure 8a). In other words, at a given axial force and foundation stiffness, there is a critical velocity at which the dynamic deflection reaches a maximum value for the case of excitation frequencies far from the fundamental frequencies. In contrast, the deflections of the beams gradually decrease with increasing velocity when the excitation frequency is near the fundamental frequencies. The influence of partial support by the elastic foundation on the dynamic response is illustrated in Figure 8 and Figure 9 for the SS and Cs beams, respectively. The curves shown in the figures are obtained for excitation frequencies considerably below the fundamental frequencies of the beams. Only the amplitude of the dynamic deflection is affected by the partial support, and it is lowered at a higher supporting parameter , regardless of the boundary conditions. The computation has also been performed for other excitation frequencies, but the result is very similar to that of the abovementioned frequencies, and it is not shown herein.

Effect of the partial foundation support

The influence of the partial foundation support on the frequency of the SS and CS beams can be seen from Table 2 and Table 3, respectively. The tables show that the frequency parameter of both beams increases by increasing the foundation supporting parameter α, regardless of the foundation stiffness and the axial force.

The influence of the partial foundation support on the dynamic response is illustrated in Figure 9 for the SS beam on the foundation with k_1=100, k_2=1, subjected to a moving harmonic load with v=15 m/s and Ω=20 rad/s. As expected, the deflection of the beam decreases with the increase in the foundation supporting parameter α. This is reasonable since the beam-foundation system becomes harder when the beam is supported by a longer foundation.

CONCLUSION

The dynamic behavior of beams partially resting on a two-parameter elastic foundation under an axial

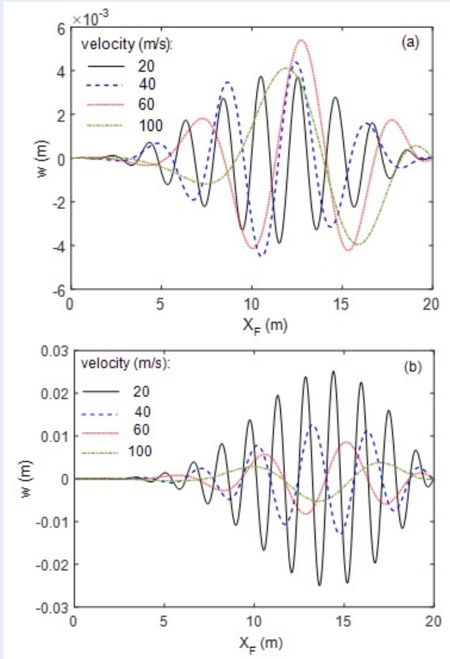

Figure 8: Effect of moving velocity on the dynamic response of the CS beam: (a) Ω = 60rad/s, (b) Ω = 80 rad/s (Q = 0.2Q_b, k_1 = 100, k_2 = 1)

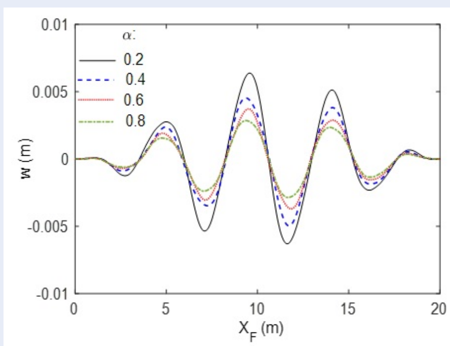

Figure 9: Effect of partial support by an elastic foundation on the dynamic response of the SS beam for v = 15 m/s, Ω = 20rad/s, Q = 0.2Q_b, k_1 = 100 and k_2 = 1.

force and a moving harmonic load has been studied. A first-order shear deformable beam element was formulated and employed in combination with the direct integration Newmark method to compute the dynamic response of the beams. The beam element was formulated by using the solution of the equilibrium equations of a beam segment to interpolate the displacement field. The vibration frequencies and the dynamic response were evaluated for the SS and CS beams under different foundation parameters and moving load parameters. The obtained numerical results reveal that the foundation supporting length, the axial compressive force and the excitation frequency play an important role in the dynamic behavior of the beams. It has been shown that the influence of the axial force on the dynamic behavior of the beams is dependent on the excitation frequency. A parametric study was carried out to highlight the effects of the foundation stiffness and moving load velocity on the dynamic behavior of the beams.

CONFLICT OF INTEREST

The author declares that he has no conflict of interest in publishing this paper.

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